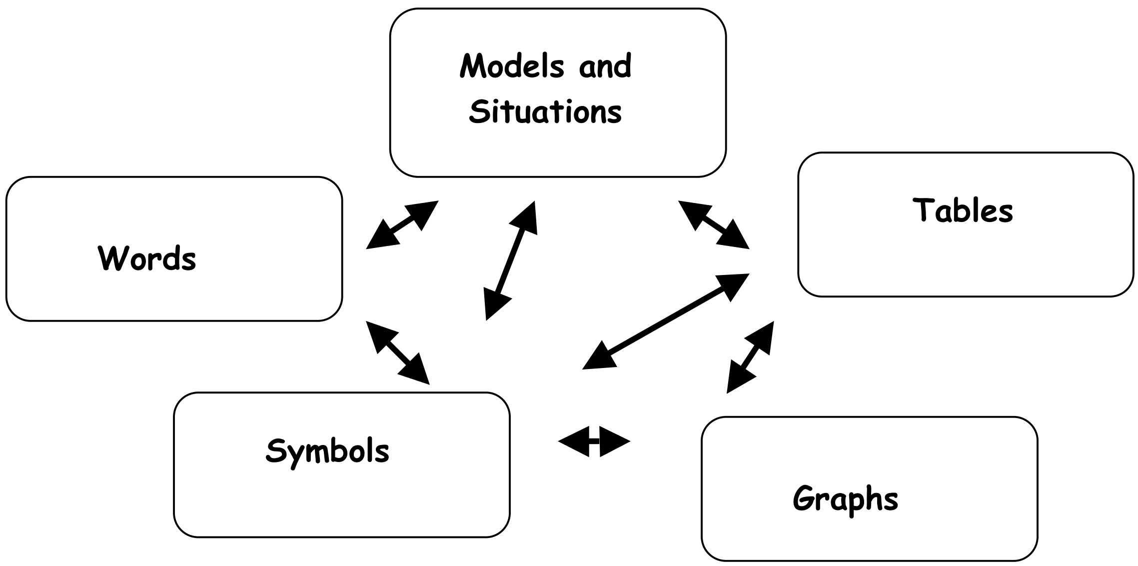
Developing Algebraic Thinking: OVERVIEW

A. Generalizing patterns across representations (one- and two- step)

This set of tasks falls in two categories. First, those that are proportional (equations look like $y = mx$). These patterns are easier than the second set. It is best to start with geometric or visual patterns, having students create tables, then look for patterns in the table and generalize what is happening in words and then in symbols. Second, are nonproportional situations. These look like $y = mx + b$ or $y = b + mx$ as equations. They are harder for students to generalize. They also are "two-step" equations, which are in the 6th grade assessment, so are the goal of this topic.

It is important to move flexibly between tables, models, equations, contexts, and graphs. This is critical foundational algebra, and the state assessment at 6th grade will often offer one representation and ask for another. If you emphasize all or most representations with each activity, you will maximize your time in addressing patterns and functions.

