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Teaching for Excellence and Equity in Mathematics

TODOS
Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

TODOS and the editors of *TEEM*
express deep appreciation to
PEARSON EDUCATION
for its generous sponsorship of this issue.

From the Editors

We have many pieces of good news in 2011 to share. First, to increase the impact and visibility of this journal, the TODOS Board passed a motion to allow past issues of *TEEM* to be accessible on the TODOS website by the entire education community without password, subscription, or membership. Whatever issue is the current issue, however, will be available only to TODOS members (i.e., the "moving wall" many journal databases use) as one of many incentives to entice those who are not yet members of our worthy organization to join. And so, with the publication now of our third issue, issues 1 and 2 become available to ALL. Spread the word!

Also, to help collect and synthesize external referee reports, *TEEM* established an **Editorial Panel** consisting of the distinguished trio of **Marta Civil** (University of North Carolina at Chapel Hill), **Alfinio Flores** (University of Delaware), and **Eric (Rico) Gutstein** (University of Illinois at Chicago). Editors and Editorial Panelists, in turn, depend upon quality work from external reviewers and we express deep appreciation to those who have reviewed so far (see page 31 for a listing as well as instructions for joining the ranks).

The 2011 issue of *TEEM* includes an invited column and two externally peer-reviewed articles. The column is a first-person essay by a leading equity advocate in mathematics education, Carol Edwards. She was presented by TODOS with this year's Iris M. Carl Leadership and Equity Award, which recognizes an individual for significant contributions to the quality of mathematics education provided to underserved students. Next, Zulmaris Diaz, Joy Esquierdo, Olga Ramirez and Isela Almaguer propose and illustrate a framework for how bilingual learners develop knowledge, language and mathematics literacy. Then, Ruth Ahn, Ji Yeong I, and Robin Wilson share their results from applying Robert Moses' Five-Step Approach to teaching English Language Learners. Finally, we have notes of congratulation and memoriam for two inspiring and distinguished advocates for excellence and equity in the mathematical sciences.

TEEM gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* serves and increases the TODOS membership, and also serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities. We welcome feedback about the journal as well as results of teachers or professional developers putting the ideas (or DARE questions) of the papers into practice. We are also happy to give initial feedback to query emails about proposed ideas, including papers specifically written by and for classroom teachers.

Lawrence M. Lesser
The University of Texas at El Paso

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The University of Arizona

IN THIS ISSUE

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students — in particular, Hispanic/Latino students — by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

Passion for Equity: An Asian-American Mathematics Educator's Journey

Carol A. Edwards

Abstract

The recipient of the 2011 TODOS Iris M. Carl Leadership and Equity Award gives a first-person account of the life experiences that shaped her passion for equity, especially in education.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What experiences influenced your attitude and beliefs regarding equitable education for all of your students?
2. What experiences influenced the manner and extent to which you work for equity beyond your classroom?

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Acknowledgements The author thanks: Cecilia Riley who, as President of the Black Student Union at Chandler-Gilbert Community College, Arizona, invited her to participate on a 2008 panel about racism and prejudice; TODOS Founding President Miriam Leiva, who provided encouragement and editorial assistance; high school classmate Kenneth Fujii, former Chief of Staffing Services for the U.S. Office of Personnel Management in Honolulu and former advisor on personnel staffing and EEO matters to federal agencies in Hawai'i, Guam and the Pacific Rim, who made helpful suggestions.

Passion for Equity:
An Asian-American Mathematics Educator's Journey

Carol A. Edwards

As a 73-year-old mathematics educator, I have lived long enough to have encountered and observed instances of prejudice, racism and discrimination during successive phases of my life. But I have also seen and experienced many changes in our society which give all of us hope for the future.

Hawai'i Beginnings and Influences

I am a third-generation American of Japanese ancestry, and my family subscribed to the American Dream. When the Empire of Japan attacked and bombed Pearl Harbor on December 7, 1941, I was 3 years old. Although I did not know it then, my family and many of our friends were subjected to what is now referred to as *racial profiling*. It was a confusing time for a child who knew only a few words of the Japanese language.

I was born and raised in the town of Hilo on the Big Island of Hawai'i in the days when the islands were a Territory of the United States. My paternal grandfather came to the islands from Japan in the late 1800s as a contract laborer for a sugar plantation. The Hawai'ian Islands were agricultural then, and Sugar was King, with plantations owned and managed by White American planters, and with field workers imported from Asian and Pacific Rim nations.

On the plantations, White managers segregated workers' housing into separate camps for Chinese, Japanese, Filipino, and other ethnicities to keep them from forming interracial bonds and cohesive labor unions. In addition, there were separate pay scales for workers of different races and ethnicities. The Japanese got $18 per month, while Portuguese and Puerto Ricans got $21-23 for the same work (Daws, 1968).

Immediately after the Pearl Harbor bombing, America lashed out at fellow citizens who had no part in the attack, as President Roosevelt issued Executive Order 9066 in February 1942 which authorized the internment of Americans of Japanese ancestry from the West Coast into camps in remote and desolate locations in the interior states. About 110,000 Japanese-Americans and legal immigrants of Japanese ancestry were forcibly evacuated from their homes and communities with little notice. They lost almost all property that belonged to them, except for what they could carry with them to camp (Conrat & Conrat, 1972). This order against Japanese-Americans remained in effect until it was revoked in 1976 by President Ford (Ford, 1976).

We in Hawai'i were mostly not interned, except for community, business, and political leaders. But we lived under close scrutiny by the government, as our civilian governor of the territory was replaced by a military commander who imposed martial law throughout the islands. That meant many civil liberties were suspended, and replaced by the often arbitrary rule of military officers in charge (Daws, 1968). It was a hard time to be a Japanese-American. My mother's relatives in California were sent to camps, which were in desolate places in California, Idaho, Utah, Arizona, Wyoming, Colorado and Arkansas. There were even small camps in Hawai'i which housed not only local Japanese, but also German and Italian community leaders.

In many camps, Japanese-Americans were housed in tarpaper-covered, unpartitioned barracks without plumbing or cooking facilities, and which were overcrowded and offered little privacy. They were initially provided only army cots and blankets and in many camps, the winter and summer weather was brutal (Wilson & Hosokawa, 1980). When the war ended and the camps were closed, many internees had nowhere to go. They had lost almost everything at the start of the War, and many of the younger Japanese-Americans fought their battles through state and federal courts and U.S. Congress to gain a measure of recompense (Wilson & Hosokawa, 1980).

Restitution was hard won and token. In 1988, the U.S. Congress passed legislation which awarded formal payments of $20,000 to each surviving internee. This victory, 40 years after the closing of the camps, came too late for many, as less than half were still alive to collect on Uncle Sam's apology (Maki, Kitano, & Berthold, 1999). Today, some surviving former internees have opposed racial profiling because they recall their own personal hardships during World War II. Following the horrific events of 9/11, then U.S. Secretary of Transportation Norman Mineta, who was interned during World War II, sent a letter to all U.S. airlines forbidding them from racial profiling (Martin, 2008).

Social Class, Ethnicity, and Language

While Hawai'i was (and remains) a multicultural society with housing and public schools integrated, there were subtle and explicit racial prejudices. The sugar and pineapple plantations were owned and run by wealthy White families. This class difference was prevalent while I was growing up. Many of the teachers were from the Midwest and tended to look down on us because we were English Language Learners. For many of us, our first language was Island pidgin, and we entered school reading at a three-year-old's level according to tests which were normed on the general U.S. school population (Daws, 1968).

As I entered public elementary school in Hilo in 1943, I knew that there was also a special public school in town, set up by the Territorial Department of Public Instruction. It was called Riverside School, and was an "English Standard School." It was supposed to be an upper tier public school for which entry was based upon passing an oral English Standard and general knowledge test. The oral English test was a pretext for admitting Caucasian (*haole)* students as they all spoke standard English, while Asians and native Hawaiians had much smaller passing rates. Riverside School, along with other English Standard Schools in Hawai'i, was disbanded in the early 1950s (Wright, 1972).

The advancement of non-White races and ethnic groups in Hawai'i can largely be seen in the elections of top executives in state and county governments. For example, we have advanced from the early 20th-century during the days when Hawai'i was a territory of the U.S. and where the governors were exclusively *haole* (White) and male, to our days after statehood when we elected a Japanese-American (George Ariyoshi), a part Hawai'ian (John Waihe'e), a Filipino-American (Ben Cayetano), and a Caucasian woman (Linda Lingle) to lead our state (Abercrombie, 2011). Hawai'i's crowning achievement is that our current President of the U.S., Barack Obama, who is of mixed race, was born and educated there.

We have in Hawai'i, like most places in the U.S., come a long way in accepting each other. During recent reunions of my high school class, our differences did not get in the way of our friendships or enjoying each other's company as adults.

High School and University Years

Some of the best science and mathematics students in our Hilo High School graduating class were both Asian and female. Our valedictorian, who was strong in mathematics, was Asian and female and was featured in *Life Magazine* for being in the first group of National Merit Scholars. Like me, she went on to achieve a doctorate in mathematics and teach in college. And to highlight further the achievements of females in my high school, there were 30 females among the 37 students elected at the end of our senior year to the Honor Society for academic achievement (Sueyoshi, 1956).

As a University of California at Berkeley undergraduate, I became friends with women from my dormitory who were mostly White from California, and one was from Argentina. I felt no racial pressures there, but I lived with being a stereotype. I was told repeatedly that, because I was Asian, I had to be good at mathematics and science. I tried not to disappoint them. It had never occurred to me that women fared worse than men in mathematics and science at that time. I was one of three women in a physics class of 600, and two of us lived in the same dormitory. The male teaching assistant who graded our papers stopped by one day. I think he wanted to be sure that we knew our stuff and that we were not copying each other's work because we were outscoring most of the men.

