Promoting the Mathematical Success of Emergent Bilinguals Through Problem Solving

CEMELA

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Session's Objectives

- Consider Cognitively Guided Instruction (CGI), a research-based approach to understanding students' mathematical thinking.
- Discuss how CGI responds to the Common Core State Standards.
- Explore how primary grade students solve a range of challenging, contextualized word problems, including multiplication and division.
- Discuss how to support Latin@ emergent bilinguals' mathematics learning.

Important Ideas….

- Latina/o young emergent bilingual students come to school with a wide range of psychological and linguistic tools, which teachers can use to help them make sense of problems posed in the classroom (Cummins, 2007; Moschkovich, 2007).

- EB not only draw from their home language and English, but between social and academic languages (Slavit & Ernst-Slavit, 2007; Moschkovich, 2002).

Discussion Question

- How do you use students’ home language and culture in the classroom to teach mathematics?

Part I: Cognitively Guided Instruction

Think-Pair-Share: What do you know about CGI?

What is Cognitively Guided Instruction (CGI)? (Carpenter et al.,1999)

- Framework for understanding children's mathematical thinking.
- Children enter school with a great deal of informal and intuitive knowledge of mathematics.
- Bridging students' experiential knowledge with formal school mathematics is critical.

CGI: Learning for Understanding

- Teachers use context-rich word problems based on knowledge of students' communities and the mathematical practices in which their families engage (Gonzales, Moll, & Amanti, 2005).

- Students develop and use their *own strategies* to solve problems.

The Common Core State Standards

- Most states have adopted the Common Core State Standards.

- The Core Standards were developed to provide a clear and consistent framework to prepare students for college and the workforce.

- They outline the most essential skills and knowledge every student needs to master.

Common Core State Standards for Mathematics

- Two corresponding and connected sets of standards:
  - Standards for Mathematical Practice: A set of 8 practices that describe the ways in which the mathematical content standards should be approached.
  - Standards for Mathematical Content: These standards define what students should understand and be able to do in their study of mathematics.

CGI and Mathematical Practices

#1. Make sense of problems and persevere in solving them.

- CGI emphasizes making sense of problems using what children know about the world.

#3. Construct viable arguments and critique the reasoning of others.

- With CGI, children listen to others and decide if the argument makes sense and supports the solution.

#4. Model with mathematics.

- There is particular emphasis in the CCSS on modeling and representation.
- In CGI, children naturally create models to find solutions.

CGI and Mathematical Practices

- #5. Use appropriate tools strategically.
  - Students choose tools that reflect the structure of the problem type.
  - In CGI, tool choices should include counters and sequential tools like number lines.

- #7. Look for and make use of structure.
  - Students look for patterns and structures within problems that relate to other problems they have solved.
  - CGI classifies problem types based on the structure and the strategies students use.

CGI and Content Standards for Primary Grades

- CCSS focus on addition and subtraction and a limited number of problem types for the primary grades.

- In CCSS, memorization of facts is encouraged. Multiplication is introduced in second grade, but is decontextualized in arrays.

- CGI expands to a wider range of problem types for even kindergartners including comparison, multiplication and division.

- CGI bases children's early conceptual learning of operations on their previous experiences.

Discussion Question

- What mathematical practices do you pay particular attention to in your own classroom?

- What challenges and opportunities do you see related to the CCSS in teaching mathematics?

Part II: CGI Problem Types

CGI Problem Types and Characteristics

- CGI provides a progression of problem types that help support students in using various strategies for engaging in problem solving.
- The problems use contextual information related to students' experiences (i.e., students' or friends' names, favorite toys, games, etc.).
- The progression of problems ranges from easier (action) to more difficult (no action). The location of the unknown also determines the complexity.

CGI Problems and Emergent Bilinguals: Language and Mathematics

JOIN

Connie had 5 marbles. Juan gave her 8 **more.**

How many does Connie have **altogether**?