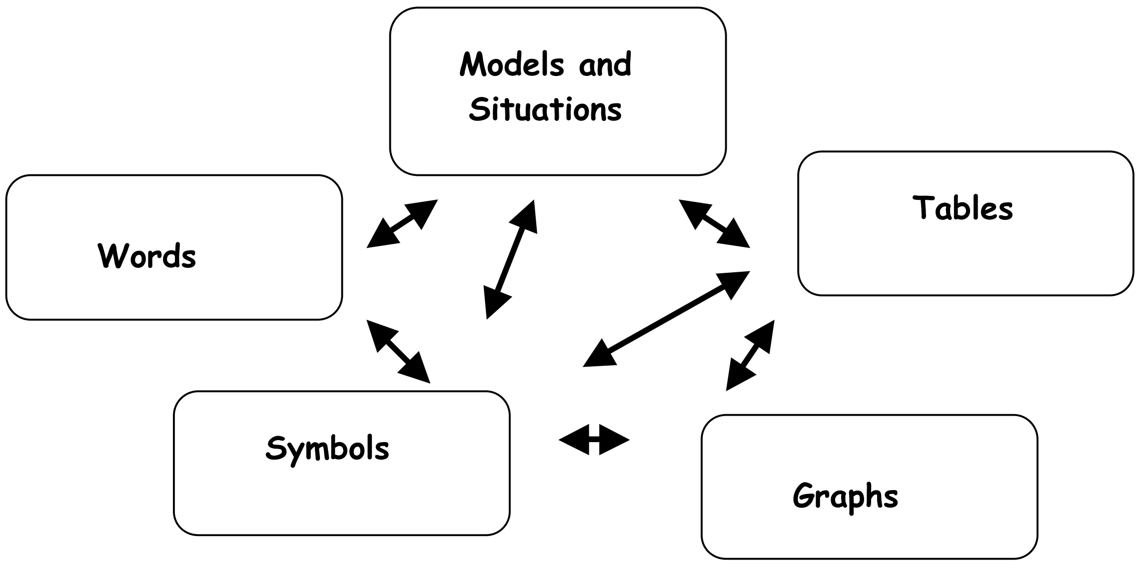


A number of excellent activities are included that work on proportional and nonproportional growing patterns, along with two lesson plan formats that could work for any one of these sets of activities (one plan for focusing on a single task, and another plan for doing stations).

B. Translating Words to Symbols

As stated above, moving among representations is very important. The hardest of these for many students is moving from words to symbols. This is also foundational and is emphasized in Kentucky at sixth grade. There are practices that should be avoided (focus on key words) and practices that can really make a difference. In this section, several resources are provided to help students work on translating words to symbols (and back again). They can be used as full lessons, as warm ups, as sponge activities. Research strongly supports the fact that a focus on reading comprehension can greatly improve student achievement and success in school.

Representations in Algebraic Thinking: Building Profound Understanding

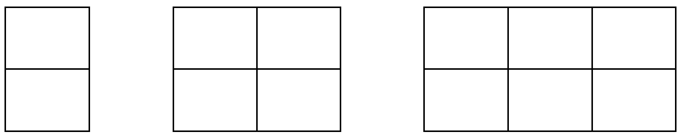


Algebra Thinking Pre-Unit Inventory

Try these problems!

Name:___________________________________ Date:____________________

What comes next?



1 2 3 4

For picture 4, the picture will have ___________________ squares.

For picture 10, the picture will have __________________ squares.

For picture 100, the picture will have _________________ squares.

2. Write as number sentences.

Example: Add 3 to 6, then multiply by 10: <u>(6 + 3) x 10</u>

Divide 45 by 9: __________________

Multiply 4 by 10, then add 35: _________________

12 is equal to 7 plus a number: _________________

3. Write equation and solve.

The sum of three numbers is 625. Two of the numbers are 80 and 184. What is the third number?

4. Solve

4 x (5 – 2) = _____

9 + 3 x 3 = _____

Two of Everything
Lesson 1

Materials:
Two of Everything by Lily Toy Hong
Two of Everything Recording Sheet
Pot or Bowl to model situation (optional)

Objectives:
Students will be able to create tables with input and output data.
Students will be able to determine a rule in words
Students will be able to write an expression for the rule (for a doubling pattern and a quadrupling pattern (2x and 4x)

Launch
Review key words in story (pot, hairpin, coin). Introduce the book, "Two of Everything" by Lily Toy Hong. Read the story to the class. Have students model the lesson. After reading the story, ask students, "What happens when something falls into the pot?"

Explore
Explain to students that they are going to be showing this pattern in a table, telling a rule (words) and in an expression (symbols). Have students share what rule and table mean in general and in math (save expression for later). Distribute the "Two of Everything" Student Page. Recount the story to aid in the start of the table. Ask them to create their own coin examples.

Place students in partners and ask them to complete 2 and 3 of the handout. Observe and ask questions using "rule". When students get the examples figured out, ask them to stop. Talk about what the word "expression" means in English (like "wow" or "very cool". Compare to what it means in math (no =, no verb). Ask students to write expressions – using symbols – to tell what the rule is.