During my graduate school years at the University of Illinois in Urbana-Champaign, my husband-to-be was arriving for the summer to enroll in classes. I called to make an appointment to see an apartment for him. The woman was very welcoming over the phone. When I showed up at her door, she would not let me in to see the apartment. As I turned to leave, a White couple came to the door, and she let them in to see the apartment. This experience happened before the landmark passage of the Civil Rights Act of 1964.

"Mixed Marriage"

My husband is a *haole* (Caucasian) and our marriage is a reminder of how far we have come. When my husband and I married in central Illinois in 1962, we could not have married in Tennessee, just a few hours' drive to the south. When we married, my parents were initially upset that I did not marry within my race; however, when our son was born, my parents could not have been more supportive and loving of our family. My late mother-in-law was wonderful and accepting, not only of our racial differences but also of our differences in religion (I am a Buddhist, and she was a devout Methodist).

In the summer of 1964 when my husband and I drove around the continental U.S., we were fearful of stopping in some states where marriages of mixed races were illegal. When we stopped at a hotel in such a state, he would go in to ask first if we could stay there. This was not a comfortable situation. One hotel clerk in Louisiana grudgingly took us in late one night but had to say "only because Asians are almost as clean as white people."

St. Louis Struggles and Successes

Before retiring and moving to Arizona, I taught mathematics and then served as Associate Dean at one of the campuses of St. Louis Community College in Missouri. During my 24 years there, this campus experienced an increasing number of Black students because of changing neighborhood demographics and enrollment patterns. The student population was one-third Black when I retired 12 years ago. Some of my colleagues had difficulty accepting this change. For example, they would lock the doors at the beginning of class to prevent late students from entering class. This was very upsetting. I asked my colleagues how they got to campus—of course, they drove there in their automobiles. Many of the students from the city came to campus by bus, sometimes making at least one transfer and then walking to buildings from the bus stop. If the bus was late, the student was late. I used to tell my students that they should come to class no matter how late; being late was better than not being there.

There are bright spots in the St. Louis part of my story. The number of students from underserved populations and the number of women enrolled in ccience, technology, engineering, and mathematics (STEM) courses were increasing. One of my former students from St. Louis Community College is Debra Dickerson, award-winning essayist whose work has appeared in *The New Republic*, *The Washington Post*, *Talk*, *Slate*, *Salon*, *Essence* and *Vibe*. She has won the New York Association of Black Journalists' first-place award for personal commentary. Her critically-acclaimed first book *An American Story* reveals her struggles to achieve an education in a way that is inspiring for students and teachers of all races.

Concluding Thoughts

I am also optimistic about the future of racial and gender equity, but feel we are not making progress fast enough. It was only in 2000 that the decennial U.S. Census began allowing respondents to identify as more than one race. We have had some recent major setbacks in our country, but our grandson and many of our great nieces and nephews are of mixed races and are living in a world more accepting of differences. Their world has a U.S. president of mixed race who was born and raised in Hawai'i where concepts and practices of equity got an early start even before required by law. This is quite a contrast to the days of my youth when no woman or African-American would have been a front-runner for her/his party's nomination for President of the United States.

My life journey has led me to volunteer for national and local organizations. I first began formally addressing multiculturalism and gender in mathematics education when, at the conclusion of my term as Program Chair for the 1994 NCTM Annual Meeting, NCTM asked me to serve on the editorial panel for the six-volume *Changing the Faces of Mathematics* series and as editor of the particular volume *Perspectives on Asian Americans and Pacific Islanders* (NCTM, 1999). This experience expanded my knowledge beyond my personal observations as I became more aware of "the roles of language and culture in the classroom.... rich traditions and contributions of Asians and Pacific Islanders to mathematics.... [and] cultural differences that influence classroom dynamics, classroom behavior and environment" (NCTM, 1999, p. v). This NCTM series came somewhat late for my generation but began to address some of the challenges we still face in mathematics education today.

Since retiring in 1999, I served 10 years on the Board of the National Council of Supervisors of Mathematics (NCSM), on a number of committees of the National Council of Teachers of Mathematics (NCTM), and on the Board of TODOS: Mathematics for ALL. They all advocate for equitable and high-quality mathematics education for all students, so my retirement is really a rededication of my life's passion. I volunteer for these organizations, especially TODOS, because the mission is so important and close to my heart.

References

Abercrombie, N. (2011). The Governor's office. http://hawaii.gov/gov/about/the-office.

Conrat, M., & Conrat, R. (1972). *Executive Order 9066, The Internment of 110,000 Japanese- Americans*, Los Angeles, CA: California Historical Society Special Publication No. 51.

Daws, G. (1968). *Shoal of Time: A History of the Hawai'ian Islands*. Honolulu, HI: The University Press of Hawai'i.

Ford, G. R. (1976). Confirming the Termination of the Executive Order Authorizing Japanese-American Internment during World War II. *Proclamation 4417, Vol. 12, No. 8*. Washington, DC.

Maki, M. T., Kitano, H. H. L., & Berthold, S. M. (1999). *Achieving the impossible dream: How Japanese-Americans obtained redress*. Chicago: University of Illinois Press.

Martin, M. (Host) (2008). Norm Mineta on Pioneering Career, America's Future at http://www.npr.org/templates/story/story.php?storyId=92579355&sc=emaf

National Council of Teachers of Mathematics (1999). *Changing the faces of mathematics: Perspectives on Asian Americans and Pacific Islanders.* Carol A. Edwards (Ed.). Reston, VA: NCTM.

Sueyoshi, P. (Ed.) (1956). *Blue and Gold 1956* [Student Annual of Hilo High School]. Hilo, HI.

Wilson, R. A., & Hosokawa, B. (1980). *East to America: A History of the Japanese in the United States.* New York: William Morrow and Company, Inc.

Wright, T. (1972). *The Disenchanted Isles: The Story of the Second Revolution in Hawai'i.* New York: The Dial Press.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are some of the ways in which racial profiling still exists today?
   (see online resources such as http://www.racialprofilinganalysis.neu.edu/)

2. How has the U.S. used statistics against minority populations, and what safeguards are now in place to avoid this?
   (see the work of Margo Anderson and/or William Seltzer)

3. What are some ways in which salary inequities still exist? What websites offer data on this?

4. How can the "Asians are good at math" stereotype be harmful as a gross generalization? How does this stereotype relate to the Chval and Pinnow article in the fall 2010 issue of *TEEM*?

5. What are ways you can make a difference in issues of equity as an individual teacher?

6. How can being active in national organizations such as TODOS, NCTM, or NCSM have an impact on issues that are important to you?

"DARE to Reach ALL Students!"

Developing Mathematics Literacy for Bilingual Learners: A Framework for Effective Learning

Zulmaris Diaz, J. Joy Esquierdo, Olga Ramirez, and Isela Almaguer

Abstract

A framework is proposed for how bilingual learners develop knowledge, language, and mathematics literacy. The framework centers on principles of learning, effective pedagogy, and second language acquisition theories, and these elements are incorporated in a mathematics lesson depicted in this article.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Does the term *bilingual learners* seem different from *English Language Learners*? Explain your reasoning and any possible influences from your experiences in teaching.

2. How do you define *mathematics literacy?*

3. What challenges might bilingual learners face when developing mathematics literacy?

4. Discuss any experience or knowledge that comes to mind connected to the components in Figure 1, page 12.

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Isela Almaguer (almagueri@utpa.edu) is an Associate Professor at The University of Texas-Pan American. Her research focuses on the teaching and learning of English Learners (ELs), instructional practices involving the literacy development of second language learners in dual language settings, and technology integration in teacher education.

Developing Mathematics Literacy for Bilingual Learners: A Framework for Effective Learning

Zulmaris Diaz, J. Joy Esquierdo, Olga Ramirez, and Isela Almaguer

The belief that mathematics is an easier subject than others to teach to students who are English Language Learners (ELLs) has misconceptions. While some view mathematics as a subject with minimum linguistic requirements that involves only numbers, many mathematics teachers would disagree. Mathematics involves specialized vocabulary, oral and written language, multiple representations of concepts, and same terminology for different concepts (Echevarria, Vogt, & Short, 2001). Moreover, research indicates that knowing how to complete computational problems and repeat definitions verbatim is insufficient for mathematical literacy.

Martin (2007) defines *mathematics literacy* as the ability to "reason, analyze, formulate and solve problems in a real-world setting" (p. 28). For students learning English and mathematics concurrently, becoming mathematically literate presents certain challenges. They need to learn not only English, but also the language of mathematics to construct meaning in mathematics (Ron, 2005); further, they need to communicate orally and in writing so as to explain solutions, provide conclusions, or present arguments (Moschkovich, 2002).

Another challenge faced by ELLs as they develop math literacy is the achievement of the Communication Standard outlined by the National Council of Teachers of Mathematics (2000), which suggest "instructional programs from prekindergarten through grade 12 should enable all students to-

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely" (p. 59).

For these reasons, we present a framework designed to support the development of mathematics literacy for ELLs. The framework will be complemented with an experimental activity that examines a lesson in a mathematics classroom through the lens of our framework.