COMPARE

Connie has 13 marbles. Juan has 5 marbles.

How many more does Connie have **than** Juan?

ACTION → NO ACTION

Problem Types and Complexity

Join	*(Result Unknown)* Connie had 5 marbles. Juan **gave** her 8 more. **How many** does Connie have altogether?	*(Change unknown)* Connie has 5 marbles. **How many** does she need to have 13 altogether?	*(Start Unknown)* Connie had some marbles. Juan gave her 5 more. Now she has 13. **How many** marbles did Connie have to start with?
Separate	*(Result Unknown)* Connie had 13 marbles. She **gave** 5 to Juan. How many marbles does Connie have left?	*(Change unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part Whole	*(Whole Unknown)* Connie **has** 5 red marbles and 8 blue marbles. How many marbles does she have?	*(Part Unknown)* Connie has 13 marbles. 8 are blue. How many red marbles does Connie have?	
Compare	*(Difference Unknown)* Connie **has** 13 marbles. Juan has 5 marbles. **How many more** does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more. How many does she have?	*(Referent Unknown)* Connie has 13 marbles. She has 5 more than Juan. How many marbles does Juan have?

Multiplication and Division Problems

Type of CGI Problem	English Version	Spanish Version
Multiplication	Elena has 5 bags of cookies. There are 3 cookies in each bag. How many cookies does Elena have altogether?	Elena tiene 5 bolsas de galletas. En cada bolsa hay 3 galletas. ¿Cuántas galletas tiene en total?
Measurement Division	Elena has 15 cookies. She puts 3 cookies in each bag. How many bags can she fill?	Elena tiene 15 galletas. Ella pone 3 galletas en cada bolsa. ¿Cuántas bolsas puede llenar?
Partitive Division	Megan has 5 bags of cookies. Altogether she has 15 cookies. There are the same number of cookies in each bag. How many cookies are in each bag?	Elena tiene 5 bolsas de galletas. Ella tiene 15 galletas en total. En cada bolsa hay la misma cantidad de galletas. ¿Cuántas galletas hay en cada bolsa?

Adapted from Carpenter, et al. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Base Ten Problem Examples
(Adapted from Carpenter et al., 1999)

Problem	Draw the bags and the extra tortillas.	Write a number sentence.	Write your solution using words to explain your thinking.
Example: You have 3 bags of tortillas and 4 extra tortillas.		*10 + 10 + 10 + 4 = 34*	*I have 3 bags of tortillas. That is the same as 30. I also have 4 extra tortillas. 30 plus 4 is the same as 34.*
1. Anali has 5 bags of tortillas and 0 extra tortillas.	Draw the bags and the extra tortillas.	Write a number sentence.	Write your solution using words to explain your thinking.

Let's Play!

- Each group has an envelope with 10 word problems.
- Your task is to determine the type of word problems you have and organize them in a sequence that progresses from easy to more difficult.
- To make this fun, let's see who can complete the task faster.

Types of Word Problems

Kinder

Mastered in 2nd

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all of the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

Problem Types Activity

- Form a group of 4 people and choose a grade level from Kindergarten through 3rd grade.
- Using the CCSS-Mathematics, create a sequence of three word problems that are appropriate for students at that grade.
- Include different types of problems
- Think about:
  - number choice
  - context
  - wording and choice of language
  - difficulty of problem structure

Problem Solving Strategies

Children's Strategies

Children's strategies for solving these problems progress:

- Direct Modeling - *La Modelización*
- Counting - *El Conteo*
- Derived Facts - *Hechos derivados*
- Recalled Facts - *Hechos numéricos*
- Algorithms – *Algoritmos*
  - Invented and Standard

Ms. Carrera's Kindergarten Classroom (March, 2008)

- Dual Language Classroom (90% Spanish/10% English)
- 95% of students were Latina/o and emergent bilinguals
- Low income Mexican immigrant families
- Ms. Carrera was from Mexico and was a beginning teacher.