Summarize
Ask students to share what the rule is, sharing different ways to say it in words and different ways to write it in symbols. For each, ask students to confirm if it is correct by checking (can use examples). Ask students to compare the two rules and two equations. How are they the same? How are they different?

Two of Everything

Magic Pot

*What happens to things that go **in** and **out** of the pot? What is the **rule**?*

1. Make a **table** to show what goes in the pot and what goes out.

	hairpins	*purses*	*coins*	*coats*	*Coins2*	*Coins3*	*Coins4*	*Coins C*
In	*1*	*1*	*5*					
Out								

2. Explain what would happen if…
- 47 Coins went in the Pot? ______________________________________
- 92 Coins went in the Pot? ______________________________________
- 1001 Coins went in the Pot? _____________________________________
- What is the rule for how many come out: ___________________________

3. What was put in if…
- 42 came out? ____
- 200 came out? ____
- 1,000,000 came out? ____
- 650 came out? ____
- What is the rule for how many go in: ______________________________

4. Now the magic pot instead quadruples (x4) what goes in. What happens if…
- 32 coins went in the Pot? _____________________________________
- 250 coins went in the Pot? ____________________________________
- 440 coins came out of the Pot?____________________________________

On the back, write (1) the <u>rule</u> in words or pictures of what happens with the magic pot – so that 5^{th} graders can understand and (2) write an <u>expression</u> to tell the rule.

Two of Everything

Lesson Day 2: What's Happening with the Pot?

Materials:
Two of Everything by Lily Toy Hong
Two of Everything Tables (p. 1 & 2 below)
Counters and bowls for students to model the problem
Pot or Bowl to model situation (optional)
Exit Slip

Objectives:
Students will be able to use a table to determine a rule in words and and expression in symbols for one-step functions involving whole numbers.
Students will be able to write an equation to describe a rule.

Launch
Review the meaning of "rule" "table" and "expression." Review the rest of Student Page #1 and have students share their thinking about the rules written as expressions. Be sure to focus on the different ways to write the expressions. Summarize by adding on the new word "equation." Explain that today the Magic Pot is doing some different things! In this lesson, you are going to study tables of what the Magic Pot did and decide what rule the Magic Pot is using – today, writing the rule (words), expression *and* equation.

Explore
Use the first table or two to model the process of completing the table and generalizing the pattern. Use the variables I and O to connect to the meaning of In the Pot and Out of the Pot (eventually to become input and output). Share with students the first partially completed table. Ask students to keep quiet, but to raise their hand if they know what the next "out" is. Then ask what the next is, and then the tenth. Ask students to talk to a partner and explain (1) what patterns they notice in the table and (2) what they think the Magic Pot rule is. Have students share. Then ask, "How could we write that using these symbols (squares and triangles)?" Record their ideas.

Be sure to ask students if all of these are correct and if they say the same thing. Ask students to work on the next table in partners. Again, focus on patterns and the rule. Students should notice that the amount added going down the table is connected to the amount the input is being multiplied by.

The second page has 2-step rules. Challenge students to try to solve these (all if time allows, otherwise just an extra for those who are wanting to try).

Summarize
Compare the situations that are similar symbolically and ask students to tell how they are different. (e.g., $x + 5$ and $5x$) Ask students to explain in words how the patterns in the table help

them to find the rule for the Magic Pot. Ask students if they know the rule, how can they find an input if they know the output. (you can use any example from the table).

Exit Slip – this is a partner exit slip that covers content from last two days. See below. It is to be done with a partner.

What is the Magic Pot's Rule?