Bilingual Learners

Throughout the general literature, students who are learning English are referred to as English Language Learners (ELLs), yet in this article we will use a more encompassing term. We refer to such students as Bilingual Learners (BLs) to emphasize that they are learning content at the same time they are developing two languages, English and their mother tongue. It is crucial when teaching BLs to keep in mind that the main focus needs to be on the cognitive development of the students and, as they are exposed to English, their linguistic abilities will develop concurrently. Teachers may sometimes become so focused on increasing language proficiency in the students' second language -- in most cases, English -- that the development of mathematics literacy can lose priority.

Framework for Teaching BLs New Content Literacy

The proposed mathematics lesson framework (in Figure 1) resulted from an extensive review of the literature on how BLs best develop content knowledge and skills. It centers on three fundamental elements: three principles of learning, effective pedagogy, and second language acquisition theories, all of which will result in a learner-centered classroom environment that supports the development of grade-level content literacy (Baker, 2006; Cook, 1992, 2002; Cummins, 1981, 1984; Krashen, 1982; Padrón & Waxman, 1999; National Research Council, 2000).

Three Key Principles of Learning

When developing the proposed mathematics lesson framework, we focused on three major principles of learning presented by the National Research Council (2000):

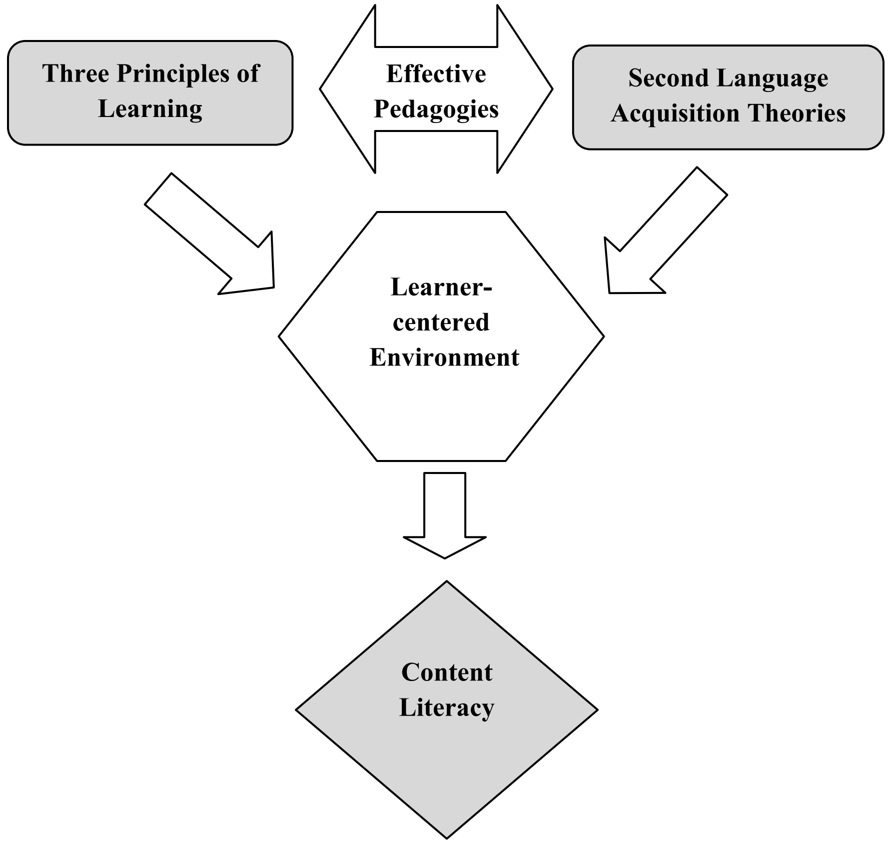

Figure 1. Framework for teaching Bilingual Learners new content literacy (modified from Diaz et al., 2010)

***Principle of Learning #1*:** All students start school with preconceived concepts of how the world functions. They bring experiences from home and from their surrounding community, which help them construct new knowledge in the classroom.

***Principle of Learning #2*:** In order for students to develop the ability to make inquiries, they must have developed basic factual knowledge and the ability to manipulate that knowledge (see Figure 2 to the right for an illustration).

***Principle of Learning #3*:** For bilingual learners to become lifelong learners, they need to take a metacognitive approach to their learning. Metacognition occurs when a student makes a conscious effort to control and monitor his/her learning through the use of various learning strategies (Brown, Bransford, Ferrara, & Campione, 1983).

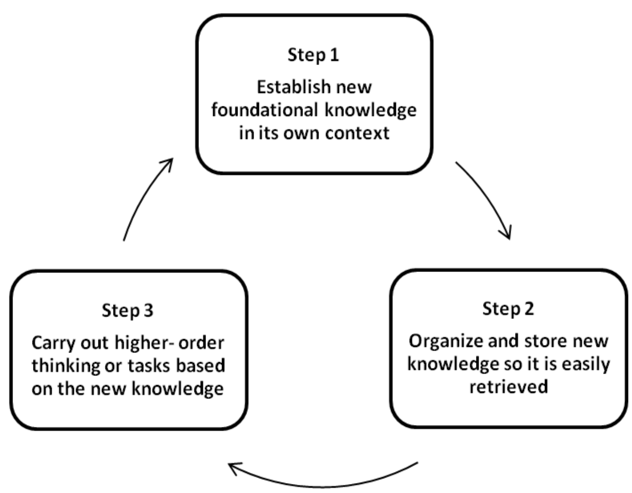

Figure 2. Learning principle #2 (Esquierdo, 2010)

Pedagogy Supportive of Language Acquisition and a Learner-Centered Classroom

When providing content instruction, mathematics teachers must be sure to employ solid pedagogy that upholds the theoretical views of language acquisition. Padrón and Waxman (1999) propose five research-based instructional practices (explained in Table 1 below) that support language acquisition, development, and knowledge for BLs: (1) Culturally Responsive Teaching; (2) Cooperative Learning; (3) Instructional Conversation; (4) Cognitively Guided Instruction; and (5) Technology-Enriched Instruction.

These instructional approaches are just a few of the research-supported strategies used to provide meaningful and effective instruction to BLs. They focus on a *learner-centered* environment, where "the students' own desire to know, to discuss, to problem solve, and to explore individually and with others" serves as the foundation of instruction rather than imparting "learning that is dictated, determined, and answered by the teacher" (Glickman, 1998, p. 52).

Table 1. Instructional practices that can support language acquisition

Pedagogy	Description
Culturally Responsive Teaching	Instruction that builds on the languages and cultures that children bring from their home and community (Slavin & Cheung, 2005).
Cooperative Learning	Instruction that involves the use of small groups as a means to optimize students' own and each other's learning. Some benefits: (1) enhances instructional conversations; (2) develops social, academic, and communication skills; and (3) develops proficiency in English (Calderón, 1991; Christian, 1995; Johnson & Johnson, 1991; Rivera & Zehler, 1991).
Instructional Conversation	Extended instructional discourse between the teacher and students (Duran, Dugan, & Weffer, 1997). Provides opportunities for extended academic conversations and allows BLs to reformulate previous concepts and attach new vocabulary to them (Christian, 1995).
Cognitively Guided Instruction (CGI)	Instruction allowing students to articulate their thinking which in turn provides teachers with a better understanding of how children learn mathematics (Carpenter, Fennema, & Franke, 1996).
Technology-Enriched Instruction	Instruction utilizing technology to help connect learning in the classroom to real-life situations (Means & Olsen, 1994) and allows students to access information in their native language as well as in their second language. Examples include the use of virtual manipulatives, web-based picture libraries, multimedia, calculators, etc.

Second Language Acquisition Theories

Teachers ought to be cognizant of students' acquisition of a second language. We will examine some principles of second-language acquisition with the intention to help teachers plan mathematics instruction for BLs. These principles take into consideration the learner as the center of his or her language and learning development and how all he/she brings into the classroom promotes language proficiency and academic achievement, specifically mathematics literacy.

Baker (2006) suggests that it is imperative that the education community stop viewing BLs as "two monolinguals in one person" (p. 10), so there is a need for a paradigm shift such that individuals are viewed as having *multi-competence* (Cook, 1992, 2002) in both languages. It is well known that language comprises four domains: listening, speaking, reading, and writing. Within these domains, there is much variation in language development and ability, spanning these stages (Baker, 2006; Krashen & Terrell, 1983):

> (a) simple or what second language acquisition scholars will call pre- and early- production stage: the person has limited comprehension of the language and uses short phrases to communicate;
>
> (b) basic or speech emergent stage: the person has an increased comprehension of the language, is less hesitant to speak and uses simple sentences to convey meaning;
>
> (c) fluent or intermediate fluency stage: speech is at greater length with the use of more complex sentence structures; and
>
> (d) accomplished or advanced proficiency: the person uses complex grammar and specialized academic vocabulary.

For BLs, the level of fluency within each of the language domain will depend largely on the need and use of a language (Grosjean, 1998). In fact, it is almost impossible for a bilingual person to be equally competent in both languages (Fishman, 1971). For example, some BLs might have a fluent or intermediate level of proficiency in speaking English when it is used in a social context (e.g., shopping, interactions with family, etc.), but demonstrate basic or speech emergent skills in reading and writing when used in formal contexts (e.g., academic lectures, work, etc.). Students might have an accomplished or advanced level of proficiency in speaking and listening to Spanish in formal context, but demonstrate fluent or intermediate skills when reading and writing Spanish in formal context.

Consequently, BLs need to be given opportunities and access to rich language environments in order to develop multi-competence in both languages at the social and formal context, or what Cummins (1984) identifies as the two key dimensions of language proficiency: (1) basic interpersonal communicative skills (BICS), language skills that are acquired easily through daily living; and (2) cognitive academic language proficiency (CALP), the language proficiency learned in an academic setting (Cummins, 1981).