Activity

Video: Multiplication

Comparing strategies

- Identify the strategies children use and what these show about their mathematical understanding.
- *I had three boxes. In each box I had five lollipops. How many lollipops did I have?*

Counting Strategies

- More efficient and abstract than direct modeling
- Gradually, children replace modeling strategies with counting strategies.
- Child recognizes that it is not necessary to physically construct and count ALL quantities in the problem.
- Physical objects (fingers) can be used to keep track of counts.

Counting…… cont’d

- Counting on from first
  - 4 + 7 = 4 [pause] 5, 6, 7, 8, 9, 10, 11.
- Counting on from larger
  - 4 + 7 = 7 [pause] 8, 9, 10, 11.
- Counting on to
  - 8 + __ = 13 8 [pause] 9, 10, 11, 12, 13. It’s 5.
- Counting down
  - 11 – 3 = 11 [pause] 10, 9, 8. It’s 8.

Pre-Post Assessment with Kindergarten Student

- Measurement Division Problem in October and May of school year
- **Pre:** *You have 10 cookies at home, and you put 2 cookies into each bag. How many bags do you need?*
- **Post:** *Amaranta has 10 cookies and some little bags. She puts 2 cookies in each bag to give to her friends. How many bags does she need?*

Derived Facts

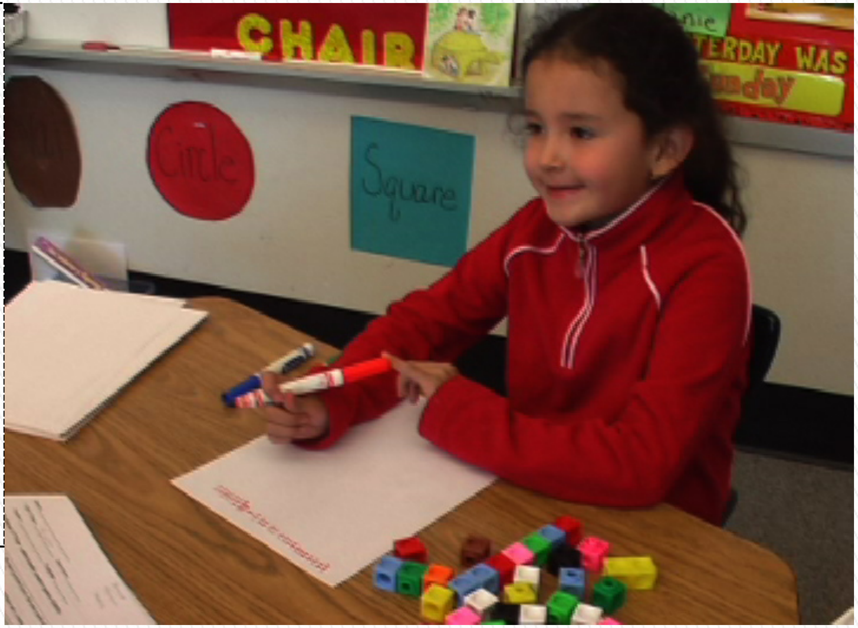
- Children learn certain combinations of number facts before others
  - Doubles
  - Combinations to 10
- Children use the facts they know to create (derive) new facts
  - To solve 5 + 6, I know 5 + 5 is 10, so 1 more is 11.
- It takes time for children to learn all the facts.
- Eventually, after children derive a fact a number of times, they will be able to recall it from memory.

Pre-Post Assessment with Kindergarten Student

- Multiplication Problem in October and May of school year
- **Pre:** *You have 3 pockets, and you put 2 pennies in each pocket. How many pennies do you have?*
- **Post:** *Jasmine has 3 bags of marbles, and there are 6 marbles in each bag. How many marbles does Jasmine have altogether?*

Partitive Division

Daniela has 84 pencils to share among 4 friends so that each friend gets the same amount. How many pencils does each friend get?