Input	Output
1	7
2	14
3	21
4	
5	
10	
100	
Rule (in words)	
I	Expression

Input	Output
1	3
2	4
3	5
4	6
5	
10	
20	
Rule (in words)	
I	Expression

Input	Output
1	6
2	7
3	8
4	9
5	
Rule (in words)	
I	Expression

Input	Output
1	5
2	10
3	15
4	
10	
Rule (in words)	
I	Expression

Input	Output
5	2
6	3
7	4
8	5
20	
Rule (in words)	
I	Expression

Input	Output
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$
4	2
10	
Rule (in words)	
I	Expression

What is the magic pot's crazy rule?
CHALLENGE

Input	Output
1	8
2	16
3	24
4	
10	
Rule (in words)	
I	Expression

Input	Output
1	5
2	6
3	7
4	8
5	
⋮	
Rule (in words)	
I	Expression

Input	Output
1	3
2	5
3	7
4	9
5	
⋮	
Rule (in words)	
I	Expression

Input	Output
1	2
2	5
3	8
4	11
5	
⋮	
Rule (in words)	
I	Expression

Input	Output
1	1 ½
2	2
3	2 ½
4	3
5	
⋮	
20	
Rule (in words)	
I	Expression

Input	Output
1	
2	
3	
4	
⋮	
Rule (in words)	
I	Expression

EXIT SLIP	EXIT SLIP
• Write names • Fill in gray Output with your secret rule.	• Write names • Fill in gray Output with your secret rule.
PASS	PASS
• Complete the table. • Write the rule • Write the expression • Write the equation. • Sign at the bottom.	• Complete the table. • Write the rule • Write the expression • Write the equation. • Sign at the bottom.

Design your Own Names:	
Input	Output
1	
2	
3	
4	
10	
100	
Rule (in words)	
I	Expression

Design your Own Names:	
Input	Output
1	
2	
3	
4	
10	
100	
Rule (in words)	
I	Expression

Equation: ______________________________

Names: _________________________________

Equation: ______________________________

Names: _________________________________

Geometric Patterns

Lesson 1: Whole Class Lesson with One Pattern

Note: Several geometric patterns follow these two lessons – they are models that can be used, but also you can create your own. The ones here get a little more challenging with each one, but the manipulative selected can be used for easy or difficult patterns (for example, pattern block patterns can use two and three colors and get quite complex, but the one here is at the beginner level.

Materials

Manipulative used in pattern you select (color tiles, cubes, etc.)
Student Page and additional paper to show work
Centimeter grid paper or calculators (to add graphing representation) - optional

Objectives

Students will be able to create tables and equations of a geometric growing pattern in the form $y = mx$.
Students will be able to explain connections among the representations (tables, models, words, and equations)

Launch

Place a simple growing pattern on the overhead or sketch on the board (first three designs), using one of the manipulatives they will use. Ask if they think they can build the next design. Ask if they can tell how many pieces are needed for the 5th design and how they figured it out. Explain that today they will be looking at geometric growing patterns and figuring out how many pieces would be needed for any design in the sequence.

Explore

Distribute Student page (for example, the triangle pattern block pattern). Explain to students that they are going to be recording this pattern in a table, in words, and in equations. Ask students to work in partners or small groups, but to keep individual recording sheets. As students work, ask questions such as the following:

- What would the eighth design look like?
- What is changing/growing with each new design?
- Is it possible for a design to use ___ number of pieces?
- What rule are they thinking about? How did they find the rule? Can they show how the rule fits with the design? The table?

Summarize

Place a completed table on the overhead or board. Ask students to share patterns they see in the table. Ask students for rules they came up with. Record all possibilities. Discuss if they are equivalent and true. Ask how the equation fits with the model and with the table. Focus attention on the way that the change shows up in each representation (three triangles get added, the table goes up by 3, the equation is times 3).

Lesson 2: Stations

NOTE: This lesson can be done several ways:
1) Teacher creates pattern. Each station uses same manipulative, but patterns are different.
2) Teacher creates pattern. Each station uses different manipulative, and patterns are different.
3) Students create the pattern, then rotate to another groups pattern.

Materials:

Manipulatives for each station
Growing pattern, with at least three designs shown (use ones below or create your own)
Recording Sheet

Objectives:

Students will be able to extend and generalize patterns

Launch

Model a growing pattern on the overhead (e.g., one that grows by 4 each time). Ask, "Is this pattern growing in a constant way?" (yes – by four each time). Place a new pattern up that grows in a nonlinear way. Ask "Is this pattern growing in a constant way?" (no). Explain that today they will be going to four (five) stations. Each one has a pattern that grows in a constant way. They will work with their group to complete the student page (table, equation, and graph) for each pattern, then move to the next pattern.