It is important for teachers not to be deceived by BLs' language fluency in the social context since they might not have the same language proficiency in the academic or formal context. If bilingual learners are not exposed to the language of the classroom, the "vocabulary and the rhetoric style that make up the academic" language (Gandara & Contreras, 2009), specifically in our case the language of mathematics, they will encounter difficulties when having to read and analyze mathematical texts, including having to support, explain, and articulate their results and ways of thinking mathematics.

Under the premises that language is acquired through social interaction in different contexts, Krashen (1982) has concluded that the key to second-language acquisition is not the quantity of exposure to the second language, but the quality of instruction in the second language. That is, when a teacher is aware of the bilingual learner's second-language development, the focus goes from a *more coverage* approach to a *more appropriate* experience. The acquisition of language competency and content literacy in the second language is a result of comprehensible input, the spoken or written message that is delivered at the student's level of comprehension, and an accommodating affective environment, where the students' level of anxiety is low.

Learner-centered Environment

The main focus of a learner-centered environment is learning with understanding, while taking into consideration the needs, abilities, and interests of the learner. A learner-centered classroom promotes active exploration and construction of meaning, while moving away from the passivity of listening to the teacher's lecture and reading the textbook. It builds on the idea that the learner is responsible for his/her own learning.

The proposed framework serves as a catalyst for the creation of a learner-centered environment. Two of the main elements of the framework ask teachers to take into consideration the learner – specifically, how BLs learn based on the three principles of learning – and understand how they acquire the second language and develop language proficiency. Moreover, using effective pedagogies in the classroom help bilingual students become motivated about their own learning. Each of these elements contributes to the evolution of mathematics literacy collectively; they do not work in isolation. In other words, a teacher cannot simply assume that, for example, using the principles of learning component of the framework will guarantee that bilingual learners develop mathematics literacy. All three components of the framework need to be considered when planning and delivering instruction to BLs so that content literacy can be acquired.

A Closer Look in a Mathematics Classroom

The following scenario is a hypothetical sixth-grade classroom applying ratios through a real-world experience. The teacher, Mr. Cruz, starts his lesson asking the students about their homework. For homework, the students had to go to the store to choose a liquid product (e.g., juices, cleaning products, milk) and record the different size containers of the chosen product and their respective prices (e.g., a gallon of milk is $4.00; a ½ gallon is $2.50, a quart is $1.40, and a pint is $0.75).

Mr. Cruz asks the students, "So what did you find out?" "I found out that the bigger the container, the higher the price," offers Michael. María raises her hand and says, "I recorded the prices for milk and noticed that one gallon of milk costs $4.00, and ½ a gallon costs $2.50, and a quart costs $1.40." Mr. Cruz adds, "That's right. The prices will vary according to the container sizes. Today we will compare how prices of milk vary depending on their container size by using ratios. Ratios are used in our everyday life. Let's look for example at the different size containers of milk and let's use the prices María recorded." The teacher then places on the board a picture of a gallon, ½ gallon, and a quart of milk and writes down the prices María wrote down as homework. He then asks "which one do you believe is a better buy and why?"

Mario answers, "I think the quart is a better buy because it's cheaper." "No but you get less fluid ounces. I think it is the gallon," responds Jesús. Lucia adds, "Yo creo que el medio-galón tiene el mejor precio porque tiene más cantidad pero solo cuesta $2.50" [*I believe that the half-gallon is the better price since it has more milk and it only costs $2.50*]. Mr. Cruz continues with the lesson by commenting, "Okay, let's see which one is a better buy. To find out, we need to calculate the price per ounce. To do this, we need to take into consideration the ratio between the price and the fluid ounces of the container." Using an organized table, Mr. Cruz guides the students on how to study and calculate ratios (see Table 2).

Table 2: Price for one gallon of milk

Capacity of Container *Capacidad del envase*	Equivalent Fluid Ounces *Equivalencia en onzas líquidas*	Price of Container *Precio del envase*	Price per Ounce *Precio por onza*
1 gallon *1 galón*	128 fluid ounces *128 onzas líquidas*	$4.00	$4.00÷128= $0.03125

Mr. Cruz continues, "Now get with your partner and find out what the price per ounce would be if the gallon would cost $4.50. As students work in partners to respond to Mr. Cruz' subsequent question, he walks around guiding them when needed and asking them to explain how they solved the problem. In the background, you can hear the students using English and Spanish to solve the problems and also discussing the different steps they take when calculating division operations with and without the use of a calculator. Mr. Cruz takes advantage of the moment and asks Lucia, who recently emigrated from México, to explain to the class how she divides since she solves the problems differently.

After the students have completed the problems he posed, Mr. Cruz asks them "What did you find out?"

Marco responds, "if the gallon costs $4.00, the price per ounce is $0.03125, but if the price per gallon costs $4.50, the price per ounce is $0.03516." "Very well, now let's use the calculators and see what the price per ounce is when the ½ gallon costs $2.50," says Mr. Cruz. The class continues with the same study format and the teacher guides the students by showing them how to link what they know about determining the price per ounce of milk when given the price per gallon to finding the price per ounce of milk when a half-gallon costs $2.50 such as in Table 3. As the students work together they use both languages to complete the assignment and to help each other.

Table 3: Price for half-gallon of milk

Capacity Container Capacidad del envase	Equivalent Fluid Ounces Equivalencia en onzas líquidas	Price of Container Precio del envase	Price per Ounce Precio por onza
$\frac{1}{2}$ gallon $\frac{1}{2}$ galón	64 fluid ounces 64 onzas líquidas	$2.50	$2.50 ÷ 64 = $0.03906

After the students have completed the work given, Mr. Cruz asks them "What did you find out?" Angela responds that "if half-gallon costs $2.50, the price per ounce is $0.039063." "So then, which one do you think is the better buy?" asks Mr. Cruz. All students shout "the gallon!" Mr. Cruz continues, "So far, the gallon of milk is our better buy, but we cannot make assumptions. We need to calculate all the ratios. Continue working with your partner and find out the price per ounce when the quart of milk is sold for $1.40." After the students complete all the calculations, Mr. Cruz asks them, "Which one is the better buy and why?" The students answer in unison "the gallon of milk, because the price per ounce is $0.03125." Subsequently, Mr. Cruz asks the students to summarize how the price per fluid ounce is determined. As the students explain their thinking, he encourages them to notice that the price of the container must be divided by the number of fluid ounces in the container. Then Mr. Cruz asks the students to write in their journals a "word square" (Winsor, 2007) for the term "ratio" (see Table 4 below).

Table 4: Word square for the word "ratio"

Ratio	Razón
A **ratio** is a relationship or comparison between two numbers **Razón** es una relación o comparación entre dos números semejantes.	Ratio = $\frac{PricePerContainer}{NumberofFluidOunces}$ Razón = $\frac{\text{Precio del envase}}{\text{onzas líquidas}}$

After having students write in their journals, Mr. Cruz asks the students to work in pairs and use the findings from their homework to calculate the ratio of the prices of each of their recorded liquid products and to decide which size container is a better buy. Then, they are to write a small paragraph explaining which size container is a better buy and why, and they are to present the findings to the class. At the end of the lesson, Mr. Cruz reminds the students that for homework they will have to compare three different brands of cereal and decide which one is the best buy.

Connection of Lesson with Framework

Three Principles of Learning

At the beginning of this lesson, Mr. Cruz makes use of his understanding of the first Principle of Learning when he asks the students to share what they found out from their homework and when he makes connections to their prior experiences and knowledge. When Mr. Cruz asks the students to work with partners to use the findings from their homework to calculate the ratio of the price per ounce for each of the liquid containers and cereal brands and to decide which container or brand is the best buy, this demonstrates how this teacher makes use of the second Principle of Learning. The students will also be required to apply the same knowledge to a different context when they will have to compare three different brands of cereal. Moreover, Mr. Cruz makes use of Learning Principle #3 when he helps the students understand the new concept by organizing the new information into a table both in English and Spanish. He also makes the connection between English and the students' native language when he asks the students to create a word square for the new term. These two approaches are cognitive strategies which help promote the development of metacognitive strategies for BLs.

Pedagogy

Throughout the lesson, the students are working cooperatively. They are working in pairs instead of larger groups because students who are not proficient in English, females, and minority students tend to participate less in cooperative learning activities when groups are larger (Webb, 1984). Therefore, in Mr. Cruz's classroom, by working in pairs, all students contribute equally and are provided with opportunities for mathematical discourse, which allows BLs to process the new information further and develop language as they discuss findings with each other. As the teacher explains the concepts and walks around when students are working in pairs, he is making use of instructional conversations to promote language development and mathematical literacy. Likewise, there are hints of CGI when the students explain how they got their answers for finding ratios and when Lucia describes the method she learned in México of solving division problems. Teachers need to understand mathematical cultural differences, accept them, and allow students to use their own strategies (Midobuche, 2001). Moreover, students had ample opportunities to participate in academic conversations with their partners, the whole class, and the teacher. They also had the opportunity to use technology – in this case, calculators -- to solve problems.

Second Language Pedagogical Strategies

In this particular lesson, the teacher works under the premise that language is acquired through social interaction, with comprehensible input in an accommodating affective environment. The students are acquiring the English language as they experience and discuss the mathematics content in this lesson. They are not learning English rigidly through the review of rules and grammatical structures, but are learning and applying their English and Spanish skills as they attain the new vocabulary and knowledge introduced in the lesson and modeled by the teacher. Additionally, Mr. Cruz uses Krashen's (1982) notion of comprehensible input by providing visuals, and modeling allowing for language and concept transfer through the use of word squares (Winsor, 2007). All of these strategies provide support to the BLs' comprehension of the mathematical content and the development of the second language. Most importantly, the lesson is designed to lower the affective filter and allow BLs to feel comfortable taking academic and linguistic risks.