Think about two ways second graders would solve this problem without using the algorithm?

Tools: 100 chart, base ten blocks, counters, number line, pencil and paper

Omar

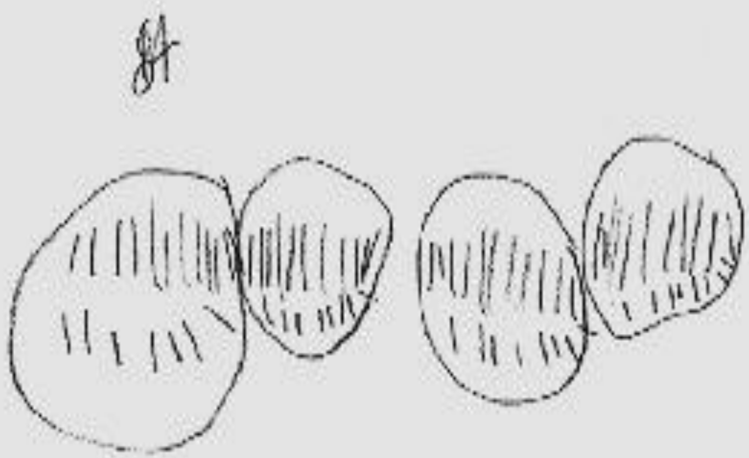


Figure 6. Omar, Partitive Division (84, 4), 2nd grade.

Excellent counting strategies, could not independently solve the problem. With scaffolding, he was able to solve it with trial and error.

Yolanda

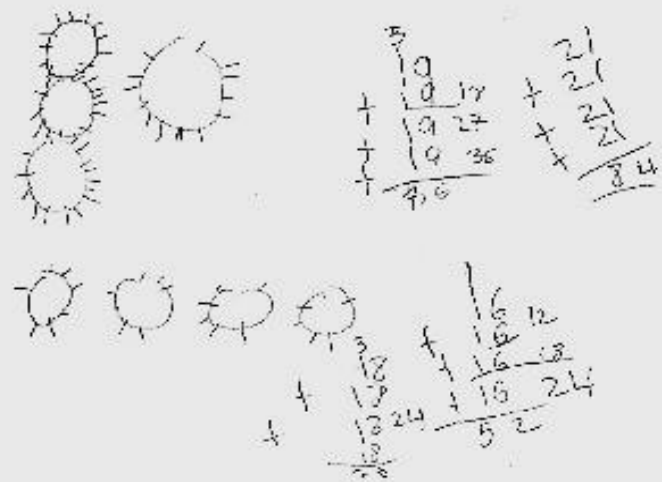


Figure 7. Yolanda, Partitive Division (84, 4), 2nd grade.

Favored algorithmic approach. Initially, she had a great deal of trouble, and had to fall back to repeated addition to find a solution. She began to think in groups

Problem Types and Examples	Direct Modeling Strategies	Advanced Strategies**	Comments
Multiplication Yolanda has three bags of marbles. There are seven marbles in each bag. How many marbles does Yolanda have altogether?	Grouping Make 3 groups with 7 objects in each group. Count all the objects to find the answer.	Skip Counting (in specific cases) If numbers in each group are easily skip counted, children will count by these and may keep track with their fingers, e.g. 5s, 10s.	This is an action problem. There is high consistency in how children directly model it, but counting strategies are observed when the size of the groups is convenient for children to skip count.
Partitive Division David has 15 marbles. He wants to share them with 3 friends so that each friend gets the same amount. How many does he give to each friend?	Partitive Divide 15 objects into 3 groups with the same number of objects in each group. Count the objects in one group to find the answer.	(no common strategy)	This is an action problem. Counting strategies are difficult because children do not know the size of the groups. They tend to use trial and error to figure out what to count by.

** For multidigit problems children will invent strategies that decompose numbers into tens and ones, increment in steps to reach a five or a ten, and/or compensate by solving with an easier number and then adjust their answer when they have reached a solution.