Explore

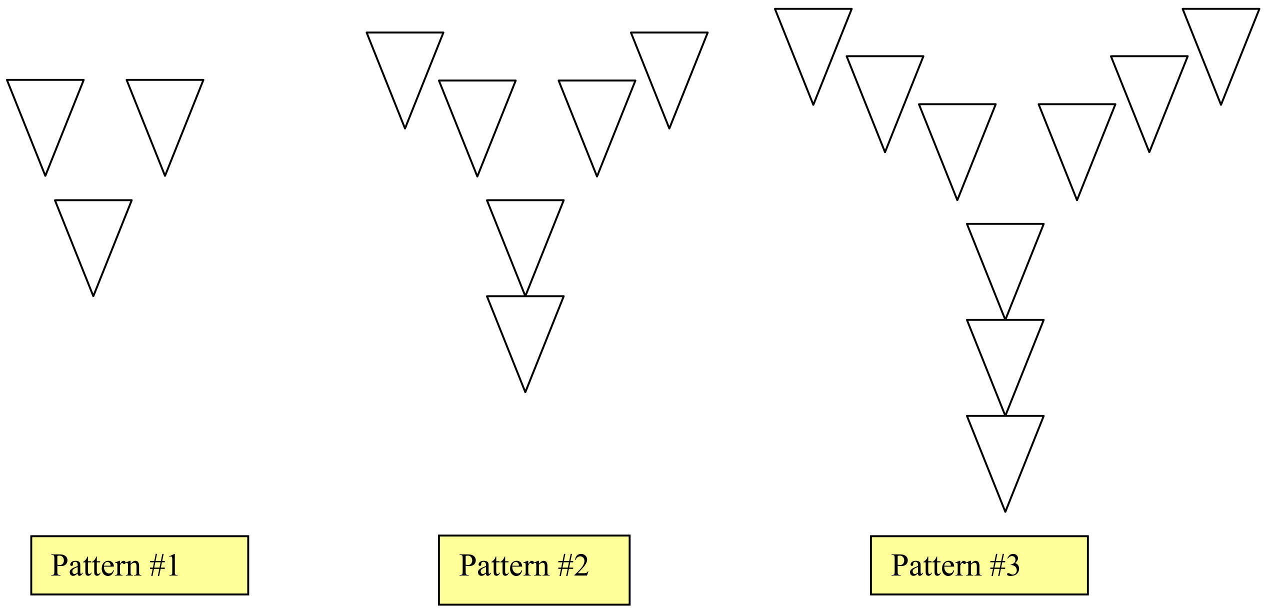
Set timer for about 12 minutes per station (more if more time is needed). As students work, ask questions such as the following:

- What would the eighth design look like?
- What is changing/growing with each new design?
- Is it possible for a design to use ____ number of pieces?
- What rule are they thinking about? How did they find the rule? Can they show how the rule fits with the design? The table?

Summarize

Ask students what helped them find the rule to the pattern. Ask, "Which ones were challenging and why?" Ask how they would help someone find an equation if they were looking at a table. Ask how they would help someone find an equation if they were looking at the pattern. Have students write their process on their paper.