Learner-centered environment

The setting of a learner-centered classroom environment is created by employing the three key Principles of Learning (NRC, 2000), effective pedagogy, and second language acquisition theories. One fundamental trend among these three major areas is that providing a learning-safe, risk-free classroom helps the development of mathematics literacy skills in BLs to flourish. In the learning framework espoused in this paper, it is clear that permitting BLs to discuss the mathematics requirements of the lesson in both English and Spanish is empowering and fundamentally important in supporting a learner-centered environment.

Conclusion

This article proposes a framework that encompasses how BLs develop knowledge, language, and mathematics literacy. It provides a functional structure for teachers on how three principles of learning, effective pedagogy, and second language acquisition theories, collectively, can lead to the development of mathematics literacy for BLs. It is based on the premise that teachers must view bilingualism as a strength and not as an obstacle to teaching and learning.

It is crucial that teachers keep aiming for BLs to perform higher-order thinking skills and tasks. We posit that when BLs have the opportunity to explore actively and construct meaning by engaging in higher-order thinking projects in a learner-centered environment, they expand their understanding of the language (first and second language), content literacy, and most importantly, transfer knowledge from one language to another. They also gain the ability to apply the information to an assortment of contexts and use language as a tool. Thus, this article provides an important message to teachers of BLs that promotes exploring, inquiring, and applying new mathematics knowledge in and out of school contexts with opportunities to use both English and their mother tongue to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (NCTM, 2000, p. 59).

References

Baker, C. (2006). *Foundations of bilingual education and bilingualism* (4th ed.). Clevedon, England: Multilingual Matters Ltd.

Brown, A. L., Bransford, J. D., Ferrara, R. A., & Campione, J. C. (1983). Learning, remembering, and understanding. In J. H. Flavell & E. M. Markman (Eds.), *Handbook of child psychology: Cognitive development, Vol. 3* (pp. 177–266). New York: Wiley.

Calderón, M. (1991). Benefits of cooperative learning for Hispanic students. *Texas Research Journal, 2* (1), 39-57.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3 -20.

Christian, D. (1995). Two-way bilingual education. In C. L. Montone (Ed.), *Teaching linguistically and culturally diverse learners: Effective programs and practices* (pp. 8-11). Santa Cruz, CA: National Center for Research on Cultural Diversity and Second Language Learning.

Cook, V. J. (1992). Evidence for multicompetence. *Language Learning, 42*(4), 557-591.

Cook, V. J. (2002). *Portraits of the L2 user.* Clevedon, England: Multilingual Matters, Ltd.

Cummins, J. (1981). The role of primary language development in promoting educational success for language minority students. In California State Department of Education (Ed.), *Schooling and language minority students: A theoretical framework* (pp. 3-50). Los Angeles: California State Department of Education.

Cummins, J. (1984). Wanted: A theoretical framework for relating language proficiency to academic achievement among bilingual students. In C. Rivera (Ed.), *Language proficiency and academic achievement* (pp. 2-18). Clevedon, England: Multilingual Matters Ltd.

Diaz, Z., Esquierdo, J. J., DeLeon, L., Almaguer, I., & Curts, J. (Eds.) (2010). *Teaching content to Latino bilingual-dual language learners: Maximizing their learning.* Dubuque, IA: Kendall Hunt.

Duran, B. J., Dugan, T., & Weffer, R. E. (1997). Increasing teacher effectiveness with language minority students. *The High School Journal, 80*(4), 238-246.

Echevarria, J., Vogt, M. E., & Short, D. J. (2001). *The SIOP model for teaching mathematics to English learners.* Boston, MA: Pearson.

Esquierdo, J. J. (2010). How bilingual/dual language learners acquire new knowledge. In Z. Diaz, J. J. Esquierdo, L. DeLeon, I. Almaguer, & J. Curts (Eds.), *Teaching content to Latino bilingual-dual language learners: Maximizing their learning* (pp. 15-27). Dubuque, IA: Kendall Hunt.

Fishman, J. A. (1971). The sociology of language. In J. A. Fishman (Ed.), *Advances in the sociology of language,* Volume 1 (pp. 217-404). The Hague: Mouton.

Gandara, P., & Contreras F. (2009). *The Latino education crisis: The consequences of failed social policies.* Cambridge, MA: Harvard University Press.

Glickman, C. D. (1998). Educational leadership for democratic purposes? What do we mean? *International Journal of Leadership in Education, 1*(1), 47-53.

Grosjean, F. (1998). Studying bilinguals: Methodological and conceptual issues. *Bilingualism: Language and Cognition, 1*(2), 131-149.

Johnson, D. W., & Johnson, R. T. (1991). Classroom instruction and cooperative grouping. In H. C. Waxman & H. J. Walberg (Eds.), *Effective teaching: Current research* (pp. 277-293). Berkeley, CA: McCutchan.

Krashen, S. (1982). *Principles and practice in second-language acquisition.* Oxford: Pergamon Press.

Krashen, S., & Terrell, T. (1983). *The natural approach: Language acquisition in the classroom.* Hayward, CA: Alemany Press.

Martin, H. (2007). Mathematical literacy. *Principal Leadership, 7*(5), 28-31.

Means, B., & Olsen, K. (1994). The link between technology and authentic learning. *Educational Leadership, 51*(7), 15-18.

Midobuche, E. (2001). Building cultural bridges between home and the mathematics classroom. *Teaching Children Mathematics, 7*(9), 500-502.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2/3), 189-212.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council (2000). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academy Press.

Padrón, Y. N., & Waxman, H. C. (1999). Effective instructional practices for English language learners. In H. C. Waxman & H. J. Walberg (Eds.), *New directions for teaching, practice, and research* (pp. 171-203). Berkeley, CA: McCutchan.

Rivera, C., & Zehler, A. M. (1991). Assuring the academic success of language minority students: Collaboration in teaching and learning. *Journal of Education, 173*(2), 52-77.

Ron, P. (2005). Spanish-English language issues in the mathematics classroom. In W. Secada, L. Ortiz-Franco, N. G. Hernandez, & Y. de la Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Slavin, R. E., & Cheung, A. (2005). A synthesis of research on language and reading instruction for English language learners. *Review of Educational Research, 75*(2), 247-284.

Webb, N. M. (1984). Sex differences in interaction and achievement in cooperative small groups. *Journal of Educational Psychology, 76*(1), 33-44.

Winsor, M. (2007). Bridging the language barrier in mathematics. *Mathematics Teacher, 101*(5), 372-8.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. Looking back at Figure 1, what did you learn about each component of the framework?

2. Which of the three Principles of Learning most resonates with your approach to teaching? Explain.

3. Review a mathematics lesson you have previously designed and/or delivered to BLs. Does it encompass any components of the framework? What modifications can you make to the lesson plan so that it can utilize the major components of this framework?

4. Create a mathematics lesson for BLs that takes into consideration the framework in this article.

5. Analyze a video lesson through the lens of the proposed framework. For example (from http://www.learner.org/catalog/browse.html?discipline=5), suggested Annenberg/CPB Mathematics Videos are "Ladybugs" or "Marshmallows" (from Teaching Math: A video library, K-4) or "The Largest Container" (from Teaching Math: A video library, 5-8).

6. How can teachers optimize mathematics learning for BLs?

"DARE to Reach ALL Students!"

Teaching Mathematics to English Language Learners Using Robert Moses' Five-Step Approach

Ruth Ahn, Ji Yeong I, and Robin T. Wilson

Abstract

An eight-week summer intervention program in a low-performing middle school in Southern California applied Robert Moses' Five-Step Approach outlined in Moses & Cobb (2001). The Teachers Radically Enhancing Education (T.R.E.E.) Project brought hands-on, experiential mathematics teaching to 20 Latino English Language Learners who failed one or more courses in the previous academic year. At the end of the eight-week program, the 20 students showed improvement in mathematical performance and behavior. An original activity plan created by the participating pre-service teachers based on the Five-Step Approach is included.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions:

1. As a teacher, what are major challenges you have experienced in teaching mathematics to students from culturally and linguistically diverse backgrounds? Describe what has (and what has not) worked well in addressing these challenges.

2. Which mathematical concepts are most frequently emphasized in K-8 mathematics standards? With which of those mathematical concepts do you think students have most trouble? Why do you think that is the case?

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Teaching Mathematics to English Language Learners Using Robert Moses' Five-Step Approach

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In response to achievement disparities of ethnic and linguistic minority K-12 students (NCES, 2009), a pre-service teacher learning community, Teachers Radically Enhancing Education (T.R.E.E.) Project, was created at a university in Southern California to prepare pre-service teachers to teach abstract mathematical concepts effectively to diverse students. Among multiple frameworks used for this project, Moses' Five-Step Approach (Moses & Cobb, 2001) became the guiding framework in creating various activity plans based on sixth-grade essential mathematical concepts identified by the National Council of Teachers of Mathematics (2006) and the California Mathematics Standards (California State Board of Education, 1997).