Partitive Division Challenges Students' Known Strategies

- Most common strategy is **trial and error.**
- In CGI theory, children move **from direct modeling** (more concrete) **to counting** (more abstract) in their strategies.
- Partitive Division is **difficult to solve with a counting strategy** because the size of the groups is not known.
- Even advanced students **need flexibility in their thinking** and strategies to solve difficult problems.

Teaching Challenging Mathematics K-3

- All students need a variety of problem types to have **expanded and flexible** thinking.
- Teachers need to **support students by**:
  - Understanding student thinking;
  - Using questions to expand student thinking about the numbers, and
  - Guiding students to choose appropriate tools and strategies.
- Teachers foster the use of multimodal approaches (i.e., pictorial, symbolic, and written) to communicate mathematical thinking.

Part III: Teaching Practices that Support EBs' Mathematical Thinking

Facilitating Productive Classroom Discussions (I) (Cirillo, 2013)

- Attend to the classroom culture.
- Choose high-level mathematics tasks.
- Anticipate strategies the students might use to solve the tasks.
- Allow student thinking to shape discussions.
- Examine and plan questions.

Facilitating Productive Classroom Discussions (II) (Cirillo, 2013)

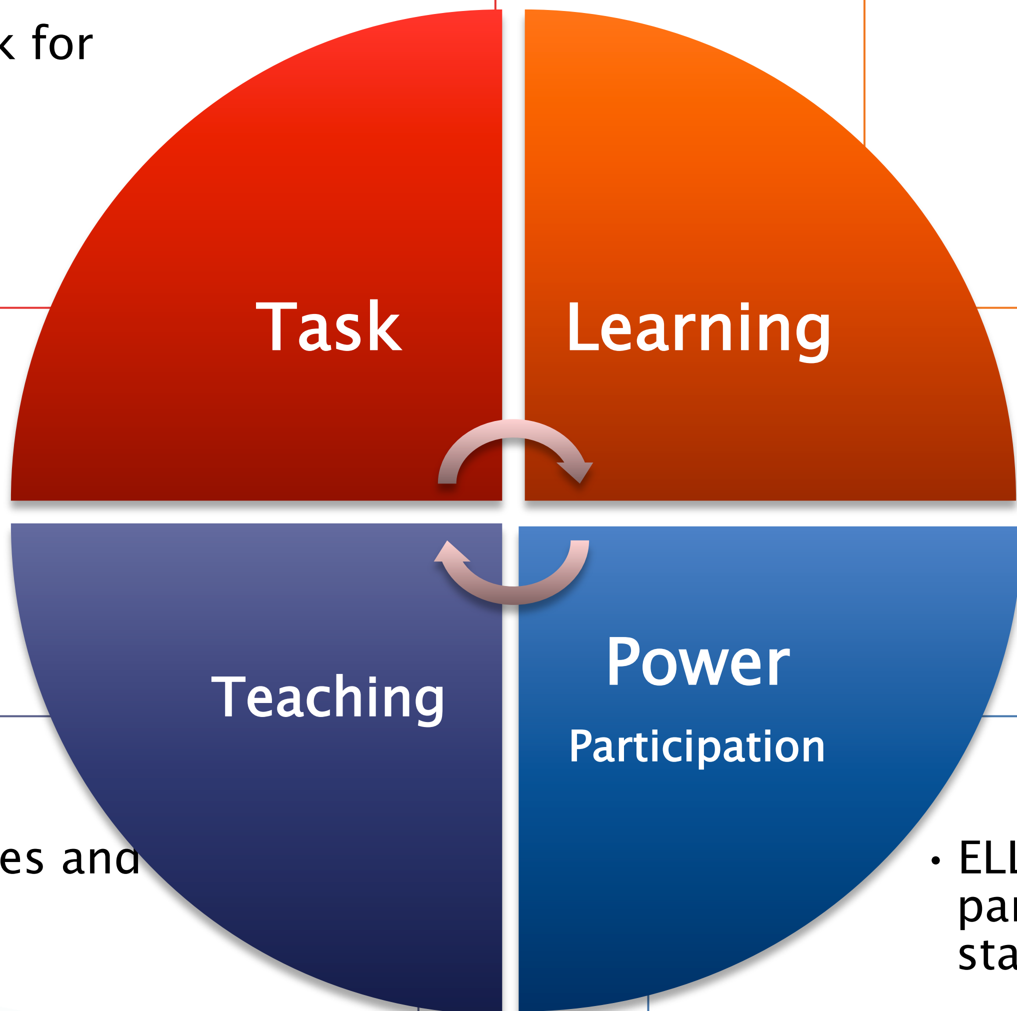
- Be strategic about "telling" new information.
- Explore incorrect solutions.
- Select and sequence ideas to be shared in the discussion.
- Use teacher discourse moves.
- Draw connections and summarize discussion.

Teacher Talk Moves (Chapin et al., 2009; Cirillo, 2013)

- Waiting
- Inviting Student Participation
- Revoicing
- Asking Students to Revoice
- Probing a Student's Thinking
- Creating Opportunities to Engage with Another's Reasoning

Multiple Mathematical Lenses

- Effectiveness of mathematical task for ELLs
- Evidence of ELLs' understanding



- Types of practices and interactions
- ELLs' engagement, participation and status

(Aguirre et al., 2012)

Exploring Lenses

- Each group has been assigned a lens—task, teaching, learning, or power and participation.
- Discuss the lens with your group. How can the lens help mathematics teachers reflect on their own teaching and planning practices?
- Be ready to share at least 3 main points of your lens.

Video Clip of Triangles Lesson with ELLs

- 4th grade class in the Southwest with 50% of students being ELL; Title I school

- Two students had IEPs.

- There are various stages of language acquisition.

- The teacher groups students so that each group has a strong English role model.

Video Clip of Triangles Lesson with ELLs (1 of 2)

- The task is to describe what they know about triangles.

- As you watch the video clip,
  - Use your own lens to discuss the video clip in relation to teaching and understanding the mathematical task.

Video Clip of Triangles Lesson with ELLs (2 of 2)

- The task is to investigate the lengths of the sides of a triangle. She asks students to mark off a strip of 4 cm, 4 cm, and 4 cm, then they cut it to experiment.
- Students conjecture whether any 3 lengths will make a triangle.
- Students work in groups to investigate.
- NOTICE:
  - How the teacher supports, clarifies, and extends students' mathematical thinking
  - How the teacher acknowledges students' contributions

Being part of a Mathematics Discourse Community

- Students were afforded opportunities to hear and use the language needed for learning mathematics, necessary for appropriation (Celedón-Pattichis, S. & Musanti, In press; Chval & Licon Khisty, 2009).

- Students progressively incorporated more accurate ways of explaining their ideas and strategies (Marshall, 2009; Musanti, & Celedón-Pattichis, 2012)

Wrapping up with some key ideas….

- Children can engage with challenging mathematics when they are able to use what they know about the world to make sense of the problems.

- It is important to understand children's solution strategies and their ways of making sense of a variety of problem types in order to move them forward in their thinking.

- First language and culture are a resource for mathematics teaching and learning (Thomas & Collier, 2012).

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First Grade Base Ten Lesson in a Dual Language Classroom

Developing and Communicating Effective Problem Solving Strategies

Teachers used questioning to:

- *Make sense of the problem and search for a solution*
  - How would you describe the problem in your own words?
  - How would you describe what you are trying to find?
  - What do you notice about...?
  - What information is given in the problem?
- *Construct arguments and explain reasoning*
  - Would you explain to me how you figured this out?
  - How did you count?
  - Which way to solve the problem is faster? Why?
  - How can we be sure that...? / How could you prove that...?