Pattern Block Patterns



1. Make Pattern #4.

2. Complete the table.

Pattern	1	2	3	4	5	10	p
triangles							

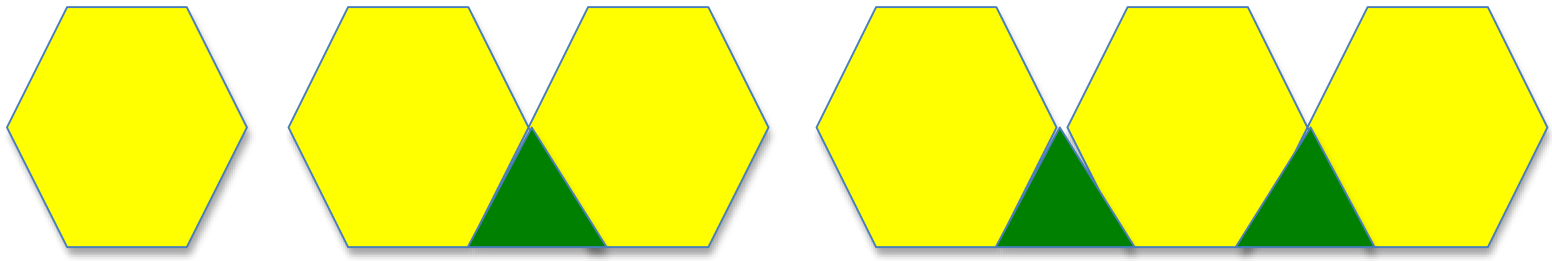
Expression

3. What is the **rule**? Write in a complete sentence.

4. What is the **equation** for finding the number of triangles (t): ___________________

5. Use the **equation** to answer these questions.
    a. How many triangles for pattern 20? ____________________
    b. How many triangles for pattern 45? ____________________
    c. 120 tiles are used for which pattern number? _____________________
    d. 312 tiles are used for which pattern number? _____________________

Pattern Block Patterns



Pattern #1　　Pattern #2　　Pattern #3

1. Make Pattern #4.

2. Complete the table.

Pattern	1	2	3	4	5	10	p
hexagons							
triangles							
total shapes							

Expression

3. What is the **rule**? Write in a complete sentence.

4. What is the **equation** for finding the number of:
   - hexagons (h): ____________________
   - triangles (t): ____________________
   - total shapes (s): ____________________

5. Use the **equation** to answer these questions.
   a. How many triangles for pattern 20? ____________________

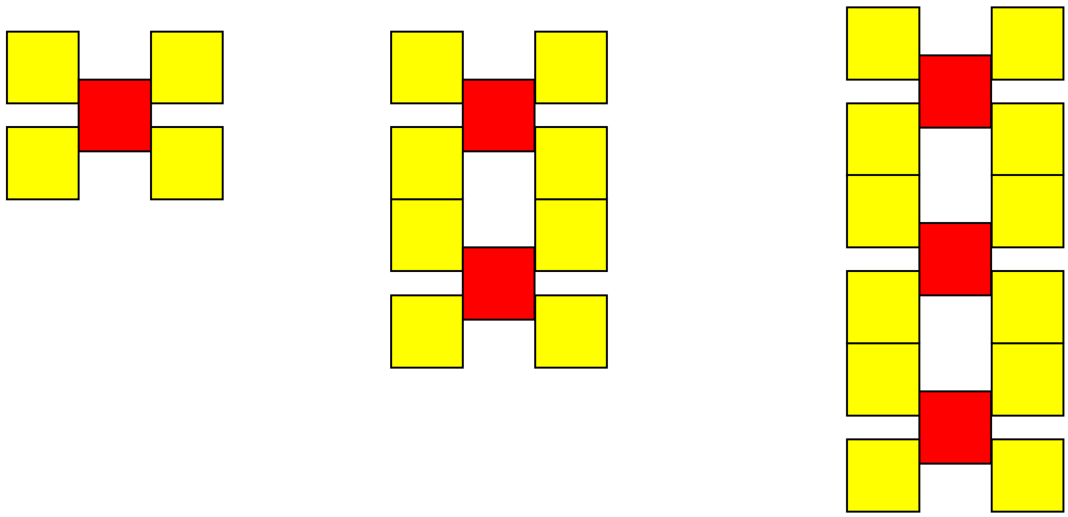
   b. How many total shapes for pattern 45? ____________________