This method demonstrates experiential learning, in which students experience concepts through familiar physical events before learning academic jargon and abstract symbols (see Figure 1). This approach reflects the ideas of the experiential learning model associated with Dewey (1938) and grounded in Quine's (1990) idea that scientific language comes from a regimentation of our ordinary discourse (Dubinsky & Wilson, under review). Taking students through the process of having a common experience, to discussing the experience in everyday language, then "mathematizing" the discourse gives all students an opportunity to use the experience as a frame of reference to engage in discussion about mathematical concepts, regardless of their previous background.

The goal of the Five-Step Approach is for the students to be guided by the instructor through the process of mathematizing the experience. As explicated by Dubinsky and Moses (2011), the first step in Moses' Five-Step Approach consists of a concrete participatory experience for students. In the second step, students draw their own pictures of the event. During this step, teachers help students identify the important features of the experience that they will need in the process of mathematizing the event.

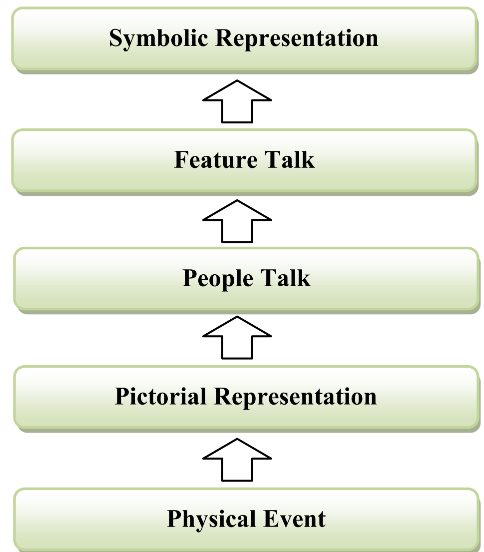


Figure 1. Robert Moses' Five-Step Approach

The third step ("people-talk") involves talking about the event and the important features with the students with everyday language that the students find familiar. The fourth step ("feature-talk") involves students moving from talking about the experience in ordinary discourse to using a more mathematical language. The fifth (final) step involves introducing the iconic representations for the mathematical symbols to complete the mathematization of the experience (Dubinsky & Moses, 2011).

Support for ELLs

The T.R.E.E. Project focused specifically on the effectiveness of Moses' Five-Step Approach with teaching mathematics to English Language Learners (ELLs). The Five-Step Approach may be a promising tool for ELLs because the process invites all students to first create their own symbolism and language to describe the mathematical objects and concepts involved in the lesson, and by doing this it gives ELLs ownership over the formal mathematical language and symbolism.

In the first step, physical events not only engage students but also support ELLs by creating multiple access points in addition to oral explanation for students to use to develop their understanding of abstract concepts. In the second step, ELLs can discuss the mathematics of the experience without formal mathematical language and symbolism (that will be introduced in steps 4 and 5). This step lowers ELLs' affective filter and offers a non-threatening opportunity to check if they are on the right track before they actually speak out.

Teachers may want to proceed to the third step in a small group setting so ELLs may participate without the added stress of speaking in public and have the opportunity to develop both their English skills and mathematical understanding by interacting with peers (Garrison & Mora, 2005). Furthermore, students' talking in everyday language or even in their first language may help teachers decide when formal mathematical terms should be introduced or connected to ordinary language.

In the fourth step, students learn the formal mathematical language for the concepts and objects involved. Therefore, the scaffolding of academic language that is implicit in the Five-Step Approach is well-suited for not only ELLs but also native speakers that struggle with the barriers of the language of mathematics. The fifth step comes after students go through the other steps by doing, drawing, and talking. This step is to guide students through the process of transitioning their acquired knowledge into a formal mathematical symbolism. This is the step where students are introduced formally to mathematical language and symbols such as +, =, or (-5) + (-2) to connect with what they experienced in the first four steps.

Teachers may fail to utilize classroom activities effectively if they believe simply conducting activities directly leads to students grasping the target concepts (Ahn, I, & Walker, under review). Jumping to the mathematical statement without proposing sufficient scaffolding and connections misguides students although they may have fun doing the activity. An appropriate sequence of scaffoldings is crucial when physical activities are used in the classroom. In this sense, the multiple steps of scaffolding in the Five-Step Approach establish a solid understanding and a safe learning environment, particularly for ELLs.

As an illustration of the five steps of Figure 1, consider this activity sequence for teaching fractions: (1) teachers brought pies of the same size and had student cut them into differing number of pieces; (2) students drew about their experience on cutting pies; (3) teachers talked with students about which pieces were bigger than others and how to share the pieces equally with others; (4) teachers had students compare and combine slices to explore the notion of "equivalent fractions" and queried which fraction representations were simplest. To introduce the concept of "equivalent fractions," teachers drew from the language that students brought up during the third step and such as "same," "equal," or "similar"; and finally, (5) teachers asked students to write and draw about these equivalent fractions in their journal.

The following sections present an original activity plan based on Moses' Five-Step Approach in teaching mathematics, created by the participating pre-service teachers. In addition, we will briefly discuss the project's results after the eight-week summer intervention program in sixth grade ELLs' understanding of basic fundamental mathematical concepts. It is beyond the scope of this paper to discuss the effect the T.R.E.E. Project had on pre-service teachers' thinking and teaching, but this is addressed by Ahn, I, & Walker (under review).

Experiential Learning in Mathematics

In applying this framework, we deliberately tapped into multi-sensory methods such as visual, auditory, kinesthetic, and tactile (VAKT) approaches (Rose & Zirkel, 2007), embedding them in the hands-on experiences of Steps 1 and 2. An emerging body of research has focused on the importance of providing hands-on, experiential learning opportunities that use VAKT approaches when teaching abstract mathematical concepts (Garrison & Mora, 2005; Gutstein, Lipman, Hernández, & Reyes, 1997; Lo Cicero, Fuson, & Allexaht-Snider, 2005; McLaughlin and Talbert, 2001; Ricks, 2010). Boakes (2009) discussed the use of tactile activities -- namely, origami -- as a teaching tool in strengthening spatial visualization skills and building general geometric understanding among 56 seventh graders in the U.S.. After instruction, students in the experimental group showed significant differences from control groups on card rotation tests. Boakes further explained that while gains in geometry knowledge were similar between the two groups, the results showed potential benefits to teaching mathematics by the use of paper folding.

Similarly, Pearn (2007) explained the use of paper folding in conjunction with fraction walls and number lines when teaching fractions to fifth through eighth grade students in Australia. Here, as students physically folded paper strips in response to fraction questions, teachers guided them to talk about how they came to their folding decisions as they worked on the problems. Next, students compared each other's strips on the fraction wall and the number line. These visual and tactile dimensions of experience helped students understand the relationship between fractions and their location on the number line while developing mathematical vocabulary. Furner (2009) discussed an approach to engage ELLs from Central America, by having them create base-20 counters with corn kernels and popsicle sticks as an aid in representing numbers in the ancient Mayan style, and deepening understanding of place value.

T.R.E.E. Project

Design

The Project was designed in response to regional and statewide need where schools with over 50% ELLs and 90% Latinos are not uncommon. The project aimed to transform pre-service teachers' thinking about how to effectively teach ethnically and linguistically diverse learners within a pre-service teacher learning community by applying Moses' framework. The T.R.E.E. Project consisted of 10 pre-service teachers, the researchers (first and third authors), and the lead teacher (second author). Under the close guidance of the researchers and lead teacher, during the eight-week summer program, the team met over four hours daily, Monday through Thursday, teaching abstract mathematical concepts to ELLs by applying the Five-Step Approach. All of the teaching integrated VAKT activities: half of the teaching provided kinesthetic learning opportunities outside the classroom, while the other half provided visual and tactile activities inside the classroom, deliberately avoiding a traditional linear "lecture" style. In addition to these teaching hours, the T.R.E.E. Project involved a 30-minute debriefing time after each day where pre-service teachers discussed and made sense of pedagogical and behavioral issues experienced. These discussions continued with expanded opportunities for interaction on an online discussion board.

Participants

Ten pre-service teachers (seven female and three male) enrolled in teacher education courses volunteered (and received from a grant a modest stipend) in this eight-week summer session in which they taught mathematics to 20 sixth-grade ELLs. Of the 10 pre-service teachers, five were Latino/a, three were Asian, one was white and one was biracial (White/Latina). The 20 ELLs (13 girls, 7 boys) who were selected by their school principal to participate in the summer intervention program either received a failing grade in mathematics or scored "Far Below Basic" or "Below Basic" on the state standardized mathematics test. Each student was assigned to a four-student group that was taught by two pre-service teachers. Group selection was based heterogeneously on test scores, grades, and gender. The middle school site was identified based on the principal's willingness to participate in the program and the school's status as a "Program Improvement School" in its fifth year under the No Child Left Behind Act. According to www.cde.ca.gov/ta/ac/sa/ (School Accountability Report Card), 85% of the school's students were Latino, 47% were ELLs, and 79% were socio-economically disadvantaged.

Assessment

Brief pre- and post-tests were given at the beginning of week 1 and end of week 8, respectively, and focused on sixth grade essential mathematical concepts in NCTM (2006): basic operations, negative numbers (integers), fractions (performing operations and writing in simplest form), one-step linear equations, and rate. The structure of questions and directions in both tests followed the sixth-grade California Standards Test (www.startest.org/cst.html) closely in order to bring standardization to the assessment. While the pre-test contained 30 questions, the post-test was reduced to 18 questions, dropping the geometry unit. This decision was made as a result of adapting to students' progress, especially when it was determined that they needed more time to work on other fundamental concepts such as basic operations, negative numbers, and fractions. The remaining questions were identical in concepts and similar in the use of wording and choice of numbers to their corresponding pre-test questions. It is important to note that because many of the students refused to take any kind of test by putting their faces down on their desks, it was decided to assess them with fewer questions per concept: five on basic operations, three on negative numbers, five on fractions, two on rate, and three on equations.

Example: Integer Addition Game

As mentioned earlier, each activity using the Five-Step Approach begins with a physical event that the students "mathematize" through a process that Moses has broken down into steps for teachers and practitioners to follow. We will describe one activity developed by pre-service teachers in the T.R.E.E. Project that adheres to this model. The physical experience that was the basis for this activity is similar to the typical hopscotch game and was originally designed for groups with four to six students. All students in the T.R.E.E. Project performed this outdoor activity at least twice, with the goal of teaching students how to add and subtract positive and negative integers. Usually two pre-service teachers worked with one student group. One teacher helped students individually while the other led the game for the whole group.

Prior to the activity, teachers and students constructed a hopscotch-like board on the ground using chalk or tape (see Figure 2). They then labeled the board with positive and negative integers to model the number line centered at '0'. Next, teachers took two different colored dice and explained to the students that one of the colors (e.g., black) represented the 'positive direction,' and the other color (e.g., red) the 'negative direction.' Each player began at '0' and rolled the dice one by one, and the students moved according to the instructions given by each roll of the dice. The students took turns until one student or team reached the designated end-points of the board or moved past them. Teachers divided the students into two groups with the same number of students and let them compete with each other.

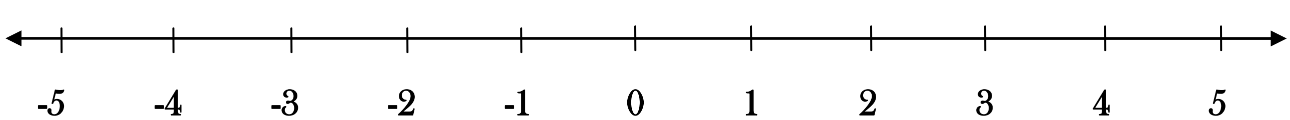
The game was designed to model addition of positive and negative numbers, with the dice representing the distance to travel and the direction. As the students played the activity in various ways, the teachers guided them to find patterns or other relationships. For example, when adding a positive number and a negative number, one of the scaffolding techniques that the teachers used with the students was to notice that when starting at the origin, if they moved farther to the left than to the right, then their ending position would be negative, and if they moved farther to the right than to the left, their ending position would be positive because equal-sized forward and backward movements were "opposites" that offset each other.

After they played the game, the teachers had students draw pictures about their experience on a piece of paper or on the ground with chalk as the second step of the Five-Step Approach. Teachers did not teach anything at this moment, but tried to remind students of the activity they just did and helped fill in details.

The third step in the Five-Step Approach involves what is referred to as "People Talk" where the teachers use language that the students use in their ordinary life, rather than mathematical or academic language, to discuss with their students what they observed and pat-

-5	-4	-3	-2	-1	0	1	2	3	4	5

Game Board 1



Game Board 2

Figure 2. Hopscotch game boards

terns they may have noticed during the experience. For example, the teacher might use "farther to the right" instead of "greater." For this particular activity, "the number to the right of 0" or "the number on the blue die" can be used for "positive number." This discussion continued until the teachers were convinced that the students understood the underlying mathematical concepts.

The fourth step in the process involves "Feature Talk" where the teachers introduced the students to the targeted mathematical terminology and symbolism used to describe the main features of the event. For this activity this step involved introducing the vocabulary of "negative" and "positive" for "movement to the left" and "movement to the right" respectively, and introducing the word addition for the operation "followed by."

For the final step in the Five-Step Approach, students were reminded of the formal mathematical symbolism for positive (+), negative (-), and addition of integers ((-5) + (-3)). They were then asked to describe patterns and rules they formulated during earlier steps in terms of the formal mathematical language and symbols.

One benefit of this particular game was that once students became familiar with the activity with addition of a positive integer and a negative integer, the game was used to teach addition of two negative numbers and subtraction of two integers as well. Figure 3 shows the visual sequence of how this activity was taught.

Results

Based on the areas in which gaps were identified in student background knowledge, the eight-week schedule was slightly modified and implemented as follows: Weeks 1 through 3 was basic operations; week 4 was negative numbers; weeks 5 and 6 were fractions; week 7 was equations; and week 8 was review and a fieldtrip.

The Five-Step Approach was not only used for teaching fractions (a major priority identified by the National Mathematics Advisory Panel, 2008), but also became the basis for teaching *all* of the essential concepts taught during the summer. After strategically applying Moses' Five-Step Approach in our teaching, it was found that the students had improved performance on CST assessment items in the targeted sixth-grade mathematical concepts including basic operations, negative numbers, fractions, rate, and solving one-step equations. Although there was gain across the board, the greatest improvement was in the area of fractions: from 24% at

Step 1.1 **Using sidewalk chalk (or tape), construct a hopscotch-like board (in the style of Figure 2) on the ground and label it from -10 to 10.**	**Step 1.2** **Take two differently-colored dice. Designate one die to be 'left' and the other to be 'right.'**	**Step 1.3** **Players begin the game standing at '0'. Each turn for a player consists of one roll of each color die.**	**Step 1.4** **Students move as the dice show and take turns until one student or team wins by going beyond either end (10 or -10) of the board.**
Step 2 **Ask students to draw their experience.**	**Step 3** **Have students discuss what they found and then guide them to make connections to integer addition.**	**Step 4** **Help students organize patterns or rules they found and express them in formal mathematical language.**	**Step 5** **Introduce the formal mathematical symbols relating to the formal language in Step 4.**

Figure 3. Hopscotch game (the first step of the Five-Step Approach spans steps 1.1-1.4)

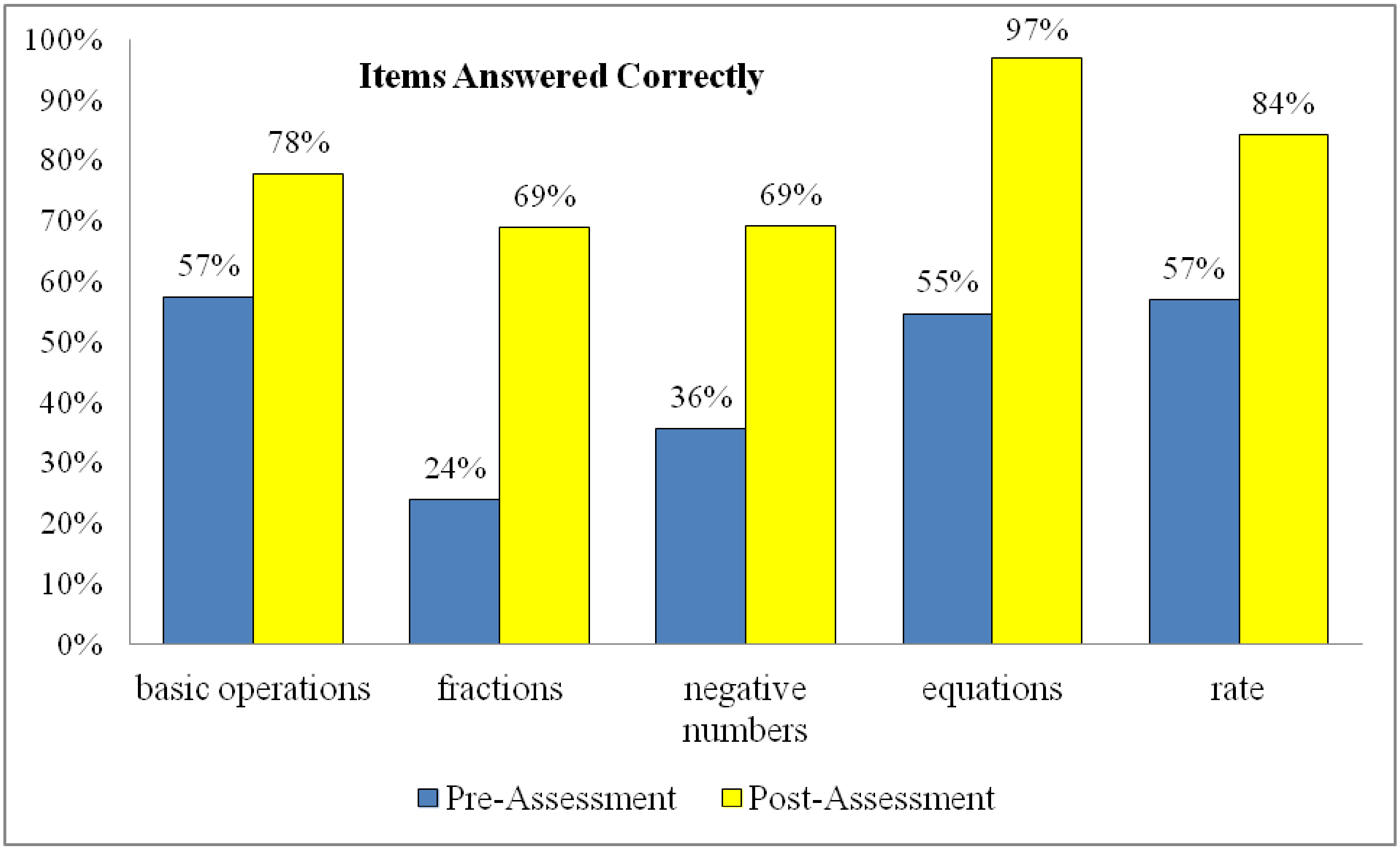


Figure 4. Pre-post results from California Standards Test items

the beginning to 69% at the end of the program, a gain of 45 percentage points (see Figure 4).

The T.R.E.E. Project illustrates an application and extension of the Five-Step Approach in teaching mathematics to ELLs. We are encouraged not only by the test results (Figure 4), but also by positive changes we observed in the students' behavior. By the end of the program, those students who had initially exhibited behaviors of "shutting down" no longer put their faces down or refused to take the tests. With its safe environment and scaffolding, Robert Moses' Five-Step Approach appears to be a promising vehicle for teaching mathematics to students, particularly ELLs.

References

Ahn, R., I, J., & Walker, P. (under review). Transforming the habits of mind of pre-service teachers: Learning to teach mathematics to English language learners in an urban middle school.

Boakes, N. (2009). Origami instruction in the middle school mathematics classroom: Its impact on spatial visualization and geometry knowledge of students. *Research in Middle Level Education, 32*(7), 1-12.

Califonia State Board of Education. (1997). *Mathematics content standards for California public schools, kindergarten through grade twelve*. Retrived from http://www.cde.ca.gov/be/st/ss/documents/mathstandards.pdf

Cirillo, M., Bruna, K., & Herbel-Eisenmann, B. (2010). Acquisition of mathematical language: Suggestions and activities for English language learners. *Multicultural Perspectives*, *12*(1), 34-41.

Dewey, J. (1938). *Experience and education*. Indianapolis, IN: Kappa Delta Pi.

Dubinsky, E., & Moses, R. P. (2011). Philosophy, math research, math ed research, K-16 education and the civil rights movement: A synthesis. *Notices of the American Mathematical Society*, *58*(3), 1-11.

Dubinsky, E., & Wilson, R. T. (under review). Developing high school students' understanding of the function concept.

Furner, J. (2009). Mayan mathematics: Connecting history and culture in the classroom. *Teaching For Excellence and Equity in Mathematics, 1*(1), 28-33.

Garrison, L., & Mora, J. (2005). Adapting mathematics instruction for English-language learners: The language-concept connection. In L. Ortiz-Franco, N. G. Hernandez, & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 35-47). Reston, VA: NCTM.

Gutstein, E., Lipman, P., Hernández, P., & Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *Journal for Research in Mathematics Education*, *28*(6), 709-737.

Lo Cicero, A., Fuson, K. C., & Allexaht-Snider, M. (2005). Mathematizing children's stories, helping children solve word problems, and supporting parental involvement. In L. Ortiz-Franco, N. G. Hernandez, & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 59-70). Reston, VA: NCTM.

McLaughlin, M., & Talbert, J. (2001). *Professional communities and the work of high school teaching.* Chicago, IL: University of Chicago Press.

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project.* Boston, MA: Beacon Press.

National Center for Education Statistics (2009). *The nation's report card mathematics and reading 2009.* Washington, DC: Institute of Education Sciences, U.S. Department of Education.

National Council of Teachers of Mathematics (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence.* Reston, VA: NCTM.

National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Pearn, C. (2007). Using paper folding, fraction walls, and number lines to develop understanding of fractions for students from years 5-8. *Australian Mathematics Teacher, 63*(4), 31-36.

Quine, W. (1990). *Pursuit of truth.* Cambridge, MA: Harvard University Press.

Ricks, T. (2010). Mathematics "is" motivating. *Mathematics Educator*, *19*(2), 2-9.

Ron, P. (2005). Spanish-English language issues in the mathematics classroom. In L. Ortiz-Franco, N. G. Hernandez, & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 23-33). Reston, VA: NCTM.

Rose, T., & Zirkel, P. (2007). Orton–Gillingham methodology for students with reading disabilities. *The Journal of Special Education, 41*(3), 171-185.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions:

1. In a university pre-service teacher education program, students are taught various theories and frameworks. This project used Robert Moses' Five-Step Approach from the Algebra Project to teaching mathematics. Why do you think it is important to have a guiding framework such as this when you plan and teach your lessons?
2. If you were given a textbook by your school district and asked to teach 50 mathematics concepts with the majority of your students being ELLs, what would you do? How would you go about identifying a sequence of lessons?
3. In reflecting on your own mathematics education, how did your teachers teach you mathematics? Did they provide concrete experiences first or the abstract symbolism in formulae or equations? In other words, did you first encounter Step 1 (experiences) or Step 5 (formulae with symbols such as +, -, =)?
4. Write a sample activity plan for teaching the addition of fractions with equal denominators by following Robert Moses' Five-Step Approach. Share with others.
5. Explore other benefits and insights games can have by reading articles such as the Jiménez-Silva, Gómez & White-Taylor article in the 2010 issue of *TEEM.*
6. Brainstorm with colleagues outside your department different ways how the Five-Step Approach may be used across Language Arts, Social Studies, Science, Music, etc.

"DARE to Reach ALL Students!"

RICHARD TAPIA WINS NATIONAL MEDAL OF SCIENCE

(Photo courtesy of the White House)

This fall, Richard A. Tapia was among 12 scientists to receive the top award the US offers its science researchers – the National Medal of Science. Tapia was honored "for his pioneering and fundamental contributions in optimization theory and numerical analysis and for his dedication and sustained efforts in fostering diversity and excellence in mathematics and science education." Tapia is known nationwide as a champion of underrepresented minorities in the sciences and one of his many hats at Rice University is Director of the Center for Excellence and Equity in Education. In 2005, Tapia received Rice's highest academic title by being named University Professor, one of only six professors in Rice history to receive this honor. In 1992, he became the first native-born Hispanic elected to the National Academy of Engineering. The medal Tapia received from President Obama is yet another item in a long list of honors to this educator born in a family of modest means where no one had gone to college.

In Memoriam:
Martha Aliaga, Advocate for Education and Equity

On October 15, 2011, the education community lost one of its dear advocates, Martha Aliaga. As director of education for the American Statistical Association (ASA) for the last eight years, Aliaga created the Educational Ambassador Program (to help statistics education reach students "in every corner of the world"), Meeting within a Meeting (held at the annual Joint Statistics Meetings, these are sustained workshops tailored for K-12 mathematics and science teachers), STEW (STatistics Education Web, a peer-reviewed repository of K-12 lesson plans), and a K-12 statistics education webinar program. She also introduced Census@School in the United States and co-authored influential reports (*Guidelines for Assessment and Instruction in Statistics Education College Report* and *Using Statistics Effectively in Mathematics Education Research*) and textbooks (*Interactive Statistics*). Aliaga was an elected council member of the International Statistical Institute, a Fellow of the American Statistical Association, and served as president of the Caucus for Women in Statistics in 2002. Just this May, Aliaga co-presented (with Larry Lesser) at the United States Conference on Teaching Statistics a well-received invited breakout session on diversity – arguably the first time that topic had been so prominently featured at a statistics education conference in this country. Aliaga is dearly missed, but her example of teaching for both excellence and equity continues to inspire. Contributions to ASA towards a scholarship fund in her name may be sent to:

The American Statistical Association, 732 North Washington Street, Alexandria, VA 22314.

The above material is adapted from various notices (mainly those in the November 2011 issues of Amstat News and Journal of Statistics Education). The rest of this Memoriam offers further perspective on what Aliaga has meant to the work of TODOS, as relayed by TODOS founding president Miriam Leiva:

Martha Aliaga was a leader and charter member in TODOS as well as a leader in the international statistics community. Born and educated in Argentina where she completed an undergraduate degree in mathematics, she earned her Ph.D. in statistics at the University of Michigan where she taught for several years as an Associate Professor and won teaching excellence awards. She was very proud of her heritage and was drawn to TODOS because of a shared commitment to underserved and underachieving students. In her candidate application for the TODOS Board election in 2005, she wrote:

> *I would like to see TODOS play a role in reducing the achievement gap in the performance of Hispanic students. The ASCD [Association for Supervision and Curriculum Development] position adopted in March 2004 says that all underserved population-high-poverty students, students with special learning needs, students of different cultural backgrounds, non native speakers and urban and rural students must have access to challenging coursework, high-quality teachers and additional resources. We need to influence the training of classroom teachers in pedagogical approaches that can be used to present mathematics and data analysis to school students as disciplined source of enlightenment. We can encourage mathematicians and statisticians to go into schools to discuss their work and their contribution to mathematics. I am very delighted ... to work with the president of TODOS and the rest of TODOS' members. I want to work to help our TODOS students to build outstanding careers. I know that we can! I know that we MUST! And I am very enthusiastic about the prospects of doing so.*

Dr. Aliaga served on the TODOS Board of Directors from 2006-08 and was instrumental in the development of our Constitution and By-Laws. She reached out to members one-on-one through our listserv as vividly shown by this 2003 email response to a TODOS member who was struggling to find the most effective way to teach mathematics at a women's prison to ESL Latina inmates:

> *I think now it is more useful to learn statistics than algebra if this a terminal course. Put away the book, then take a newspaper and help them learn how to interpret the data published (pie charts, bar charts, etc). Teach them how to answer their own social issues looking for data on the web, and analyze the data. Teach them how to write a questionnaire to ask the other women in the prison so they can collect real data to present to the prison officers to receive better living standards, school for their children, etc. Make mathematics meaningful to them.*

Acknowledgement of Referees

We thank these reviewers for their service in 2010 and/or 2011 and we are always interested in having more people available to review (one manuscript per year). If willing, please email teem@todos-math.org with your (1) contact information, (2) grade levels of teaching experience, and (3) thematic areas of greatest interest and expertise. And if there is someone you would like to nominate, please relay the necessary information for the editors to be able to send an invitation.

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