   c. 41 tiles are used for which pattern number? ____________________

   d. 101 tiles are used for which pattern number? ____________________

Color Tile Patterns

Name: ___________________________



Pattern #1 Pattern #2 Pattern #3

1. Make Pattern #4.

2. Complete the table.

Pattern	1	2	3	4	5	10	p
light Squares							
dark Squares							
Total Squares							

Expression

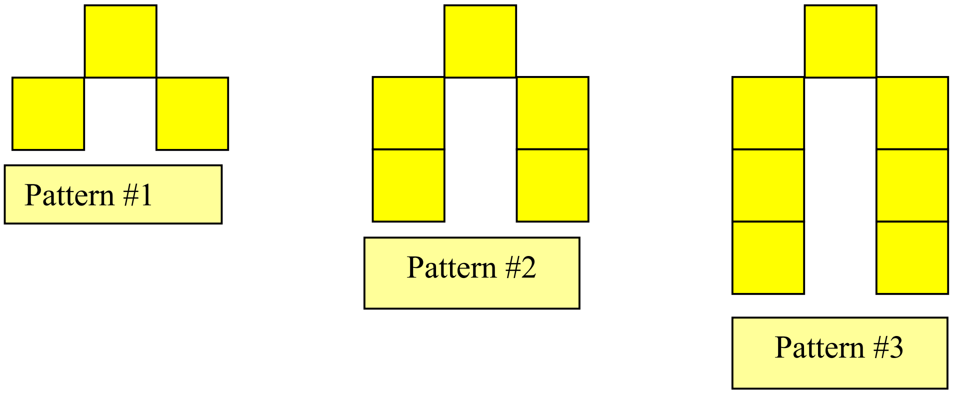
3. What is the **rule**? Write in a complete sentence.

4. What is the **equation** for finding the number of:
   - Light Squares (l): ___________________
   - Dark Squares (d): ___________________
   - Total Squares (t): ___________________

5. Use the **equation** to answer these questions.
   a. How many light squares for pattern 20?
   b. How many total squares for pattern 45?
   c. 125 tiles are used for which pattern number?

Color Tile Patterns

Name: ___________________________



1. Make Pattern #4.

2. Complete the table.

Design	1	2	3	4	5	10	p
Total Squares							

Expression

3. What is the **rule**? Write in a complete sentence.

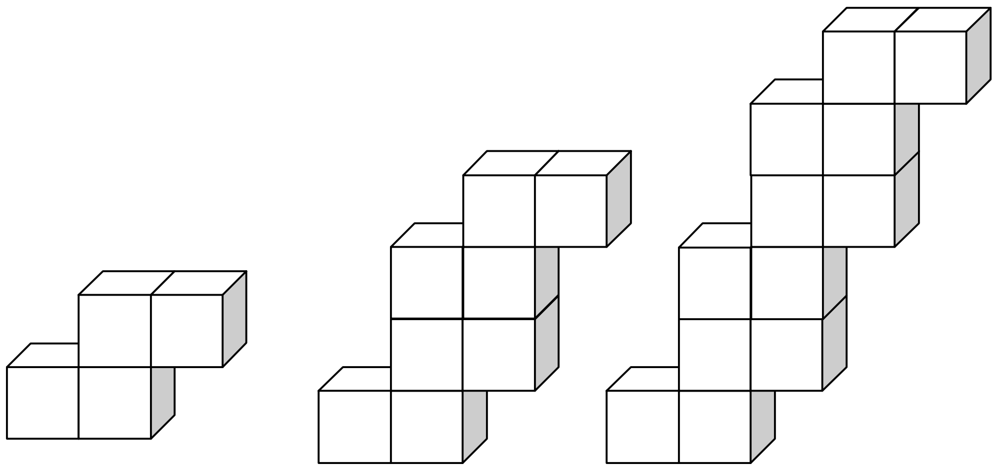
4. What is the **equation** for finding the number of Total Squares (t):

____

5. Use the **equation** to answer these questions.
   a. How many squares for pattern 20?
   b. How many squares for pattern 45?
   c. 25 tiles are used for which pattern number?
   d. 51 tiles are used for which pattern number?

Multilink Cube Patterns

Use the Drawings below to answer the questions.



Pattern #1 | Pattern #2 | Pattern #3

1. Make Pattern #4.

2. Complete the table.

Design	1	2	3	4	5	10	p
Total Cubes							

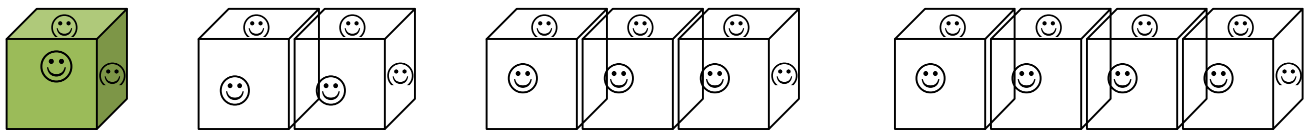
Expression

3. What is the **rule**? Write in a complete sentence.

4. What is the **equation** for finding the number of cubes (c): _______________

5. Use the **equation** to answer these questions.
   a. How many cubes for pattern 15?
   b. How many cubes for pattern 20?
   c. 32 tiles are used for which pattern number?
   d. 100 tiles are used for which pattern number?

Cube Stamping



Train 1 Train 2 Train 3 Train 4

Each face has a smiley face ☺ on each face of the train, how many smileys would you need for any sized train?

1. Make Train #5.

2. Complete the table.

Train	1	2	3	4	5	10	t
Smiley Faces							

Expression

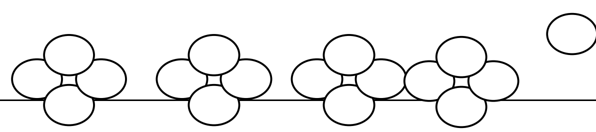
3. What is the **rule**? Write in a complete sentence.

4. What is the **equation** for finding the number of cubes (c): _______________

5. Use the **equation** to answer these questions.
   a. How many smiley faces for train 15?
   b. How many smiley faces for train 100?
   c. 50 tiles are used for which train number?
   d. 130 tiles are used for which pattern number?

Name:________________

May 18, 2012

Expression	Picture	Solution
$5 + 8 \times 6$		
$2 \times 4 - 5 \times 2$		
		
		24
$2 + 3 \times 4 + 5 \times 2 - 5$		

Each Orange Has 8 Slices Recording Table

Story	Table
Rule	**Equation**

M & M Equations using Variables

Name____

1. I would have to add (or eat) _____ red candies to have the same number of red candies as the teacher. How many red candies do I have?_______	*Variable:* *Equation:*
2. I would have to add (or eat) _____ orange candies to have the same number of orange candies as the teacher. How many orange candies do I have?______	*Variable:* *Equation:*
3. If I had 2 times the number of tan candies I have, then I would have _____ tan candies. How many tan candies do I have?______	*Variable:* *Equation:*
4. If I had ½ the number of brown candies that I have, I would have ____ brown candies. How many brown candies do I have?_____	*Variable:* *Equation:*
5. If I had 3 times the number of green candies I have, then I would have ____ more (or less) than the teacher. How many green candies do I have?_______	*Variable:* *Equation:*
6. If I added 15 yellow candies to my bag, the teacher would have to add ______ yellow candies to his or her bag for us to have the same number of yellow candies. How many yellow candies do I have?______	*Variable:* *Equation:*
7. If I double my blue M&Ms, then I would have ______ more (or less) than the teacher. How many blue M&Ms do I have?______	*Variable:* *Equation:*

M & M Equations: Equation Challenge!!

Name(s)____

1. If I tripled the number of yellow candies I have, I would have _____ more yellow candies than the teacher. How many yellow candies do I have?

Variable:

Equation:

2. If I ate 3 of my orange candies, then put my orange candies together with the teacher's orange candies, we would have ______orange candies. How many orange candies did I start with originally in my bag?

Variable:

Equation:

3. Suppose another student had a bag of M&Ms exactly like mine. So we each started with the same number of each color candy. If we combined our candy, then I ate 5 of our red candies, we would have ______ red candies left. How many red candies did I start with originally in my bag?

Variable:

Equation:

4. My brown, yellow, and green candies total ______. I have _______ more (or fewer) brown candies than yellow candies. I have _______fewer (or more) green candies than yellow candies. How many brown candies do I have? How many yellow? How many green?

Variable:

Equation: