CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Tucson, Arizona March 4 -6, 2010

CEMELA

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This conference was supported in part by the National Science Foundation under grants Nos. ESI-0227586 and ESI-0424983. Any opinions, findings, and conclusions or recommendations expressed in these materials are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Conference Overview, Agenda & Plenary Session

Section 1 of 9

Gil Cuevas, Texas State University - San Marcos

Tucson, Arizona March 4 -6, 2010

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CEMELA-CPTM-TODOS Conference

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

The Center for the Mathematics Education of Latinos/as (CEMELA) co-sponsored along with the Center for Proficiency in Teaching Mathematics (CPTM) and TODOS: Mathematics for All a two-and-a-half-day working conference, *Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as* held March 4-6, 2010 in Tucson, Arizona. The conference focused on issues of language, culture, and policy in the context of teaching and learning mathematics. The goal of the conference was to engage a broad community of researchers and practitioners in considering key issues related to the teaching and learning of Latino students.

This working conference brought together 150 participants: teachers; administrators; mathematics educators; mathematicians; bilingual/ESL educators; graduate students; and policy makers for the purposes of learning from each other's research and experiences, considering implications to their field, and identifying additional research areas, instructional materials or supports needed.

The conference served to showcase and disseminate the work and outcomes of CEMELA through its most visible ambassadors, its Fellows and alumni as well as to highlight the findings of ten other researchers and the knowledge of practitioners grounded in teaching. CEMELA's attention to the development of theory and practice on how to turn language and cultural diversity into educational assets is relevant to the mathematics education of all students including all groups of linguistically and culturally diverse students.

The conference opened with plenary speaker Gil Cuevas presenting "Maintaining a balance: Culture, pedagogy, and subject matter. Reflections on preparing teachers of Latino/a students." His presentation was followed by a reception sponsored by the Helios Education Foundation. For the rest of the conference, participants were organized into groups to address eight research strands: Assessment; Curriculum; Visions from the Classroom: Focus on Teachers; Family Engagement; Teacher Education and Professional Development; Policy; Visions from the Classroom: Focus on Students; and Transforming Mathematical Identities through After School Settings. Each strand began with presentations from a researcher in the field followed by CEMELA Fellows and alumni, then a practitioner panel and finally a reactor. To conclude the conference a panel presented comments, impressions, and possible future actions.

15 CEMELA Fellows and CEMELA alumni presented papers on research that reflected the culminating work of CEMELA regarding mathematics education with Latinos/as. Working sessions followed presentations where participants gathered into groups of approximately 8 to 10 and addressed: what we now know; what are the implications or recommendations for practice and research; and what else do we need to know. These discussions were augmented further by 26 poster session presentations, 18 of which were presented by CEMELA Fellows and alumni.

Recorders in each small group took notes on the discussions that followed and these discussions are summarized here. Also included in these proceedings are the unpublished papers presented at the conference, along with transcriptions from the reactors in each strand (Visions from the Classroom sessions were structured differently and did not include a reactor).

CEMELA-CPTM-TODOS Conference

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Agenda

Thursday, March 4, 2010	
Opening Session	Gil Cuevas (Texas State University - San Marcos)
Reception	Sponsored by Helios Education Foundation

Friday, March 5, 2010	
Strands	**Speakers and Facilitators**
K-12 Assessment Chair: Rick Kitchen (University of New Mexico)	Speakers: • Maria Martiniello (Educational Testing Service) • Laura Burr (University of New Mexico) • Cynthia Anhalt (University of Arizona) and Anthony Fernandes (University of North Carolina-Charlotte) Practitioner Panel: • Fanny Castillo-Gonzalez (Hernandez Elementary, Española, NM) • Ana England (University of California-Santa Cruz, CA) • Heidi Aranda (Ochoa Elementary, Tucson, AZ) Panel Facilitator: Kathleen Ross (University of Arizona) Reactor: Jeremy Kilpatrick (University of Georgia)
Visions from the Classroom: Focus on Teachers Chairs: Lena Licón Khisty (University of Illinois-Chicago) and Sylvia Celedón-Pattichis (University of New Mexico)	Speakers: • Alma Ramirez (WestEd) • Rodrigo Gutiérrez (University of Arizona) and Charles Collingwood (Rincon High School, Tucson, AZ) • Craig Willey (University of Illinois-Chicago) and Julie Rodriguez (Chicago Public Schools) Practitioners as Reactors: • Bob McDonald (Atkinson Middle School, Phoenix, AZ) • Cindy Chapman (Albuquerque, NM) • José David Fonseca (Math Specialist, GEAR UP, University of Arizona)

Friday (continued)	
Strands	**Speakers and Facilitators**
Curriculum Chair: Nora Ramirez (Arizona State University)	Speakers: • Kathryn Chval (University of Missouri) • Jerry Lipka (University of Alaska, Fairbanks) • Kathleen Pitvorec (University of Illinois-Chicago) Practitioner Panel: • Donna Gaarder (WestEd) • Edna Alvarado (La Mesa Elementary, Albuquerque, NM) • Cathy Amanti (Hohokam Middle School, Tucson, AZ) Panel Facilitator: Ksenija Simic-Muller (Pacific Lutheran University) Reactor: Sharon Nelson-Barber (WestEd)
Family Engagement Chair: Marta Civil (University of Arizona)	Speakers: • Martha Allexsaht-Snider (University of Georgia) • Higinio Dominguez (Texas State University) • Javier Díez Palomar (Universitat Autònoma de Barcelona) Parent Panel: • Itzel Ceballos (Albuquerque, NM) • Edith Alvarado (Chicago, IL) • Leticia Jimenez (Chicago, IL) • Margarita Canela (Tucson, AZ) • Bethel Martinez (Tucson, AZ) • Mirta Osuna (Tucson, AZ) Panel Facilitators: Jesús Acosta-Iriqui (University of Arizona) José María Menéndez (Radford University) Reactor: Enrique Murillo (California State University-San Bernardino)

CEMELA-CPTM-TODOS Conference

Saturday, March 6, 2010	
Strands	**Speakers and Facilitators**
Teacher Education and Professional Development Chairs: Virginia Horak (University of Arizona) and Kip Téllez (University of California – Santa Cruz)	Speakers: • Rochelle Gutiérrez (University of Illinois, Urbana-Champaign) • Eugenia Vomvoridi-Ivanović (University of South Florida) • Laura Kondek McLeman (Willamette University) Practitioner Panel: • Brian Burns (Castro Middle School, Phoenix, AZ) • Rosemary Klein (Bailey Middle School, Davidson, NC) • Larisa Velasco (Drexel Elementary, Tucson, AZ) Panel Facilitator: Angela Thompson (University of California – Santa Cruz) Reactor: Dorothy White (University of Georgia)
Policy Chair: Luis Moll (University of Arizona)	Speakers: • Pedro Portes (University of Georgia) • Barbara Trujillo (University of New Mexico) Practitioner Panel: • Deby Valadez, (Holiday Park School, Phoenix, AZ) • Maria Santos, (NYC DOE) • Norma Torres Martinez (TX DOE-TX Education Agency) Panel Facilitator: Nora Ramirez (TODOS/Arizona State University) Reactor: Harold Asturias (University of California-Berkeley)

Saturday (continued)	
Strands	**Speakers and Facilitators**
Transforming Mathematical Identities Through After School Settings Chair: Aria Razfar (University of Illinois-Chicago)	Speakers: • Olga Vasquez (University of California-San Diego) • Maura Varley Gutiérrez (University of Arizona) • Carlos López Leiva (University of Illinois-Chicago) Practitioner Panel: • Sagrario Guadalupe Moreno (University of Illinois-Chicago) • Susan Foree (Safford Middle School, Tucson, AZ) • Grace Dávila Coates (University of California-Berkeley) Panel Facilitator: Tal Sutton (University of Arizona) Reactor: Shelley Goldman (Stanford University)
Visions from the Classroom: Focus on Students Chair: Judit Moschkovich (University of California-Santa Cruz)	Speakers: • Sandra Crespo (Michigan State University) • Bill Zahner (University of California-Santa Cruz) • Mary Marshall (University of New Mexico) Practitioners as Reactors: • Sharon Hoffert (James River High School, Midlothian, VA) • Arnulfo Velasquez (Wakefield Middle School, Tucson, AZ) • Chris Confer (Tucson, AZ)
Dinner **Closing Panel** Moderator: Jeremey Kilpatrick (University of Georgia)	Panelists: • Jere Confrey (North Carolina State University) • Kathy Escamilla (University of Colorado – Boulder) • Matt Ondrus (Weber State University) • Richard Ruiz (University of Arizona) • Olga Torres (Mathematics Education Consultant, Tucson, AZ)
Closing Remarks	Miriam Leiva (University of North Carolina – Charlotte)

MAINTAINING A BALANCE: CULTURE, PEDAGOGY, AND CONTENT
REFLECTIONS ON PREPARING TEACHERS OF LATINO/A STUDENTS

Gilbert J. Cuevas, Professor

Mathematics Department
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The appropriate professional development of teachers at all levels is critical in the efforts to provide effective mathematics education to Latino/a students. The design and implementation of degree and in-service programs that address pedagogy, subject matter content and culture are challenging endeavors. The paper addresses salient issues in each of these areas and ways to achieve a balance among them. The proposed idea is to prepare teachers who are knowledgeable of mathematics, have the pedagogical skills to teach the subject in ways that are linguistically and culturally relevant.

The National Center on Education Statistics (2008) reported that in 2009 nearly 3.3 million teachers taught in public schools in the U.S. In another report, the NCES (2002) highlighted that fact that "less than 13% of teachers have received any kind of professional development to prepare them for teaching linguistically and culturally diverse students." (2002, p. x). We find evidence in the literature that quality teacher education and professional development can convert schools into places where students in general and English language learners in particular become deeply engaged in learning. While we strive to achieve the necessary changes needed, teachers face mounting accountability pressures and regulations. To illustrate the frustrations many teachers feel, I would like to share what a teacher said working in a school with a large concentration of English language learners:
I look at the standards, at what I am supposed to teach, and then ... I have to develop the activity based on what I know the kids need. ... Will the students have opportunities to produce ***language****? Will they have choices? Is there a meaningful context? Am I modeling for them?* (Stillman, 2009, p. x). Many of the teachers I work with echo such feelings.

To address the challenges the education of English language learners present, there is a critical need to continue our efforts to design and implement teacher in-service curriculum and professional development programs that integrate existing research and practice about learning and teaching that address cultural issues, language and subject matter needs.

Before I embark on a discussion of teacher professional development, I would like to point out that a subtitle of this paper has **reflection** as a theme. A colleague of mine, Louis Schmier, history professor at Valdosta State University in Georgia, shared with me the following thoughts about reflection:

> *It is said that among Native Americans the medicine men ask three questions of the sick: 'When was the last time you sang? When was the last time you danced? When was the last time you told your story?*

I understand the first two questions I responded to Louis, but, I asked him why is it that telling your story so important? Louis' answer was that:

Each of us is someone who has learned something and who, by telling that part of his or her experiences, can benefit others. ...Our experiences, our discoveries, our ideas, our visions are all meant to be shared if for no other reason than we never live or work or love alone.

The message here is that each of us can bring the knowledge and experiences we have to meet the challenges we face in the education of Latino/a students.

Therefore, consider my presentation a part of my professional "story". It is a brief summary of the knowledge I have acquired through study and experiences working with teachers, English language learners, and many colleagues whose endeavors have contributed to my personal and professional growth. My remarks will center on ways we can prepare teachers to promote high levels of achievement among Latino/a students in general and to English language learners in particular. A basic question I would like to address is: What knowledge, understandings, beliefs, and resources do teachers need in order to create and maintain high quality learning environments for Latino/a mathematics learners? [Note: This is one the basic research questions of CEMELA]. To give structure to this "story," I have organized my comments into four sections patterned after the work of Lee Shulman, Ball and others on what accomplished teachers know (Shulman, 1986, 1987; Wilson et al., 1987; Ball, xxxx): cultural knowledge, language knowledge, content knowledge, and pedagogical content knowledge. I will close with remarks for achieving a balance among these components of teacher professional development programs.

CULTURAL KNOWLEDGE

Among the questions my colleagues and I have addressed in designing teacher development programs are: How do we provide teachers with the knowledge and skills to effectively interact with students from cultures other than their own? To what extent are courses, workshops and experiences included in these programs? How are these learning opportunities made part of teacher professional development?

The ideas underlying the processes of learning vs. acquisition have guided my thinking in addressing these questions. Second language educators are very familiar with these two concepts. The research literature in this area contains numerous studies describing the processes involved in each and the effectiveness of strategies for facilitating them. Learning, from a language perspective, involves a "formal" process to gain facility in a language. Language acquisition refers to the gaining of language skills through exposure in meaningful settings. Some language experts, such as Chomsky (1968), suggest that children do not "learn" language in any systematic way, but instead "acquire" it via exposure and existing mental structures. One of the goals concerning culture in teacher preparation is to have educators who can link the students' home culture and school knowledge. Many suggest that this cannot be accomplished by solely learning about student culture; it must be acquired through natural exposure. From experience, I have found that a strong feature of the cultural component of a teacher development programs involves the interaction of teachers with the cultures of target groups. I would like to stress here the critical importance of providing pre-service students during the early stages of their degree program with field experiences in community agencies where they have opportunities to interact with members of various cultural groups. From such experiences, pre-service teachers can learn first hand the challenges faced by those from a different culture, many of whom are not proficient in English and may have recently arrived to the United States. The same idea applies to practicing teachers. I am reminded of an experience I had during my first year of teaching fifth grade in a small rural community in southwest Florida. At this school, it was a frequent occurrence for some children to miss their bus home. The principal would "suggest" that new teachers "volunteer" to give these students a ride home. One day I was the "designated" driver. I drove two students to their home at a migrant camp. What I saw that day and the interactions I had with their families provided a rich source of information about poverty, racism and the educational needs of these children that the experience is still with me and has shaped many of my professional endeavors. So even though there are many demands placed on the time teachers have in their professional lives, I would suggest asking these questions: How well do I know my students? When was the last time I met with one of my students' parents? (or when did I visit a student home who was in need?). I can assure you that such experiences will leave you with long-lasting impressions about the students, and make you a more effective teacher because of the social understandings you have acquired.

One of the questions I am frequently asked is: “How can we infuse culture in the day-to-day classroom activities?” From a broad perspective, we may draw from the literature which deals with “culturally relevant pedagogy.” [Insert brief discussion – see Delpit, 1995, Gutstein, 2006, Ladson-Billings, 2000]

At the classroom level I have shared with preservice students and teachers at least four ways to integrate culture into mathematics learning activities: history, games, people, and cultural facts. To illustrate, in one of my methods classes, one of my students had planned a lesson on money. He wanted the students to examine a monetary system from other countries. He asked the children in a third-grade class to bring coins from their country of origin (if the parents had them). The children brought quite a few, and my student spent time during the social studies period discussing the symbols engraved on the coins. For example, in coins from Guatemala, the meaning of the Quetzal (national bird) was discussed. During the mathematics class the children carried out activities that provided reinforcement for “making change.” These strategies give Latino and children from other cultures a legitimacy of their backgrounds and provide opportunities for learning in meaningful contexts.

In general, we need to be reminded that “teacher education for diversity involves much more than the transfer of information from teacher educators to their students. It involves the profound transformation of people and of the world views and assumptions that they have carried with them for their entire lives.” (Melnik and Zeichner (1998, p.90).

The work of Hollins (1990), Ladson-Billings (1998), and Beyer (1991), to name a few, has shown that under certain conditions such approaches have facilitated pre-service teachers to gain knowledge about various cultural groups and to change their attitudes toward members of these groups. Overall, Gonzalez and Moll’s (2002) recommend that teacher preparation programs value students’ cultures and recognize the need to address those cultures in teaching practices.

SECOND LANGUAGE KNOWLEDGE

In a 2003 report, The National Center for Education Statistics indicated that of the 13 million 13-24 year olds in the U.S. who speak a language other than English, 72 percent speak Spanish, and a large percentage of those report that they speak English with difficulty. Consequently, language learning must be a primary concern of those who teach Latino students, and those who prepare teachers of Latino/a students. We cannot longer delegate this task to those educators whose specialization is English as a second language teaching (ESL) as was done in the past. The typical classroom in the U.S. today includes ELLs. Unfortunately, many of these teachers are not prepared to work with these students. The National Center for Education Statistics (2002) reports that 42% of the teachers surveyed responded that they had ELLs in their classrooms. Of these teachers only 12% had received more than eight hours of professional development specifically related to ELLs.

We all agree that teachers need to understand the foundations of language instruction as well as specific strategies for helping ELLs develop English language skills in the context of the learning of subject matter. According to Moshkovich (2002), if we are to improve the achievement of ELLs we need reform teacher preparation to include current perspectives of learning with current perspectives of language, bilingualism, and classroom discourse.

The use of language occurs in the context of communication. In any classroom, negotiating meaning, explaining ideas and procedures, justifying results or points of view are among the many uses of language. The term "language" also implies the forms language takes. In mathematics, terms, verbal expressions, symbols, representations are all part of the language we use to communicate mathematical ideas. In the design of teacher professional development programs we can begin by incorporating the ideas presented in the NCTM Professional Standards for Teaching Mathematics (1991) related to three dimensions of learning: worthwhile tasks, classroom discourse, and classroom environment.

- *Tasks* are the projects, questions, exercises, problems, and applications in which the students engage. They provide the intellectual context for students' development in the subject matter.
- *Discourse* refers to the ways language is used by teachers and students to think, talk, agree and disagree as they engage in the tasks.
- *Environment* represents the setting for learning. It is the unique interplay of intellectual, social, and physical characteristics that shape the ways of knowing and working that are encouraged and expected in the classroom (NCTM, 1991, p.20).

Teachers must develop a sense that language learning and the learning of subject matter must be closely integrated. There is an analogy that I have shared with my students concerning this idea. Think of a fiber optics cable. It contains bundles of intertwined wires surrounded by a protective covering. The function of the cable is to carry many forms of information accurately and effectively. One of the wires itself cannot accomplish the task of information transfer. All wires act in accord to achieve this goal. We need to think of the relationship between the learning of mathematics and language in the same manner. Mathematical content and process strands are bound together with the language components needed to successfully achieve effective communication. And, the wrapping that keeps these bundles together represents the socio-cultural context of the classroom, or according to Moschkovich (2002), “the social linguistic, and material resources that allow students to participate in mathematical practices.”

Our teacher development programs must provide a clear sense of the role of language in the learning of subject matter in general, and more specifically help teachers to develop the necessary knowledge and skills to incorporate language development strategies in the teaching the academic content. To accomplish this goal, institutions can follow approaches parallel to those used to develop cultural knowledge, attitudes, and skills. The teacher preparation programs can include second language teaching courses, and/or infuse second language principles and teaching strategies throughout all the course and field experiences. In the context of individual courses, instructors help students apply this knowledge in the context of specific subject matter teaching strategies.

To conclude this section, we all agree that teachers need to know the processes involved in first and second language acquisition and how these processes relate to students’ learning of the language needed in the subject areas. These include ways of adapting materials for ELLs and addressing the language demands of assessment.

CONTENT AND PEDAGOGICAL KNOWLEDGE

The National Center for Educational Statistics (as cited in Rodriguez, 2003) reports that one-third of teachers lack college preparation in the main subject areas they teach, and even less have the skills to use second language strategies in the subject areas. There is increased pressure for teachers to demonstrate competency in the subject matter they teach. This is a result of state and federal legislation requiring districts to provide students with “highly qualified teachers.” We all believe that teachers must possess thorough competence in subject matter. In recent years, writers have recommended that these pre-service teachers must go beyond this competency to develop and learn to cultivate in their future students, true subject matter proficiency. This translates into having a strong liberal arts subject matter component in the teacher preparation curriculum. In addition, subject matter courses need to be interconnected with methods classes and field experiences as well as peer coaching and mentoring.

Shulman, as early as 1986 advanced the notion of pedagogical content knowledge. He described this idea as “representations of specific content ideas, as well as an understanding of what makes the learning of specific topics easy or difficult for students. More recently in the context of mathematics education, Ball and her colleagues have expanded Shulman’s idea defining *Mathematical*

Knowledge for Teaching (MKT) as the "mathematical knowledge used to carry out the work of teaching mathematics (Hill, Rowan and Ball, 2005). Examples include "explaining terms and concepts to students, interpreting students' statements and solutions accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs" (Hill, Rowan and Ball, 2005, p. 4).

[Note: Insert adaptation of Hill & Ball (2008) MKT model that reflects ELLs learning needs.]

One of the lessons I have learned is that subject matter content and subject matter pedagogy together with cultural and language knowledge and instructional skills are inseparable when preparing teachers of Latino/a students.

MAINTAINING A BALANCE

As we can see, there is a balance that must be maintained among these components of teacher professional development – cultural knowledge, language knowledge, subject matter knowledge and pedagogical knowledge (or mathematics knowledge for teaching). This balance can be maintained through the careful design, implementation, and evaluation of such programs.

In my closing remarks I would like to echo the recommendations of my colleagues at the Center for Research on Education, Diversity and Excellence. In their report on A National Study of Teacher Education Preparation for Diverse Student Populations (Walton, Baca, & Escamilla, 2002) list the following among a list of recommendations:

- All teachers should be prepared to address the social, cultural, linguistic and economic backgrounds of the entire spectrum of American students.
- All teacher preparation programs should include in their curricula study of the nature of language development and first and second language acquisition and dialect.
- All teachers need to learn teaching methodologies that are specially designed to teach ELLs and dialect speakers. Methodologies should include methods that provide access to academic content in English, as well as access to learning the language.
- All teachers must learn strategies for literacy development.

I would add:

- All teachers need to be proficient in the subject matter they teach, and be able to teach the subject in an appropriate and effective manner.

We must demand these competencies of all teachers of Latino students.

I would like to leave you with this thought:

> *By three methods we may learn wisdom: First, by reflection, which is noblest; second, by imitation,*

> *which is easiest; and third by experience, which is the bitterest.*
>
> *Confucius*

And this, my dear colleagues is part of my ‘story’. I thank you for the opportunity to share it with you.

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K – 12 Assessment

Section 2 of 9

Chair: Rick Kitchen, University of New Mexico

Tucson, Arizona March 4 -6, 2010

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Language and the Performance of English-Language Learners in Math Word Problems

Maria Martiniello
Educational Testing Service

The paper presented by Maria Martiniello was published in the Harvard Education Review. The abstract for her paper can be found at http://www.hepg.org/her/abstract/652.

Martiniello, M. (2008). "Language and the Performance of English-Language Learners in Math Word Problems." *Harvard Educational Review* Vol. 78 No. 2 Summer, 333-368.

The Interaction of Interviewer Questions and Students' Mathematical Explanations: A Study of Seventh Grade ELL Students

Laura Burr

The University of New Mexico

This qualitative study explores the interaction of interviewer questions and students' mathematical explanations of seventh grade English Language Learners (ELLs). Students were videotaped in dyads as they interacted with the interviewer and each other when solving proportional reasoning problems. An analysis of the results demonstrated that ELLs effectively analyzed and solved complex mathematical problems and explained their procedures when responding to probing and guiding questions. Based on the socio-cultural perspective of acquiring knowledge through interactions with others, students developed solutions based on discussions and idea sharing that advanced their knowledge beyond the level attained prior to the interactions.

INTRODUCTION

Problem Statement

> Because mathematics holds the key to leadership in our information-based society, the widening gap between those who are mathematically literate and those who are not coincides, to a frightening degree, with racial and economic categories. We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority disproportionately Hispanic and Black, find economic and political power beyond reach. Unless corrected, innumeracy and illiteracy will drive America apart. (National Research Council, 1989, p. 14)

Twenty years after the above quote was published, our schools and communities are struggling with the problem of educating ethnic minority students in mathematics. The changing demographics of the United States makes the mission of providing a high quality education that empowers all students to become leaders even more urgent. For example, the rapid increase of ELL students in schools (NCELA, 2007) demands educational changes in order to support these new students. Educators have realized that accomodations must be made in instruction and assessment to support the diverse learning needs of ELLs (Abedi & Gándara, 2006). Researchers studied accountability issues and standardized assessments and found that, by changing how language was used, questions were more accessible to ELLs (Abedi & Gándara, 2006).

There has been a persistent gap in scores on standardized, large-scale assessments among ethnic minorities and white and Asian students (Williams, 2003). Traditional approaches locate the problem within the student (Boaler, 2002) rather than in either the assessments or in the instructional methodology. Traditional, or expository, instruction places the teacher as the authority who teaches students concepts and processes needed to do mathematics (Shulman, 2004). Reform mathematics is based on the constructivist viewpoint where students discover mathematics concepts by socially constructing their understanding as a member of a learning community. The impact of reform mathematics curriculum on ELLs has been a source of disagreement among researchers. Boaler (2002) found that reform mathematics classes reduced disparities in achievement resulting from forms of cultural congruency. Other researchers have found that Special Education students, Limited English Proficient students (LEPs), and low socio-economic status (SES) students are disadvantaged on written assessments in reform mathematics classrooms (Lampert & Cobb, 2003; Morgan & Watson, 2002). The challenge faced by educators is how to understand as accurately as possible what students know, understand, and are able to do in mathematics.

The problem being addressed in this study is how questioning techniques, during an interactive interview, affected ELL seventh graders who were attempting to make sense of proportional reasoning in mathematics. Knowledge of the performance of diverse students vis-à-vis traditional academic disciplines, including ELLs, tends to be fragmented because they generally constitute different research agendas (Kitchen, DePree, Celedón-Pattichis, & Brinkerhoff, 2007). This study specifically targeted ELLs because the language used in mathematics questions often disadvantaged them while changing the syntax (Abedi & Gándara, 2006) or context (Moll, Amanti, Neff & González, 2005) of the problem enabled students to gain access to the meaning of the task. The interactive interview protocol made it possible to respond to the students' cues during the problem solving experience ensuring their full understanding of the task.

Significance of the Study

The importance of this study is to add to the body of knowledge in formative assessment of ELLs in mathematics. At the classroom level, teachers can effectively implement formative assessments, "assessment that focuses on teacher's responses to student learning data they encounter on a daily basis" (Wilson & Kenney, 2003, p. 55) to shed light on student understanding and subsequently inform instruction. As stated in Principles and Standards in School Mathematics (NCTM, 2000), "To maximize the instructional value of assessment, teachers need to move beyond a superficial 'right or wrong' analysis of tasks to a focus on how students are thinking about the tasks" (p. 24). This research will help practitioners and researchers better understand how ELL students' are making sense of complex mathematical concepts.

Review of the Literature

The National Council of Teachers of Mathematics (NCTM, 2000) published a document entitled Principles and Standards for School Mathematics setting forth the important aspects of mathematics that students should know. One of the principles, the Assessment Principle states that

> Assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students; rather it should also be done *for* students, to guide and enhance their learning. (p. 22)

Assessment is the window into the student's mind; through assessment, educators can gain an accurate representation of the student's understanding of mathematical concepts. It is imperative that the educational community use effective strategies to determine what students know so effective teaching strategies can be implemented to move students from their present state of understanding to the next level (Franke, Kazemi, & Battey, 2007).

Traditional classroom format invokes the initiation-response-evaluation (IRE) sequence whereby the teacher initiates the communication often resulting in a yes/no or short response using a cloze type phrase (Pimm, as cited in Piccolo et al., 2008). The student responds and the teacher evaluates by indicating whether or not the response is correct (O'Conner & Michaels, 1993). Another method to determine student understanding is based on constructivism which emphasizes the process of constructing knowledge (Vygotsky, 1978). This occurs when the student connects new knowledge to prior knowledge, a process that often is not linear. This approach provides students with the opportunity to learn concepts in a way that makes sense to them rather than simply applying procedures others have developed. To approach learning using a constructivist paradigm, teaching techniques and classroom procedures must change so students take on the responsibility for learning (NCTM, 2000; Shepard, 2000). How teachers and students communicate is an important aspect of that change.

Questioning techniques play a significant role in helping students learn to explain their approaches and thinking as they solve mathematical tasks using verbal and non-verbal tools (Franke, Kazemi, & Battey, 2007; Heibert & Carpenter, 1992). While knowing specialized math vocabulary is not a prerequisite to doing complex mathematics, students can use gestures, drawings and descriptive phrases to communicate their thinking (Moschkovich, 2007). By using a variety of ways of communicating, ELLs can demonstrate their ability to solve complex problems.

Piccolo, et al. (2008), in their research, grouped teachers' questions into two categories, probing and guiding. Probing questions encouraged expression of knowledge and understanding or asked students to clarify, justify, interpret, or represent their solutions. Guiding questions helped students by giving real world examples and providing hints or suggestions. While Piccolo's study included white, middle class students and this study included ELL's with a low socio-economic status, the researcher decided to use the question categories as a starting point.

In another study, Way (2008) used Badham's (cited in Way, 2008) categorization of four question types in her research—starter questions, questions to stimulate mathematical thinking, assessment questions and final discussion questions. While there were similarities in the categories used by Piccolo, et al. (2008) and Way, (2008), the researcher, for this study, began by using the categories described by Piccolo, et al. (2008). Using probing and guiding questions lead to a formative assessment that represented how the students were making meaning of the mathematical concepts.

Formative assessments can provide information used to determine where the student resides on the trajectory toward obtaining proficiency on a standard or group of standards. Formative assessments can include the following: observation, inquiry, group work, whole class discussions, peer assessment, written work, individual interviews, student self-assessment, and portfolio assessment (Gearhart & Saxe, 2004; Stiggins, 2001). There are many features of the listed classroom assessments that are effective, albeit in differing ways resulting in different types of information. The realities of time and resource demands of implementation have to be considered or, if necessary, mitigated, as an educator decides how to get the most useful and accurate assessment information. Because of the depth and breadth of information gained through interviews, this research study focused on interactive interviews of dyads to reveal students' thinking.

Research Questions

This study was designed to examine how formative assessment in a reform mathematics classroom with ELLs can be implemented to shed light on student understanding of proportional reasoning. The study sought answers to the following research question and sub-questions:

In what ways do the types of questions in an interactive interview promote detailed mathematical explanations of proportional reasoning tasks?

a. How do the types of questions encourage students to expand on their explanations of task solutions?
b. How do the types of questions affect student's willingness to persist in their attempts to make sense of and solve problems?

DESCRIPTION OF THE CASE

School Setting

Students who participated in the research project attended a private school, El Mundo, located in the southwestern United States. It was a religious-based neighborhood school that was funded through donations. The school opened its doors in the school year 2007-2008 with one class of 6th graders—in 2008-2009, there were two grade levels, 6th and 7th. During the first year, all the teachers, the director, the after-school tutors, the people who made repairs and fixed the buildings were all volunteers. One person, an assistant, was paid a nominal salary. In 2008-2009, there were two paid teachers.

The school enrollment included six sixth graders and twelve seventh graders where all but three of the students were ELLs. Prior to attending El Mundo, students attended the local public school where some of the classes were taught in Spanish, the research participants' first language.

The people who have worked together to establish the school are the parents, students and members of the local ministries. The students who attend the school are from the neighborhood—one African-American student, one Anglo student, and 16 Latino/a students. The majority of the Latino/a students are from Mexico or their parents are from Mexico; the students speak Spanish and English during the school day both in social settings and during academic activities.

Participants

The participants in this study, done in 2009, included three girls and one boy who were in seventh grade and all attended El Mundo in 2007-2008. As described by the school's director, all participants' are ELLs whose first language is Spanish and who have a functional command of the English language (based on observation). Their socio-economic status (SES) is low, based on information provided by the director. Students' levels of performance in mathematics are as follows: Marisol tended to perform at a below average level, Andres at an average level, and Zenia and Veronica at an above average level. The levels are based on the math teacher's observation of the students' oral and written work,

DESIGN OF THE STUDY

Methodological Framework

The researcher used the multiple case studies methodological design (Creswell, 2007) where she focused on four students, which provided insight into how the assessment reflected Latino/a students' mathematical understanding. The researcher focused on four students to gain an in-depth understanding of their internal perceptions by observing the external representations

(Heibert & Carpenter, 1992) and listening to the explanations of their logic and processes as they sought to build connections and fully comprehend the concepts. The case study approach worked well because it explored "multiple bounded systems (cases) over time, through detailed, in-depth data collection involving *multiple sources of information* . . . and report[ed] a case *description* and case-based themes" (Creswell, 2007, p. 73). The multiple sources of information provided rich information on individuals who were performing at varying levels academically, solved problems in multiple ways, or reacted to interviewers in particularistic ways. The case study approach allowed the interviewer to pursue avenues of questioning based on student responses, to follow the student's lead. Then, the information gathered provided the data for researchers to analyze the question/response interaction.

Conceptual Framework

Social-Constructivism. The emergent social-constructivist paradigm borrows from cognitive, socio-cultural and constructivist theories (Shepard, 2000). Within the cognitive psychology paradigm, scholars seek to understand an individual's learning in terms of internal cognitive structures and processes (Cobb, 2007). The learning of mathematics is viewed as an active process of mental construction and sense making. Within this paradigm, frameworks have been developed to locate students' thinking within specific mathematical domains such as multiplicative reasoning (e.g., Confrey & Smith, 1995). A potential pitfall of domain-specific cognitive frameworks is that they may not take into account cultural and social issues such as the cultural practices of the communities in which the learner lives.

In the socio-cultural perspective, learning is developed through socially supported interactions (Vygotsky, 1979) and that learning and child development is brought about from the beginning through communication. "Instruction and development do not meet for the first time at school age; rather, they are in fact connected with each other from the very first day of a child's life" (Vygotsky, 1956, cited in Lerman, 2001, p. 5). From this perspective, cognition is inherently social and learning is viewed as an element of a system of cultural practices (Cobb, 2007). Vygotsky advocated that we not only look at mental activity but at situated practices and that the process must be studied, not just the outcome of activities (Forman, 2003). Thus, socio-cultural theory provides a means to explain the complex relationship between social context and learning.

Constructivism views learning as a process whereby learners build upon a foundation of knowledge, making additions, connections, and revisions based on new knowledge (Vygotsky, 1979). Learning is grounded in making connections to that which is previously known, not memorizing information in isolation. The social-constructivist framework informed our work during the data collection process.

Methods of Data Collection

Data for this study was collected in multiple ways including obtaining Internet documents from the school's website, collecting written work samples, videotaping interviews, and doing classroom observations. During visits to the site, the researcher took field notes describing the physical setting, personnel and students. The researcher also took field notes describing classroom interactions during math class. Students' work products, along with their descriptions of the strategies used to understand mathematical concepts were collected and analyzed.

Time in the field consisted of taking fieldnotes describing classroom practice and the dynamics of student-student and student-teacher interactions during multiple visits. Many of the reform mathematics lessons included working in small groups or in pairs enabling students to verbalize their thinking, make modifications in strategies based on other students' understanding and negotiate their final decisions on how to solve the problem. When in the field, the researcher recorded explanations the students framed around some of the math work they had completed, asking them to elaborate on their thinking thereby providing greater depth of understanding of their thought processes.

To prepare for the interview portion of the research project, the researcher selected three proportional reasoning problems from the reform mathematics materials being used in the class and an additional problem on ratio (see Appendix A). She then formulated open-ended type questions (see Appendix B) to be used as initial questions for the interviews. Subsequent questions were from the suggested questions or were formulated by the interviewers in response to students' answers and comments. The researcher and interviewers collected data as described in the following section.

The researcher and interviewers collected videotaped data of the four students as they estimated, calculated, and explained their solutions to proportional reasoning tasks. A four-stage interactive interview protocol was designed and used throughout the interviews: Stage 1—individual estimation interview, Stage 2—individual written response, Stage 3—explanation interview in dyads, Stage 4—simulated phone interview in dyads (see figure 1). Three interviewers and four students participated in the study.

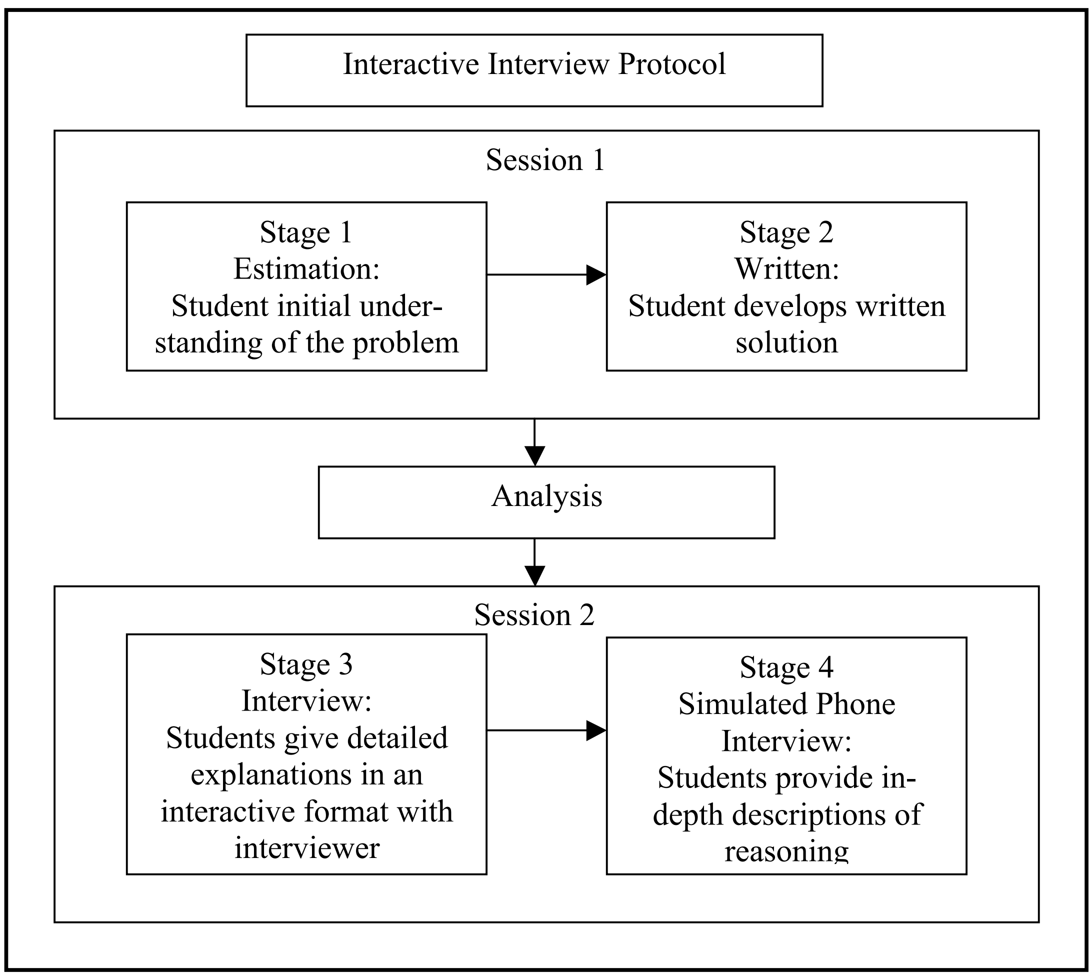

Figure 1: Interactive Interview Protocol

In the first stage of the interactive interview, the student was presented with a mathematics task written in English and was asked if she/he wanted it translated or explained in Spanish. After being read the task by an interviewer, the student was asked to estimate a solution without having the opportunity to utilize any tools (e.g., ruler, paper and pencil, calculator, etc.). This was done to encourage the student to use mental math to approximate an answer and to discourage them from simply calculating the exact answer by invoking an algorithm. Throughout this initial stage, the interviewer could ask clarifying questions based on the students' responses. In the second stage, the student went to a separate room where, working independently, she/he developed written solutions to all the tasks for which she/he had developed estimates for previously. In the third stage, the students were asked by an interviewer to first discuss the ways each of them had solved the problem with each other and if needed, help their partner make sense of the problem and its solution. The interviewer asked them to explain their processes and logic used to solve each task. The students were encouraged to write on a dry-erase board to demonstrate their mathematical thinking. The interviewer also asked probing or guiding questions, re-voiced students' explanations, and/or referenced aspects of the students' work. In the fourth and final stage, the students were asked to explain their mathematical thinking for each task to one of the other interviewers, one who was not in the room for the interview stage, in a simulated telephone interview. The interviewer could not view any of the students' written work during this stage because the goal was to motivate the student to provide rich descriptions of their mathematical reasoning used to solve the task. While the interviewer in stage 4 could not see what students wrote, the students often used the dry-erase board as a means to recall the process used to solve the task previously or used the board to develop a new solution. Similar to stage three, the interviewer could ask probing or guiding questions, and re-voice explanations. The interviewer additionally may have asked students to extend their thinking by providing a different way of solving the problem. The interviewers took turns in the role of interviewer and phone interviewer in stages 3 and 4 of the interview protocol.

During stages 3 and 4, students were allowed to review and reference their written solutions produced during stage 2. Students, at times, modified their earlier solutions as they interacted with each other and the interviewers.

Students were videotaped during stages 1, 3, and 4 and transcripts were created for each of the videotaped sessions. In addition to considering the student's reasoning to solve assigned tasks, the researcher analyzed each student's interactions with the interviewers during the explanation and phone simulation stages (stages 1, 3, and 4). The participants were described earlier in this paper; the following paragraphs describe the interviewers.

Vick, the director and a professor at a university, ran the school in conjunction with a board. In 2007-2008, he taught the math class for the students who were participating in the study. While Vick is a middle class, Anglo male, he communicates effectively in Spanish with students and their parents. He has worked with Latino/a populations throughout his career.

Lina was generally unfamiliar to the students; she entered the process as an unknown. Lina is a middle class female of Asian descent, whose first language is English. She has had extensive experience with Native American populations and has taught in preschool settings.

Laura, the researcher, had developed a relationship with the students in 2007-2008, but hadn't had much contact with them this school year. The relationship was that of a researcher and occasional teacher/tutor and caring adult. The researcher is a middle-class Anglo female whose first language is English with a rudimentary knowledge of Spanish. During her entire career, she has been involved with Latino/a students, communities and families. There were differences in the researchers' and participants' ethnicity and language, characteristics that warranted consideration in the data analysis.

The researcher was concerned that the differences in the interviewer and students' characteristics may have influenced the answers and interactions. When analyzing the transcripts, the researcher was unable to find any patterns in the transcripts or field notes that revealed what that influence might be.

The data was collected from February 9, 2009 to April 8, 2009 during 10 visits totaling approximately 25 hours, each lasting from 0.5 to 3 hours. During one visit, the researcher focused on collecting information on the site, its location, features and general layout. The researcher observed the participants and took fieldnotes in the outdoor play area and in the classroom during math lessons on two occasions. Seven sessions were spent interviewing and videotaping the students. Student pairs were videotaped from 45 minutes to an hour during each session.

Methods of analysis

Organization. The data was organized into general categories: interviewer questioning—probing and guiding questions and participant responses—short answer, extended, those indicating a willingness to persevere. The data being analyzed consisted of written work, the interactive interview, and the simulated phone interview.

Coding scheme. The coding scheme the researcher employed was that described in Corbin & Strauss (1998): open, axial and selective coding. She used an open coding approach to gain a general view of the properties or characteristics of the data. At this stage, the researcher fragmented the data by analyzing and categorizing each question and response from the videotapes. She then reassembled the data by identifying categories that related to each other. Finally, she employed selective coding to determine the central categories.

Categories and themes. There were two groups of people on which this analysis was based— the interviewers and the students. During the open coding phase, the researcher first analyzed interviewer questions, which resulted in the following categories: short answer questions, probing questions, and guiding questions. Additionally, there were instances of revoicing, encouraging, and verifying. The researcher then analyzed the student responses resulting in two categories—short answer and expanded answer. She also noted instances when students indicated they didn't know or didn't understand.

The researcher then employed axial coding to further refine the categories. Subcategories emerged under the probing question category as described in Figure 3, including open, reference, additional solution, what if, explain to peer and explain to interviewer. Additional interviewer comments included revoicing, encouraging, and verifying.

Revoicing was the repetition of a student's idea either using the student's words or words that encompassed the student's meaning but used the interviewer's words. Revoicing was implemented throughout the interviews as a way of confirming what students meant or as a locator identifying where they resided in the conversational path—describing what had been done so far and what still needed to be addressed in solving the problem. Encouraging comments included words and phrases that praised the students and encouraged them to continue their work and verifying statements confirmed their answer or trajectory being employed to find a solution. While revoicing and encouraging comments were part of the interactions, the part they played in the interviews will not be addressed in this paper.

Definitions of Question Types	
Short answer questions	Questions requiring a one word or short phrase answer
Probing questions	Asked students to clarify, expand, justify, add details, find alternate solutions
Open	Asked students to explain their work or thinking about a problem
Reference	Questions referred to something students said or did when attempting to solve the problem
Additional Solution	Asked students to solve the problem in a different way than their previously described solution
What if	Asked students what would happen if . . . -- the interviewer suggested a different number or procedure
Explain to peer	Asked students to explain their procedure to the other student
Explain to interviewer	Asked students to explain the solution to the interviewer as a means to check for understanding
Guiding questions	Interviewer suggested a strategy or way of approaching a problem

Figure 3: Definitions of Question Types

Next, the researcher analyzed student responses for commonalities. The student responses were expanded to four types, short answer, extended answer, exploration, and terminating as described in Figure 4. One question type, extended answer, was found to include two subcategories, alternate solution and expansion, also described in Figure 4.

Response Type	Characteristics
Short answer	One word or short phrase answer, label or operation
Extended answer	Extended explanations of student work or thinking
Alternate solution	Developed a different procedure for solving the problem
Expansion	Explanation goes beyond the right or wrong answer
Exploration	Exploration of the use of a procedure or idea to solve a problem, rejecting or accepting the results
Terminating	Expression of inability to understand or explain their thinking

Figure 4: Student Responses

Another category surfaced at this time—student to student communication. Three types of communication emerged from the data as described in Figure 5.

Type of interaction	Characteristics
Individual explanation	One student explains the processes and thinking they used to solve the problem
Collaboration	Students offer ideas and make suggestions to each other as they solve a problem together
Clarification	Students make comments to each other as they explain an agreed upon solution to the interviewer

Figure 5: Student to student interactions

Finally, the researcher used selective coding to refine and integrate the categories. The central categories that emerged were student-interviewer interactions, or student-student interactions. The student-interviewer interactions included: types of questions asked by the interviewer, types of answers given by the students, comments indicating a student didn't know what to do next. The student-student interactions included students' discussions with each other explaining how each of them solved the problem, comparing their solutions and socially constructing knowledge by interacting and sharing ideas and collaboration when explaining a solution.

INTERPRETATION OF THE DATA

Student to Interviewer Interactions

The interviewers used probing questions when initiating interviews resulting in extended explanations given by the students thereby shedding light on what the students knew and understood in relationship to proportional reasoning. Guiding questions were inserted when students were struggling and the interviewer thought the student had experience with and might be able to make connections to previously learned mathematics concepts helping them pursue a solution to the problem.

The following proportional reasoning task was given to the students to solve:

> Rita wants to estimate the number of beans in a large jar. She takes 100 beans and marks them and then she returns them into the jar and mixes them with the unmarked beans. She then gathers data by taking a sample of beans from the jar. Use her data to predict the number of beans in the jar.
>
> She got 2 marked beans and there were 30 beans in the sample.

In the following vignette, Vick, the interviewer, used probing questions to elicit student thinking, to go beyond the original answer. The responses demonstrated how Andres and Marisol explained their solution in a concrete way, not only determining an answer based on the ratio but explaining the concept of sampling as well.

M- I think she took out 30, and then, I think there's going to be 50 piles of 30 because she took out 30 and there was 2 marked so I think that's going to keep on happening.

Vick- Okay, for how long?

M- 'Til there's no more beans.

Vick- [to Andres] Do you understand what she's saying?

A- Kind of, not really.

Vick- So, like she takes out 30 and puts it down right, and then she takes out 30 again and puts them down and she keeps going for how long?

A- Until she gets to 1500.

In an attempt to elicit how the students were making sense of the sampling, the interviewer asked probing questions referencing their explanations as follows:

Vick- So would there always be 2 marked beans in each pile of 30?

M- Well, that's my guess, because if it happens on the first pile I think its going to happen on the second.

Vick- Always the same, [to Andres] what do you think of that?

A- No.

Vick- Why not?

A- Well, I'm not sure

Vick- So maybe we need to do this experiment and see if it works, we take out 30 at a time and you're saying there is always going to be two marked beans in that--why?

M- There is a chance like 2 in 30.

Vick- Do you think there would be exactly 1500 beans in the jar?

M- Probably.

A- Yeah, no, I don't know.

Vick- How come, what do you think?

A- I don't know.

Vick- So let's think about it for a minute so you just keep pulling out 30 in your way you just keep pulling out 30 and pretty much every time 2 will be marked will there be exactly 1500 in the end?

M- Not exactly, but close.

Vick- Why do you say that?

A- I think it's just an estimate

Vick- Why?

A- Because I don't think that every time we take out 30 beans from the jar that we will find 2 that are marked.

As he envisioned the piles of beans being made from those being removed from the jar, Andres used his logic when he said he did not think there would be exactly 1500 beans in the jar. When asked for an explanation, he persevered after saying "I don't know" twice; he explained it was just an estimate because they would probably not get exactly 2 every time they made a pile.

This vignette illustrates three important aspects of the findings. First, after solving the problem, students described the meaning of their calculations with a concrete description. Second, they used their logic to figure out that the results were only an estimate. Third, this vignette demonstrates Andres willingness to persevere, pushing past "I don't know" to explaining that the samples won't necessarily be the same every time. The next finding focuses on student to student interactions.

Student to student interactions

During the interview process, students were encouraged to discuss their solutions, explain their thinking to each other and socially construct an explanation of their agreed upon solution. During the interviews, each student would explain their written solution to the other, discuss differences in their solutions and then agree on a solution.

In a student to student interaction, with the author as the interviewer, Veronica and Marisol, an above average and below average performer in mathematics, respectively, were solving the following problem: A box of cereal cost $2.50 and contains 680 grams of cereal and then a larger box of the same cereal cost $3.50 and contains 900 grams, which is the better value? Each student solved it in a different way as detailed in the following vignette.

V- I did grams per dollar and then you divide that [680 ÷ 2.50] and then you get 272 grams per dollar and then for the other one, [900 ÷ 3.50] was 257 per dollar so this one was the better buy [pointing to the 2.50].

La - Could you explain your thinking to Marisol and how you set it up?

V- [To Marisol] So I was thinking how many grams that was in each box per dollar so I just put the grams with the grams and the dollars with the dollars I just divided and I got it.

[Veronica wrote the following equation on the white board:]

$$\frac{680\ grams}{2.50\ dollars} = 272\ grams/dollar \quad \frac{900\ grams}{3.50\ dollars} = 257\ grams/dollar$$

La – Okay, Marisol, do you have any . . . ?

M- So this is the better value [pointing to the 272 grams].

V- Yeah because this one has 272 grams.

La - So why do you think the 272 vs. the 257 is the better value?

V- Because you get more than here. [pointing to the 257].

To further delve into the students understanding of this problem, the interviewer asked the participants to find another way to solve this problem:

La – Okay, you really got a handle on this, is there any other way to figure this out?

V- [To Marisol] Did you do it my way?

M- Yeah, but I got a different answer.

At this point, Marisol had agreed with Veronica's answer, had indicated that she had found the solution in the same way, yet, as she explained her method, her approach and answer were different. As she explained her thinking, she realized that part of her process was not logical.

La - How were you thinking about it?

M- Well, I got the 2.50 and then and I added 1 dollar which is 3.50 so they are both the same cost and so I am going to do the same to grams and I am going to take out the 100 because that's what I added here so its 780 and so they have the same cost and in the other one is 900 grams for 3.50 and right here is 780 grams for 3.50 so this one was the better value [pointing to the 900 grams for $3.50 on the worksheet].

[Marisol wrote the following equations on the white board:]

$$2.50 + 1.00 = 3.50 \qquad 680 + 100 = 780$$

La - You got that one? [Pointing to the reference to the box with 900 grams]

M- Yeah.

Because the students' answers were different, the interviewer made the following comment:

La - For the two of you something was kind of a disconnect as you were going through this. Can you figure out where that broke down, that path?

M- I think it was right here [pointing to the 100 in the 680 + 100 = 780].

La – Good.

Vick- [who was running the video camera]- How did it break down?

M- I wasn't sure what to add here so I just guessed 100 because I just took out the decimal. [Pointing to the 1.00 she had added to the 2.50]

La – Well, if you label these different things, because there's some merit to what you're thinking about – okay, so you decided there is something wrong with that 100 there, so is there any other way that might have worked if you were adding the money, what do you think -----do you think there is some way for this to work- so you have the 680 grams and then you have 900 grams

V- [To Marisol] You can add, if you added 1 dollar you could find the grams per dollar from that.

La - So you would then, you said how many grams per dollar.

M- Oh yeah I get it now, yeah because like if I added one dollar here it was like [to Veronica] how many did you say grams per dollar.

V- It was 272.

M-So like if I added one more here, because 2.50 equals 680 grams so one dollar equals 272 grams, so I should have added those two and not 100. [On the white board, M writes 680 + 272 = 952]

La – Okay.

M- So, yeah, this is the better value [pointing to the 2.50].

This exchange between the interviewer and students illustrates how a guiding question can help a student identify that her reasoning was faulty giving her the opportunity to make needed changes with the help of another student. Other comments made by the interviewer consisted of predominately revoicing and encouraging utterances. While the interviewer pointed out that Marisol's approach had some merit and that there was a disconnect, Marisol identified adding 100 grams to 680 as the error, then it was Veronica that suggested using the grams per dollar amount to arrive at a solution. From the socio-cultural perspective, this demonstrates how two students would be viewed as making meaning by communicating their ideas to collaboratively come to a consensus on solutions to a problem.

CONCLUSIONS

The study demonstrated that probing questions used in the interactive interview protocol proved to be valuable in the formative assessment format to "gather evidence of (a student's) knowledge and then infer what the student knows" (Wilson & Keeney, 2003, p. 53). Three types of probing questions tended to provoke extended answers that expanded on the original responses given on the written portion of the interview protocol. They included those referencing student work, those asking students to develop another way to solve the problem, and those asking the student to explain their thinking to each other.

The first type, probing questions that referenced student work, resulted in students revisiting their solutions in search of extended meanings. For example, after having solved the ratio problem, Zenia and Marisol responded to probing questions resulting in relating their answer to the concept of sampling techniques. In a problem where students were measuring figures with two different sizes of paper clips (see appendix), Zenia described her solution, "Because we still don't know what it is in little paper clips but we know the big paperclips and so I just divided 6 by 4 and it gave me 1.5 and I multiplied 1.5 times 6 which gave me 9." When the interviewer asked a probing question referencing the explanation, "Why did you multiply 1.5 by 6?" the student responded, "That's like the scale factor so it needs to be equivalent and the scale factor that 1.5 gave me." In this instance, the student went beyond determining the height of the figure in paper clips to describing the ratio using mathematical language, "scale factor". These two examples show the power of the questions to move students beyond a basic solution to an in-depth understanding of the math and how it is applied to solve real world problems.

The second type of probing question, asking students to develop an alternate way of solving problems, also extended student understanding of proportional reasoning concepts. The effect of this type of question was to promote further inquiry into possible ways of understanding the elements of the question and manipulate them to reveal different numerical answers that still enabled the students to determine the answer. Marisol and Veronica developed different ways of determining the values of the boxes of cereal, one method used an algorithm while the other combined logic with information from the algorithmic solution. While the first method determined an accurate answer, the second method made more sense to Marisol. Veronica solved that same problem in a third way demonstrating that dollars per gram would also provide accurate information on which to determine the better value. Being able to make sense of a problem in multiple ways shows an in-depth understanding of the underlying concepts.

The third type of probing question encouraged communication between peers moving the interaction from primarily student—interviewer, to student—student. Probing questions that asked students to explain their thinking to their peers resulted in socially constructed responses (Shepard, 2000). When Marisol and Veronica were asked to develop an alternate solution, and to explain their thinking to each other, they combined what they knew, identified the error and solved the problem. When Andres and Marisol were solving the sampling problem, there was some confusion as to whether the answer would be exactly 1500. By having Marisol describe a concrete representation of what they had done, Andres realized that not every pile would necessarily have 2 of the marked beans, and therefore, the answer was just an estimate.

Students generally showed a willingness to stick with a problem until it was solved. The response, “I don’t know,” appeared in several of the interviews. In most cases, after declaring that they did not know, the students went on to figure out the answer. Andres said he didn’t know three times when solving the bean problem. Veronica said she didn’t know why she divided by 1.5 when measuring figures with paper clips and then she explained that 1.5 little paper clips equal one big one. Evidence that students were willing to persist, and thereby pursue a deeper level of understanding, after finding a solution was apparent as they gave detailed explanations of their answers as interviewers attempted to determine their level of understanding.

Employing probing questions in the interactive interview as a formative assessment method with ELL students informed interviewers what students knew and could do in solving proportional reasoning problems. ELLs were able to solve complex problems when positioned as competent (Turner,Celedón-Pattichis, Marshall, 2008) and were able to express what they knew in a non-threatening setting. When students were encouraged to think aloud and interact with either a peer or knowledgeable other in response to probing questions, new knowledge and understandings were formulated (Vygotsky, 1978). Because assessment is part of the educational landscape with high stakes attached to the results, developing accurate assessments for ELLs and using the information to address their needs will enable ELLs to increase their level of performance in mathematics. This will then position these students to participate in advanced mathematics courses, opportunities not afforded them in that past due to the low level of cognitive demand in many of their mathematics courses (Gutierrez, 2007).

Interactive interviews enabled students to use gestures, drawings and phrases, allowing them to reveal what they knew without having to use specialized math vocabulary (Moschkovich, 2007). In some cases, however, this assessment method supported students in a way that resulted in their moving from a simple explanation of the procedure used to one of using specific mathematical terms. Expansion of the original explanation was motivated by the use of probing questions.

From the socio-cultural perspective, students interacted with interviewers and other students as they made sense of the mathematics. Vygotsky (1978) noted the importance of analyzing the process, not just the outcome of activities. The interactive interview uncovered the process where students communicated with each other, asked for explanations, shared their understanding and collaborated to make sense of the mathematics going beyond the right or wrong answer to one of revealing their understanding of the multiple aspects of the tasks. Students were not taught an algorithm and asked to practice that algorithm until memorized, a method used in an expository classroom. Instead, a constructivist approach was used whereby students learned mathematics with understanding.

RECOMMENDATIONS

Assessment of ELLs must undergo change from current methods that disadvantage them to methods that ensure students' mathematical knowledge is accurately represented. The information acquired needs to be used to plan instruction that meets ELLs needs and prepare them for the challenges of advanced mathematics. Without such changes, the dire consequences of becoming a divided nation with Hispanics and African-Americans as a mathematically semiliterate majority, will come to pass.

Because current assessments are not effective, further research into assessment options for ELLs coupled with resources to implement those identified methods is needed to change the current course of mathematics assessment. For example, the interactive interview, while effective, is time intensive and requires additional resources to enable teachers to conduct and analyze interviews. A commitment must be made to pursue a research agenda to ensure accurate assessments are discovered, developed and implemented, thereby providing formerly disadvantaged ELLs an opportunity for a high quality, challenging mathematics programs and a chance to participate fully in today's society.

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Appendix A

Proportional reasoning problems used in the research

Find solutions to the following problems. Give a detailed explanation of your reasoning used in finding the solution.

1. Participation in Team Sports at XYZ Middle School

Sport	**Girls**	**Boys**
Basketball	30	80
Football	10	60
Soccer	120	85
Total Surveyed	**160**	**225**

The participation in these team sports is about the same for students at Key Middle School.

a. Suppose 250 boys play sports. How many would you expect to play each of the three sports?
b. Suppose 240 girls at Key play sports. How many would you expect to play each of the three sports?

2. Rita wants to estimate the number of beans in a large jar. She takes out 100 beans and marks them. Then she returns them to the jar and mixes them with the unmarked beans. She then gathers some data by taking a sample of beans from the jar. Use her data to predict the number of beans in the jar.

Sample
Number of marked beans: 2
Beans in sample: 30

3. A box of cereal costs $2.50 and contains 680 grams and a larger box of the same cereal costs $3.50 and contains 900 grams. Which is the better value?

4. What strategies can you use to solve proportions such as $5/8 = 12/x$ and $5/8 = x/24$?

5. Here is a picture of Mr. Short (A stick figure that is drawn on a sheet of paper is given to the students.) Mr. Short's height in little paper clips is 6 little paper clips tall. What is Mr. Short's height in big paper clips?
6. Mr. Short has a friend names Ms. Tall. Ms. Tall's height in big paper clips is 6 big paper clips tall. What would Ms. Tall's height be in little paper clips?

7. Ms. Tall's car is 15 little paper clips long. How long is her car if we measure it in big paper clips?

Appendix B

Questions used by researchers.

My research involves having students solve proportional reasoning mathematical problems. The interview questions will begin with general, open-ended questions followed by questions based on the students' answers.

1. Will you read the problem and explain what is being asked in your own words?
2. What strategies did you use to solve the problem?
3. How did you decide to do that?
4. How does your solution answer the question?
5. Why does your solution make sense?
6. Could you solve the problem in a different way? How?

BEYOND MULTIPLE CHOICE: TASK-BASED INTERVIEWS WITH MEXICAN-AMERICAN STUDENTS ON NAEP ASSESSMENT ITEMS

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This study explores the thinking of a group of Mexican-American students when solving NAEP measurement problems via task-based interviews. Using a combination of sociocultural and cognitive perspectives we probed the ways students represented and communicated their mathematical ideas. We found that the linguistic complexity of a problem influenced the strategies employed by students. In both correct and incorrect responses, we found that most students were able to verbally communicate their thinking process.

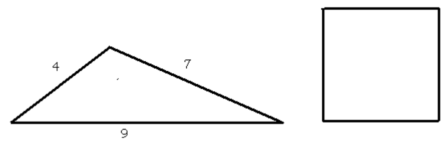
Large scale summative assessments like the National Assessment of Educational Progress (NAEP) are used to certify what students know and can do in a given content area. These assessments contrast with formative assessments that act as a bridge between teaching and learning and provide the teacher with crucial information that can be fed back into teaching (Wiliam, 2007). Even though summative and formative assessments are employed for different purposes, we will illustrate how NAEP items can be used to extract information about English Language Learning (ELL) students, which can then be used for formative purposes. The purpose of this study is to illustrate that items from the NAEP could be used to get a better understanding of issues of language and mathematics that impact Latino[1] students, who comprise the majority of ELLs. In particular, the goals of this study are to a) gain an understanding of how a group of 15 Mexican-American students approached selected NAEP measurement items and; b) uncover some of the challenges that these items presented this group of students.

Lubienski (2003) noted that the largest achievement gaps between White and Hispanic scores at the 8th grade (in NAEP 2000) were in the area of measurement. We conjectured that these gaps in performance would point to certain test items that seemed more challenging to Latino students; probing their understanding on these items would uncover some unintended conceptions and ways of thinking.

In this article we focus on what we learned from interviews with 15 Mexican-American students – drawn from grades 4 through 6 in schools serving working class communities – as they solved two NAEP measurement tasks. These items were a perimeter problem and an area comparison problem, both from the fourth grade 1996 NAEP (see Fig. 1). We chose these interview questions since they were the ones that yielded big differences in the percent correct when comparing White and Hispanic scores in NAEP. Note that our intention is not to explain the underperformance of Hispanic students on these two items, but to uncover the potential of these questions for formative assessment.

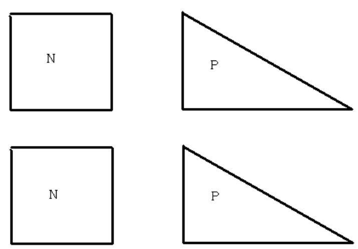
Figure 1: The NAEP problems

The Perimeter Problem



If both the square and the triangle above have the same perimeter, what is the length of the side of the square?
(a) 4
(b) 5
(c) 6
(d) 7

The Area Comparison Problem



(Cut outs of N and P were provided with the base of P being twice the side of the square) Bob, Carmen and Tyler were comparing the areas of N and P. They each conclude the following:

Bob:	N and P have the same area;
Carmen:	The area of N is larger;
Tyler:	The area of P is larger.

Who was correct? Use pictures and words to explain why.

The Area Comparison problem has been discussed by other researchers. Lubienski (2003) highlighted the Area Comparison problem as being illustrative of gaps in performance between White and Hispanic students as seen on other NAEP multi-step problems. She conjectured that these gaps revealed a lack of opportunity for Hispanic students to solve multi-step problems, which teachers could redress by providing these students with the appropriate opportunities. Strutchens, Martin, and Kenney (2003) also looked closely at the students' strategies in the Area Comparison problem as part of a larger analysis of student performance on the NAEP measurement strand and concluded that the students had different levels of understanding of area. Students with a conceptual understanding of area could discover the relationships between the sides of the triangle and square and solve the problem flexibly without recourse to the use of area formulas. On the other hand, they did not rule out the possibility that the manipulation of the shapes may have been more efficient for the students who found it difficult to remember the formulas. Based on their overall analysis, they recommend teachers to introduce the concept of area through informal activities that involved area comparison of shapes without the presence of specific numerical measurements.

THEORETICAL PERSPECTIVES

This study is part of the research agenda of the Center for the Mathematics Education of Latinos/as (CEMELA), which aims to understand the interplay of mathematics, language, and culture among Latino[1] students. Our perspective is essentially a combination of a sociocultural perspective and a cognitive perspective (Brenner, 1998; Civil, 2006; Cobb & Yackel, 1996). Our student interviews were cognitively-based, and our analysis of these interviews was guided by a sociocultural perspective of mathematics cognition and language (Moschkovich, 2002, 2007b) with an emphasis on students' communication *about,* and *in* mathematics (Brenner, 1994).

Communication was central to our study in that we were investigating students' use of language as they interpreted the tasks and explained their thinking, even though this is not the intention of NAEP tasks; in some cases, our students were bilingual (English and Spanish), but more proficient in one of their two languages. Moschkovich (2002) points out that communication is multifaceted involving gestures, expressions, drawings, and objects as resources to simultaneously communicate mathematical ideas. These resources are especially crucial for students who may be less proficient in English, but are being educated in the U.S. in an English-only instruction classroom, even though the students may have been able to use Spanish as a resource.

Brenner's (1994) Communication Framework for Mathematics advocates for students to actively participate in mathematical discourse during classroom discussions. The framework provides categorization of three different kinds of discourse, namely: (a) communication *about* mathematics, which entails the need for description of problem solving processes and their own thoughts about these processes; (b) mathematical communication *in* mathematics, which entails using the language and symbols of mathematical conventions; and (c) communication *with* mathematics, which refers to the uses of mathematics that empower students by enabling them to deal with meaningful problems. Brenner emphasizes that all three kinds of mathematical communication are needed in the classroom for developing useful mathematical understanding. The third kind, communication *with* mathematics, was not part of our coding of the data since our focus was on NAEP assessment items, which are overall context-free and carried little meaning for the students.

METHODS

We conducted and videotaped task-based interviews (Goldin, 2000) with the 15 Mexican-American students in predominantly working class neighborhoods and each interview lasted approximately 30 minutes. The students first solved the problems independently and then explained their thinking. We asked probing questions based on their responses and our interview script. For example, if a student successfully solved the perimeter problem we asked clarifying questions about their calculations like "Why did you divide 20 by 5?" or "What do you know about a square?" On the other hand, if a student was unsuccessful with the same problem, we tried to uncover their understanding of the language and concept by asking questions like "Do you know what the word perimeter means?" or "What do you know about a square?" In some cases, the students' interactions with the researcher prompted them to revise their initial solution.

DATA ANALYSIS

Initially, we individually reviewed the videotapes of the students solving the two problems, and then reviewed and discussed key segments together to reach agreement on interpretation of interactions. We summarized the students' interactions, based on the agreed upon interpretations, by using key words and short descriptions in a table format. The organization of the data allowed for a holistic view of themes that we found across the problems and students.

Our first task was to categorize the solutions as being correct or incorrect based on the students' explanations. In doing this, we ignored careless mistakes (e.g. choosing an incorrect answer in the multiple choice question only if the student had verbally given a correct explanation). A solution was considered incorrect if the student had the correct answer but could not back it up with a correct explanation. After classifying each solution attempt as correct or incorrect, we analyzed the videos closely for issues of language and communication.

We made a note of instances when the language in the problem was not clear to the student. For example, one cue was when students read the problem slowly and did so multiple times to understand it. These instances were coded as *linguistic complexity.* In coding for *linguistic complexity,* we focused attention on the students' comprehension of the problem. Students' communication of their thinking was noted -- coded either as *area and perimeter* or *visual* -- as we tracked patterns across students and across problems. For example, the code *area and perimeter* pointed to instances when the students confused either one or both of these concepts in their solution and later in their communication of this solution. The *visual* code focused on the students' use of grids and visual approximations of lengths in their explanations of their thought processes. These codes informed us about the students' communication *about* mathematics (Brenner, 1994).

In tracking the student's communication *in* mathematics, we paid attention to the use of definitions like perimeter, area, square, triangle, height, width etc., their conception of geometric figures, and their translations between representations and noted these instances in the table. We also looked at the instances when the students participated in a mathematical discussion. We associated the code *discourse* if the students made sense of the mathematical arguments and statements that were being discussed and tried to communicate their ideas. Another code, *connections,* captured the fluidity of translation between representations (physical, verbal and symbolic) displayed by the students.

Once the data were coded, we individually cycled between the video clips and the codes to ensure that our coding was accurate. Further, all the researchers were in agreement over the data analysis and the themes that emerged.

RESULTS

We first give a brief overview of the students' independent solutions on the two problems, focusing primarily on the difficulties students had with these problems; again, our emphasis is not on what students could not do, but in learning from these students' approaches to the problems to inform assessment and instruction (see Fernandes, Anhalt, & Civil, 2009). After discussing the students' performance on the problems, we discuss our findings in terms of linguistic complexity, communication *about* mathematics, and communication *in* mathematics.

About the Perimeter Problem

Six of the fifteen students solved the Perimeter Problem correctly. All of the nine students who gave the incorrect response displayed, at some point in the interview, that they did not have a clear understanding of the question. Among these nine students, seven relied on what we refer to as a visual approach. The visual approach consisted of either "seeing" that the side of the triangle with length 4 was about the same as the side of the square or creating an arbitrary grid to estimate the perimeter by counting the number of squares on the border. The other two of the nine students did not choose an answer, and said that they could not understand the question. In one of these cases, providing a context to the problem actually helped the student to solve it. When asked about the meaning of perimeter, six of the nine students could explain the meaning of perimeter, one student could not, and two had a wrong concept of perimeter (e.g., one student described the perimeter as the number of sides of a polygon).

Three students among the nine who gave the incorrect response had difficulty connecting the two pieces of information in the problem, namely that the perimeter of the square was 20 and that a square had sides of equal length. These students put arbitrary numbers that added up to 20 as the sides of the square when asked about the length of its side. On the other hand, when queried about the properties of a square, some of them mentioned that the sides had to be of equal length. Two 6th grade students confused area and perimeter. These students tried to enclose the triangle in a rectangle and attempted to compare the area of this rectangle to the area of the square. One of the students mentioned that he was recalling a procedure taught by his teacher and was not sure if it was for the area or the perimeter. He eventually realized that he had the perimeter and area mixed up.

The six students who arrived at the correct solution appeared to be comfortable with both conceptual and procedural understanding needed to answer this question. They knew the concept of perimeter and how to apply it to the problem, and were also proficient with the arithmetic operations required in the task.

About the Area Comparison Problem

Eight of the fifteen students solved the problem correctly. In five of the seven incorrect responses, students used visual differences to draw conclusions. One of the five students mentioned that the square had a larger area since it had four sides as opposed to three of the triangle. A second student relied on grids and counted the squares, a third student conjectured that the triangle had a larger area since there was a part of the triangle that was sticking out after placing the shapes on top of each other. A fourth student assumed that the triangle was larger since the rectangle formed by the two triangles had sides that were larger to those of the square. The fifth student concluded that the square was larger since there was a part of the square that did not overlap with the triangle when they were superimposed. The confusion between perimeter and area accounted for the incorrect response of a sixth grade student who moved the square around the triangle and compared the perimeters. Finally, one sixth grader reasoned that the area of the triangle had to be less than that of the square since there was a factor of half in the formula (1/2 x base x height as opposed to base x height).

Of the eight students who successfully solved this problem, in most cases, they compared one triangle and one square and used the area preservation property to work through the problem as shown below (Fig. 2). The arrow indicated a "cut" and "paste" operation, in which part of the triangle was cut and arranged in the dark area. This operation helped the students draw the conclusion that the areas of the triangle and the square were the same. Four of the eight students came up with a method that used the pair of triangles and squares. These students observed that by placing the two triangles together and the two squares together, the two newly formed rectangles were equal in area, and each triangle and square represented half the area of the respective rectangles (Fig. 3).

Figure 2: The "cut" and "paste" strategy in the Area Comparison Problem

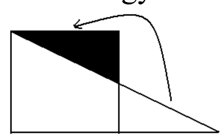
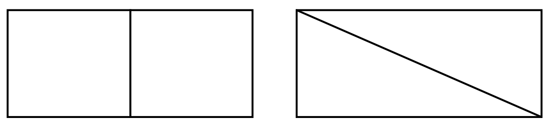


Figure 3: Strategy involving all four cut-outs in the Area Comparison Problem



Linguistic Complexity

We observed two forms of linguistic complexity, one with regard to reading and comprehending the test items, and the other with regard to the written answers. Note that the students were expected to write an explanation only for the Area Comparison Problem (as per NAEP instructions). The first form of linguistic complexity that we observed was in phrasing of the Perimeter Problem. The problem read, "If both the square and the triangle above have the same perimeter, what is the length of each side of the square?" One of the fourth grade students interpreted the "if" statement as "they do not have the same perimeter." When probed, she said "but they do not because it says IF (emphasis added)." This child was interpreting the "if" statement as a negation statement, therefore, the square and the triangle could not possibly have the same perimeter. Her facial expression indicated that she was faced with conflict and was not able to engage with the mathematics as the problem intended.

Student: [Reads the problem] If both the square and the triangle above have the same perimeter, what is the length of each side of the square?
Interviewer: So does that triangle and that square, do they have the same perimeter? (pause)
Student: No.
Interviewer: And what does the problem say? The first part.
Student: If both the square and the triangle above have the same perimeter...
Interviewer: Okay, stop. What does that mean? What are they telling you?
Student: That if both of them have the same perimeter, what is the length of each side?
Interviewer: So, are they supposed to have the same perimeter?
Student: Well, yeah.

Interviewer: According to the question, are they telling you that they have the same perimeter? [Student shakes head 'no'] No, how is it? Read that part again.
Student: If both the square and the triangle above have the same perimeter,…
Interviewer: Okay, stop right there, if both triangle and the square have the same perimeter, are they telling you that they have the same perimeter?
Student: [shakes head no]
Interviewer: No, how come they're not?
Student: Because they're saying, "IF both."
Interviewer: What does that mean?
Student: That means that…like…if…if both of them have the same perimeter.
Interviewer: So are they saying, "If both of them, meaning they don't have it, but if they did," that's how you interpret the question that they're asking?
Student: [nods yes]

By the student's interpretation of the language used in this problem, it was difficult to assess her mathematical understanding of two shapes having the same perimeter. In this case, the interviewer decided to drop the word "if" and rephrased the question to, "The square and the triangle above have the same perimeter," and then asked the student to proceed with the problem. However, this same student was unable to solve the problem as she continued because she made an assumption that the square had "four sides" and not four *equal* sides. Thus in her discussion of her solution, she noted that the sides of the square were 1,3,7,9. Our overall conclusion was that this student's difficulties with this problem involved both linguistic aspects and mathematical concepts. We contrast this episode to our next example of a sixth grade student who solved the same problem.

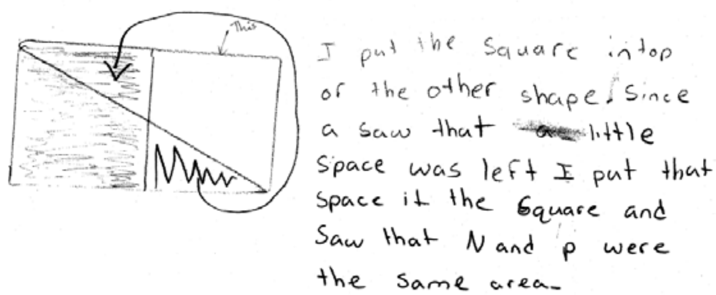
The sixth grade student first added the sides of the triangle to get 20 and then mentioned that she did not understand the question. After the interviewer explained the question to her, she was able to figure out the side of the square mentally.

Student: [First she adds the lengths of the sides of the triangle and gets 20] I don't understand it.
Interviewer: Okay, do you want me to read it or do you want to read it aloud? [the student reads the question]. Do you know what the word perimeter means?
Student: The outside of umm [points to shapes]
Interviewer: Okay, and so they give you this triangle and give you the measurements of the sides of this triangle and they give you a square and they don't give you the sides. So they are telling you that "If the square and the triangle have the same (emphasis) perimeter what is the length of the sides of this square?"
Student: Five!
Interviewer: How'd you do that?
Student: This together is 20 [points to the triangle and the work she did before] and this has 4 sides [pointing to the square] and 5 times 4 is 20.

In this case it seems that the student had not seen the relevance of the word "same" in "same perimeter." By emphasizing the word "same", the interviewer assisted the student in comprehending the problem, and after that she could successfully solve the problem. We wonder if the student's difficulty in the problem was not noticing the word "same" or again an issue with the "if-then" statement.

Overall, the students found the Perimeter Problem more linguistically challenging than the Area Comparison Problem. For this second problem, the main linguistic complexity occurred when the students were asked to give a written explanation for their work. They were able to explain their work orally but, in general, had difficulties providing a written explanation. Although this may be the case for many students, we argue that this is more likely with ELL students who in communicating orally were able to use (and did use) gestures and were able to interact with the interviewer about the mathematics. Moschkovich (2007a) also found that ELL students were better at orally expressing their ideas that were not mathematically precise in writing. Only one fifth grade student provided a complete explanation in words and a diagram to go along with her verbal interactions with the researcher (Fig. 4). She wrote, "I put the square on top of the other shape. Since [I] saw that one little space was left I put that space in the square and saw that N and P were the same area."

Figure 4: An example of a clear written solution of the Area Comparison Problem



Communication About Mathematics

According to Brenner (1994), communication *about* mathematics, entails the need for the students to describe the problem solving processes and their own thoughts about these processes. The majority of the students in the study were able to describe the process when asked to explain their thinking, even if their reasoning was not complete. There were a few cases in which the students gave a response, but could not provide their reasoning, but most were able to convey their ideas either on their own or with some probing.

In communicating their solution, some students confused area and perimeter conceptually in their explanations. For example in the Area Comparison Problem, a fifth grade student concluded that the area of the triangle was larger than the square. On being asked to explain her thinking, she rotated the square cut-out around the triangular cut-out, thus comparing perimeters instead of areas. On being asked about area and perimeter, the student said that the perimeter "was the outside of the shape" and the area was "the inside of the shape." However, she was confused when trying to use these ideas in solving the problem.

The students who were successful in explaining their solutions and reasoning were able to make connections during their discussions *about* mathematics. Two successful students, on the Area Comparison Problem, reasoned that the area of the triangle and the square were the same by assigning numbers to the unknown lengths and areas in a way that accurately reflected the relationships. One of these students (a sixth grader) assumed that the rectangles formed by the

two triangles and the two squares had an area of 100 and so one triangle and one square each had an area of 50.
The unsuccessful students usually concluded that the two rectangles formed with the two squares and the two triangles had equal area but were unable to use this to conclude that the individual triangle and square had equal areas. It was this latter point that the sixth-grader communicated so elegantly through his use of the numbers 100 and 50.

Communication In Mathematics

Communication *in* mathematics (Brenner, 1994) referred to the student's proper use of language and symbols of mathematical conventions. Students who were successful with the Perimeter and Area Comparison Problems displayed competent communication *in* mathematics. These students could calculate the perimeter, work out the side of the square, either using division or addition strategies. Further, in the Area Comparison Problem, the students understood the concept of area preservation and were adept at showing that the square and the triangle had the same area by cutting and rearranging either a portion of the square or the triangle.

We found that the students who were successful in solving these problems were fluid in their translations between representations. For example, a successful sixth grade student simultaneously represented his manipulations of the concrete shapes in the Area Comparison Problem with the equation 2P=2N ("P" represented the triangle and "N" represented the square). He reasoned that if 2P=2N, then half of each rectangle is the triangle P and the square N, therefore, P=N and the areas were equal

Proper mathematical communication was displayed in the interactions with a sixth grade student who successfully solved the Perimeter Problem. The student was very clear in the process that he used to arrive at the solution and could justify his steps when asked by the researcher. Further, the student knew the mathematical terms that were part of the problem like perimeter, square etc. He was also proficient in translating between the various representations of diagrams, verbal, and symbolic representations. This was representative of the conversations between other successful students and the interviewer. Here this sixth grade student was being queried about his method in solving the Perimeter Problem.

Student; [chooses the '4' as his solution]
Interviewer: Okay, do you want to explain your thinking?
Student: Ahh, I added, I added, uhh 4, 7 and 9, I got 20 and then I divided 20 by 4 and I got (hesitates), I got ...oh I messed up [erases his choice of 4]
Interviewer: Oh, you are changing your answer?
Student: Yeah, I messed up on this one.
Interviewer: Why did you change it? What happened?
Student: Because its 20, the, the [indicates all around the square] the (unclear) whole perimeter of the square is 20, so I know, umm...there are four, umm…divided by 20 equals 5. So it's 5 for each side.
Interviewer: Okay, tell me about the math sentence that you just mentioned…umm…you added this and got 20 [points to the triangle] and then tell me what you did over here?
Student: I, I, umm, divided 20 by 4.
Interviewer: Oh, okay, and 20 divided by 4…
Student: Equals 5.

Interviewer: Equals 5. Okay, I see what you did. Initially, when you got the 4, what were you thinking? Did you just …
Student: Divided by 5 instead of 4…I messed up.
Interviewer: Oh, okay...and how did you know to divide by 4?
Student: Well, because I know that 4 times 5 equals 20.
Interviewer: Okay,…umm, how did you know to take the 20 and divide it by 4?
Student: Because there's 4 sides.
Interviewer: Oh, what do you know about the sides of a square?
Student: It has 4…4 equal sides.
Interviewer: Ahh…equal, okay, I see your thinking.

The student showed a good understanding of participating in mathematical communication. He knew how to go about solving the problem and explains the process to the interviewer. He was also able to recognize and rectify his error independent of the interviewer. The student understood the concept of perimeter and made proper use of the mathematical terminology.

DISCUSSION

Examining the overall results from the interviews, the linguistic complexity was a challenge that seemed to be prominent with this group of Latino students, especially in the case of the Perimeter Problem. These students' struggle with the hypothetical assumption involved with the 'if' in the statement of the Perimeter Problem is also mirrored by Fillimore (2007) who described similar struggles of the students with the word 'suppose' in the sentence "For example, suppose you are randomly choosing marbles one after another,…" (p. 340). Fillimore pointed out that in some cases these students were categorized as English 'proficient' and yet struggled with the academic discourse that was needed. We agree that hypothetical assumptions such as "if-then" and "suppose" may be difficult for native speakers of English, and we argue that these may even be more difficult for ELLs or, at the very least, should be an area for teachers and assessment developers to give serious consideration.

Our findings show that most students were able to communicate *about* and *in* mathematics. In some cases, the students provided an incorrect solution, but they could explain their thought process, and in doing so, found the error in their initial solution. Although we saw a number of cases where the students could verbally express their thinking, it was much more difficult to convey their thinking in writing. Our findings underscore the need to provide students with multiple opportunities to express their thinking in writing and to reinforce their learning of academic English; though this is necessary for all students, it becomes particularly important for ELL students.

We conjecture that the linguistic difficulties that the students faced in understanding the questions impacted their approach to the problem. Students who did not understand the question completely relied on other resources like visual cues, concrete objects, past classroom activities, and partial information from the question. Although these strategies may also be observed in non-ELL students, we posit that ELL students in English-only classrooms take a longer time to understand all the subtleties of mathematical experiences given that they are grappling with the language and learning concepts at the same time (also expressed by Chamot & O' Malley, 1994).

In Closing

This study gives a closer look at the thinking of Latino students, which is often masked by paper-and-pencil tests. For example, in our study, the linguistic complexity of the question interfered with students' mathematical thinking, and the nature of this interference was only clear after interacting with the students. Further, paper-and-pencil tests rely solely on the written work of the students (in the case of constructed responses), however we have seen that the students' written responses were difficult to understand, and therefore, needed verbal explanations. In our interactions with the students, we observed that it was important to consider the students' use of language resources to explain their thinking, and thus it is important to broaden our conception of competency in mathematics. The use of resources has also been discussed by Moschkovich (2002, 2007) and Radford, Bardini, and Sabena (2007), and they are in agreement that it is not possible to capture the students' mathematical thinking only by examining their written work.

We conjecture that interviewing students with NAEP mathematical tasks that show large gaps in performances of different groups by ethnicities could be used to draw out interesting thinking from the students. This information could improve instruction and has the potential for promoting equity for all students in terms of reaching more students of diverse linguistic backgrounds. The interviewing process and the newly gained perspectives have heightened our understanding of student thinking, especially in the case of ELL and multilingual students. Improvement in instruction is more likely to happen with increased teacher understanding of students' thinking of the mathematics through dialogue that engages students in communication *about* and *in* mathematics.

Notes

[1]We use the term *Latinos* to refer to the student population in the U.S. whose origins are of Cuban, Mexican, Puerto Rican, South or Central American, or other Spanish culture regardless of race as defined by *The Oxford Encyclopedia of Latinos and Latinas in the United States*, 4 vls, Oxford University Press 2006. In our local context most Latino students are of Mexican origin.

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K-12 Assessment-Reactor: Jeremy Kilpatrick

WHAT: CEMELA-CPTM-TODOS Conference
WHO: Jeremy Kilpatrick (University of Georgia)
PLACE: Tucson, AZ
DATE: March 5, 2010

TRANSCRIPTION

Jeremy

It's a pleasure to follow up of the bright comments that have been made this morning regarding this aspect, and I have a number of things to talk about. I'm going to first of all just say a word or two about mathematical proficiency because that's what we're going after, and we need to keep a couple of things in mind there, but I want to spend most of my time talking about what it is that makes a mathematical problem difficult and in particular why are some problems more difficult for ELLs, and I'll end with a (incomprehensible) about this word "gap".

So, just a word or two about mathematical proficiency… I just want to remind us, and of course, (Someone's name) mentioned last night, thank you very much, that (incomprehensible) that we chose is strategic. It was a strategic choice to portray mathematical proficiency as a braid. We came out with five strands in the braid, but it really isn't important that it's five, and in fact, some people have been saying we need at least a sixth strand to deal with culture and the historical parts of mathematics, and that has critiqued ever since we published this thing, but the issue isn't the particular five, it's not those five. The idea is, there are a couple of ideas that belong in this. One is that mathematical proficiency is not something that comes in eighth grade, or twelfth grade or graduate school. It's something that you can have and be developing all the way along, so that's part of the reason we chose this model, but the more important reason is to say that it's intertwined and you should need to be working on all the time. So if you're trying to construct any assessment in our field, you can't just assess one thing. You have to be thinking about mathematics as a multifaceted enterprise and if people are going to be proficient in it, there are a lot of aspects to that. That stuff gets (incomprehensible) all the time by psycho-... Some of my best friends are psychometricians, but psychometricians really like to think things as unidimensional if possible, and we have to keep reminding them that what we're interested in is something that it's multidimensional.

At the same time we get our strata model, we discover later that in Singapore they came out with five items too. They portray theirs as a pentagon and theirs doesn't exactly correspond to ours, but there are some connections. We came out with these things completely independently, but we were pleased to see that they saw some of the same things in mathematics proficiency that we saw, but my main idea is we have to keep our eye on the ball here, that mathematics is a multidimensional thing, and any assessment that we make has to try to represent at least some of that complexity, some of that multidimensionality, so that's just a reminder to us to think about our subject as a complicated one.

OK. Now I want to talk about what makes a problem difficult and I was interested that in her review of the literature, Maria picked up on one idea which is that some of this work that's been done has looked at questions of how readable is a problem. Can people actually read this thing? And that actually reminded me… I've written a little history in here, because fifty years ago, five oh, fifty years ago I was teaching ninth grade geometry in junior high school in Berkeley, California, and I was also working on a master's degree and I was interested in the problem because my kids were struggling with the word problems in algebra. I was interested in problems. What makes problems difficult for kids? And so I had taken some courses that involved readability formulas, so I took twenty-one algebra problems. They all required setting up a couple of equations and I defined readability as being able to come up with the equations. Solvability as being able to actually get the solution from that, and I developed... I just went back and looked at this thing for the first time... in fifty years. (Laughter)

I can't believe I did this, but anyway, this is history, folks. (Laughter) And what I found were some of the same things that we've been talking about today, and that is that vocabulary makes it tougher, longer problems are tougher, prepositional phrases make things a little bit easier; that sort of thing. I just want you to know that this stuff has a history. Doctor (Inaudible), in his dissertation and the article published in '86, talks about adjusting readability and what does that do in affecting the difficulty, and it's an interesting study (incomprehensible), systematically tried to adjust the readability and discovered that actually adjusting it didn't change the difficulty of the problem. He had a very homogeneous group. This was done in Ohio over the really homogeneous group of a thousand kids, but it raises the question, "What's readability all about? What makes problems difficult for kids?" and I want... Is Carl here? We haven't met, but I noticed that your poster that's coming up this afternoon takes this one step further, I think, and asks the question, "Well, what about readability for the... I don't know that you used readability formulas, but what about the reading that kids need to engage in if they are English Language Learners and doing algebra?" So fifty years earlier I was working on it and now you're working on it. I'm looking forward to seeing your poster.

OK, I made a list of some of the things that seem to come out of the papers and in the discussion and in my own thinking about what is it that makes problems difficult, because once you start listening to these things you realize that there are a lot of dimensions to problem difficulty, obviously the mathematical content. We heard a little bit about it this morning; that problems that involve statistics and probability tend to be somewhat more difficult and so do measurement problems. I think that comes about because these problems are set in a real-world context, which also often brings in vocabulary and ideas that kids haven't been learning, but also there are issues of linguistic complexity. We heard about that, which are both lexical and syntactic. There is this issue of cognitive demand; what is the demand of the problem? And I listed some words there are often used: identify, solve, explain, and prove. Three weeks ago I spent four days and a weekend in a hotel in Atlanta working with people from the Educational Testing Service and the National Assessment Governing Board, trying to figure out how we're gonna talk about what these levels are for twelfth grade NAEP. If you say that the kid is "advanced" rather than being "proficient", what can that person do? We were looking at the performance that the kids had shown us; or if they're called "proficient," how are they different from the people who are called "basic"? This was an intense and frustrating activity, but it raises the question of "What's the demand of the problem?" The reason I bring this up is one of the most difficult things that occurs in national assessment is that every time they ask kids to explain or, worse yet, to prove, they can't get good performance at all. Even if they get part of the proof and say "What's the next step on the proof?" the kids just bum out, the twelve graders, and presumably even the younger ones, too. So we have a problem of how do we assess kids' ability to explain; and I think that a lot of what came out this morning is we can get some of that if we move from a written to an oral approach, and I think we need to think harder. Let's just not make a formative assessment where we (incomprehensible) the oral explanation. Let’s figure out a way to get oral explanation to be part of a summative assessment.

I've also added both the form in which the item is presented and the form which the response is made. One of the lessons, I think, for me, from these papers that we heard this morning, is that you only get so much information from written stuff and the kids have a lot more they can give you if you can figure out a way to get it back orally. So we haven't been working, I think, hard enough on that problem. We need to ask ourselves "What could we do to get systematically oral information back from the kids and use that as part of their assessment?" Also I've added a couple of other things: aids, different kinds of technology, objects, language resources... These are some of the things that are mentioned in the paper; and finally, and not least, the experience that the kids have, the instruction that they had, the coaching, the familiarity that they had with the ideas in the assessment... and this is just the starting list, but it helps us see how complicated this question of assessment is, and how hard it is to get a handle on the specific problems that English Language Learners are having, and that's the thing that I want to address next.

Why are some problems more difficult for English Language Learners? Well, I just ran across, recently, (name) Williams' article on what counts as evidence of educational achievement, the role of constructs and the pursuit of equity in assessment. It's an article I recommend to you. It's in the recent Review of Research in Education, 2010. Just came out; very nice article, and the kinds of things he points out, “Bias is not a property of assessments but of the inferences that are made on the basis of their outcomes.” We all know that, but it helps to be reminded of it. “The debate should be focused on the issue of construct definition, and the consequences of the definition rather than on technical issues…” He brings up construct irrelevant variance and construct under-representation. Those are just technical terms; forget about those. He wants us to forget about those. Let's not worry about the technical aspects of validation. Let's worry about the construct we are trying to measure. That's the point he wants to drive home; and then he says, "If a particular construct that is defined for one population is not suitable for another, it's much better to define a construct that it's appropriate for that second population rather than simply modifying the assessment." It's got some really good ideas in this article and I recommend it very much to you, particularly those of you who are concerned about validity issues in assessing English Language Learners.

OK. Two groups differ in performance on a test. Is that a sign of bias? And the response we should make is "We can't tell. Let's look at the content of the test items." It's important to understand that a difference in total score need not indicate bias. Understand that. A randomly chosen item would be expected to show the same difference, so it's not an indication of bias. I want to applaud the use of differential item functioning, even though this highly technical idea. If you've not heard differential item functioning before today, read this measuring up book by Dan Koretz, who actually, Maria, you thanked him for reading your paper. He was your advisor.

Good. Well, he's written a wonderful book that is not technical. It's a great communication device for teachers. If they read the part of differential item functioning in this book, they'll understand it much better. Here's how he defines it: "It refers to group differences in performance on a particular test item among students who are comparable in terms of their overall proficiency." So why do students perform differently? Well, it could be bias. It could be bias, but it could also, as he points out, and he gives some examples, it also could be coming from differences in instruction, but we can't really know without going further into the specific case, because differential item functioning is only a clue that we need to look further, and I appreciate the work that Maria did in looking further in the articles and the items that she looked at.

OK. So I have, for each of the presentations this morning, some questions. These are not very good questions and in some sense they can't be answered, but it's something to think about. For Maria, I would say, it's great to use differential item functioning, but the question that lurks in the back of my mind is, when we use that construct, we're assuming that everybody who gets the same total test score has the same mathematical proficiency, and I keep wondering to myself, if I'm an English Language Learner, maybe I'm drawing on other knowledge, other abilities to get the same score as my classmate who's sitting next to me who is not learning English the way I am. I think it's interesting that it's an assumption that DIF rests on, and it's a good assumption for its purpose, but we should always question our assumptions, and that's when we can think of that; and the other one, you used the term "excessive linguistic complexity" and I asked "What is excessive linguistic complexity for a math item?" and that goes to the heart of a lot of the work that we're doing. We have linguistic complexity; we have mathematical complexity. We like to think of these in our minds as separate, but in my view they are completely intertwined and pulling them apart is a really tough (inaudible) and we need to keep thinking about how we can't pull them apart.

For Laura, in your paper, after I read it, I said to myself, one of the things you might pursue a little further, I don't know if you had the energy or interest... `(Laughter)`... but I was thinking to myself, OK. These kids were working in different diets (?). I wonder how their responses differ depending on which diet they were in. I don't know if you have data to answer that, but I also wonder. They got interviewed by different people, and I wonder how the interviewer, how their differences in responses, how they were different across interviews, so that's just a little question you might pursue. The second one, you can't pursue, but it may be a (incomprehensible) in the future somewhere, you found probing questions to have a certain effect with these English Language Learners. Would they have the same effect with non-English Language Learners? That's a question down the road.

For Cynthia and Anthony, my first question is, "Do the NAEP items, which you looked at, were the largest differences in percent correct, do they also have the greatest differential item function? That's a question you might want to look at sometime; and I got confused about linguistic complexity. Some of what Anthony said straightened it out for me today. "Linguistic complexity", what do we mean by that? Is that a property of the test item and can experts come in and evaluate it? From what you said here today, you were looking at it as a feature of the response the students were making. I didn't realize that from reading your paper. I thought you were saying it's a property of the students' comprehension. I guess my point is, about linguistic complexity, we as a group need to figure out what do we mean by linguistic complexity and who has it. That's a question for all of us. Not just for you.

Finally... This word "gap," all of our papers used it; all others papers use it. I want to argue that we should not use this word "gap" in relation to an achievement gap or a performance gap. It's the wrong word. What do you think of when you think of gap? An open, something that's not there, right? So I can imagine people when they... `(Shows a picture on the screen - off camera) (Laughter)` … they think, "OK. There's nobody in there." `(Laughter)` Right? It's the wrong word. You can't see that very well, but there are a lot of people, here they are, who are in the middle, right? So, when we use the word "gap," we're portraying the wrong situation. These actually don't overlap as much as the usual case. `(Shows a bell graph with "ELLs" and "Non-ELLs" at both ends of the curve)` I had made my graph very good, but the point is, you have a lot of folks who are in the middle. The variation within each group is much bigger than this difference between the two. In an article that my colleague Denise Newborn and I have written, it's going to be published by NCTM in this book on teaching and learning mathematics, translating research into the classroom for administrators, I'm trying to get the message across, here's what we said, and you can't read it, I guess, in the back, but, the term "gap," commonly used to describe a difference in achievement between groups of students classified by race, gender, or socioeconomic class conveys the false image of an empty space separating the group. One might easily conclude that there must be break in the common distribution of achievement: no one in the group with lower average achievement did as well as anyone in the group with having a higher average. In reality, however, the distributions of achievement in such groups greatly overlap. Any difference between averages is dwarfed by the dispersion of achievement within each group. "Gap" is the wrong word to use to describe a difference in group averages. So let's stop using the word "gap."

END OF TRANSCRIPT

K-12 Assessment Discussion

Following the Assessment research presentations, a practitioner panel, a reactor, and the participants met in small groups of six to eight for discussions. The groups included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The connections question became embedded in the discussions of the other questions. This summary represents common themes identified within and across the working groups.

What do we know?

Combining the presented research studies from the Assessment Strand, the provided poster sessions, and with our own professional knowledge and experiences, we are able to state what we believe to be the issues involved in assessment of mathematics with English Language Learners (ELLs) and particularly with Latino/as.

Although the use of standardized testing in mathematics of K-12 students is widely accepted as a valid measure of their competency in the subject, it is becoming more apparent that the current mathematics assessments in use at state and national levels do not accurately measure mathematical knowledge of ELL students. This becomes particularly problematic due to the current focus on high-stakes assessment that determines a school's rating of adequate yearly progress (AYP) as defined by the No Child Left Behind Act, and the corrective action districts, schools, administrators and teachers face if they are rated as failing. The high-stakes atmosphere has resulted in more testing in schools with high poverty and has changed mathematics instruction for these students. Teachers no longer present students with complex mathematical problems that require extended periods of time because these are not the type of problems on the assessment tests. Mathematics curriculum for ELL students has been preempted as a vocabulary development and language course instead of focusing on deep mathematics curricula.

From the research presented at this conference along with the growing body of research on ELLs, it is evidenced that these assessments are first and foremost a test of language. However, as the National Research Council stated in their 2000 report, "A test [of proficiency in a content area] cannot provide valid information about a student's knowledge or skills if a language barrier prevents the students from demonstrating what they know and can do" (Martiniello, 2008, p. 334). As was pointed out by Assessment Strand reactor, Jeremy Kilpatrick (University of Georgia), there is only so much information you can gain about a student's proficiency with written assessment and that oral explanation needs to become a part of summative assessments, and not just used for formative assessment.

Any written assessment in English is also an English proficiency test. Several factors contribute to assessment issues specific to ELLs. Complex language issues are certainly important along with others that also need to be considered: failure to recognize linguistic complexity embedded in items that are not associated with the mathematics being tested; complex vocabulary; use of social contexts that are unfamiliar to the student; language demand required for answering; failure to recognize and allow ELL students use of their cognitive resources and communicative representations; and assessment formats. There are many significant complex interactions between language, culture, cognitive demand, mathematics construct/context, and representations when ELLs make meaning of assessment items.

Mathematical proficiency is multidimensional and complex, involving an intertwined group of strands. Psychometricians want to think of it in unit dimensionality when trying to measure proficiency a strand at a time. We also know that "linguistic complexity" of assessment items is not one-dimensional--it is an interaction of the item, the student, and interpretations of the problem. Linguistic complexity and mathematics complexity are not separate. Although an item may be linguistically demanding, it may or may not have a high cognitive demand. Items often have "nonmathematical" linguistic complexity. Also, "linguistic complexity" in mathematics assessments is not unique to ELLs, but can also "trip up" native English speakers. We must recognize that the category of English Learners (ELs) is fluid, although as ELs become more proficient they are reclassified. Everybody is an English learner on some level.

Length of sentences, sentence structure and use of clauses, verb tense, and complex non-mathematical vocabulary all contribute to an item's comprehensibility for an ELL. Being able to make meaning of the problem allows ELLs entry to engage with the required mathematics to solve the problem. As one example demonstrated, when ELLs were given the same problem in Spanish some would change their answer to the correct one because they now understood the problem (Lager, 2010). To further complicate the issue, word meaning and usage in English, and from one language to another may differ. Interviews with students also revealed misinterpretations of problems due to vocabulary and mathematical register (Anhalt & Fernandes, 2010).

Findings also document construct-irrelevant variance (difference in test scores attributable to factors unrelated to the construct of interest, William, 2010) in test items due to contexts chosen. Although it may not seem as though common contexts should be a significant factor, without familiarity with a student's funds of knowledge or culture, the subtleties of context may interfere with a student's meaning making process. As one example demonstrated, when ELLs were given the same mathematical construct using a familiar context, their rate of success was parallel to English speakers with similar mathematical scores. But in another item with unfamiliar context, differential item functioning was exhibited (Martiniello, 2008).

Awareness of language demand required for a response also needs to be considered. When students are asked to explain, prove or justify reasoning, the language demand is increased. When given options in forms of communication through an interview process, (i.e. oral, gesture, diagrams, native language, etc.) students' mathematical proficiency becomes more apparent. This points to the fact that our current assessment arrangements for ELLs limit their communicative resources and do not adequately address the constructs being assessed.

To further exacerbate the ELLs difficulty in learning mathematics, language policies in place in several states contribute to limiting ELLs' access to mathematics curriculum thus creating issues of equity. English Only policies have been translated into not ever using the native language, even for clarification, thus denying ELLs access to resources they have to understand the mathematics. Language policies also reinforce tracking even though research shows that when there is curriculum tracking the low performing students are disadvantaged. There also is research that shows that higher performing students are not held behind when students are mixed. Teachers have little or no training in linguistics and have difficulty recognizing what language demands are being placed on ELLs.

English speaking students also have the home as a resource for learning mathematics. Because many ELL students come from homes where English is not spoken, there is becoming a void in communication between parents and students, and ELL students are not able to communicate with parents about mathematics or other topics since English is the language of instruction. Students become more and more reluctant to use Spanish since the message they are receiving in school is that their language is not valued and it is important for them to assimilate. However, there is evidence that if parents are involved in a variety of ways there is more positive learning.

What are the implications for practice and research?

Based on these findings we recommend the following implications for assessment policy, teaching, research and home/school connections. First and foremost, we need to have clear definitions of what proficiency looks like for the mathematical constructs to be assessed at various grade levels. It needs to be determined if our construct of mathematical proficiency includes being able to solve mathematical problems embedded in a context which may be unfamiliar to ELL students or being able to understand complex linguistic structures that are not part of the mathematical complexity related to solving the problem (William, 2010). We must also honestly acknowledge what is the purpose of the assessment. Is it to stratify students into groups by their culture and English proficiency, to measure their mathematics proficiency for instructional purposes, or to identify and label how they compare to each other based on a frame of middle-class white culture? Presently the assessments in place do two out of three of these goals, but do not accurately measure mathematics proficiency for instructional purposes.

Because of the adverse impact the current assessments have on ELLs and other groups due to deficiencies in the assessments, we must question their validity. Given the high cost of assessments in both time and money, we must also decide if this is how we want our available education resources to be spent.

From the research presented the format of summative assessments needs to change to include an interview process. This will offer one way to better tap into ELLs' knowledge and skills. Work on assessments needs to be done in order to recognize and validate other modes of communication such as gesture, representation, use of native language, reframe their explanations, etc. These methods will give a better window into what the students understand, know and can do, moving away from the deficit model that is currently in place. Also contributing to this deficit model is the term “gap” that is commonly used to describe a difference in achievement between groups of students classified by race, gender or socioeconomic status. This term conveys an image of a “void” between groups. There is in reality a greater variation within groups than between groups and distribution of achievement between groups greatly overlap. The term “gap” is the wrong word to describe a difference in group averages (Kilpatrick, 2010).

Preservice and inservice teachers need instruction on identifying cognitive demand, understanding language complexity and identifying language demands for ELLs as they work with instruction and assessments in the classroom. They also must be aware of how bias can come into the assessment event. Work needs to be done with them on designing and using good assessments (written and oral) to understand what ELLs know, understand, and can do. Teachers’ judgments need to be valued and more trust needs to be placed on what teachers know about students. Practitioners also need inservice in good questioning strategies and must have a deep understanding of the mathematics content. Teachers know the value of strong classroom discussions, but in reality it is difficult to sustain meaningful discussion on a regular basis for students due to the many demands placed on instructional time. We need to consider how we structure and organize our classrooms and school day to incorporate thoughtful instruction. There needs to be a stronger connection between practitioners and researchers, where practitioners are involved with equal status in the research.

The connection between home and school should be used as a resource, especially for ELLs. Parent involvement may take many forms and its definition needs to be expanded. Activities should include parent/student nights working on concepts that students are studying in school using their native language, working with parents to understand how learning happens and is a social activity, and discussing different educational experiences, pedagogy, problem solving methods and algorithms from other countries.

All stakeholders, item writers, policy makers, teachers, researchers, etc. need to know the complexities of assessing ELLs’ mathematics proficiency (discussed above) and need to equitably address them. They also must recognize the limitations of current assessment practices. Practices of segregating ELLs due to states’ language proficiency policies have contributed to ELLs not having an equal opportunity to learn deep mathematics content. These practices need to end. It needs to be recognized that language and culture play a role in learning mathematics. Assessment has become a civil rights issue for students and teachers, and must be addressed.

The national standards movement presents an opportunity to integrate assessment design principles for English Learners into the national policy discussion. There could be a framework created for assessing specific mathematical domains while simultaneously accounting for students’ level of English language acquisition. This would require quite a bit of technical skill (in terms of assessment theory), but it would also allow for a way to take insights from smaller interview-based studies and use them to develop rich mathematics items, construct maps, scoring guides, and reporting systems that have language integrated throughout.

What else do we need to know?

Here is a list of questions that will require further research in assessment with ELLs.

- Do assessment arrangements adequately address the mathematics constructs?
- How do Latinos and African Americans solve problems from a frame that fits them?
- What is going on with Latinos who are not ELLs? How does culture go together with language? Where does knowing about their background experiences fit?
- How can lumping large groups all together be stopped? We need a diversity readiness scale.
- Some ELs are placed in special education courses. How does this affect their outcomes?
- Are there models we can use for teachers to show them what this looks like in practice? For example, provide video cases of teachers working with students in classrooms and doing good formative assessment.
- Is adding context or using word problems at all fair for ELLs? But if we try to remove all contexts, whatever happened to doing meaningful mathematics? Specific tests need to be carefully analyzed for bias. When a difference is found in items it could be because of bias, but it also may be an indicator that students need different instruction. We cannot just say there is bias because there is a difference.
- What is the relationship between linguistic complexity and mathematical complexity and also specific mathematics topics and complexity? Data analysis and statistics and measurement context seem most assessable to students.
- How can we address the interplay between language complexity and language demand?
- What is the role of mathematical representations in assessment?
- What is the role of policy in assessment issues?
- How can we include parents in assessment?
- What other roles can parents play in assessment besides interpreting their children's scores?
- Can we design assessments that have two dimensions—mathematical development and second language acquisition?

- We need to have strong content-focused construct maps (e.g. a sense of how students learn a mathematical topic such as measurement).
- We also need a strong developmental trajectory of second language acquisition—something we should get from linguists.
- Finally we need some assessment gurus to help us put this together.
- But the good news is that there are models, e.g. Confrey's work, BEAR Assessment, etc.

- At the student level, are there other parts of personal history/trajectory that influence assessment performance?

- Supposing we can design better assessments that get at math and language development, how can we help teachers interpret assessment results in a way that leads to effective practices?

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CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Visions from the Classroom: Focus on Teachers

Section 3 of 9

Chairs: Lena Licón Khisty, University of Illinois-Chicago
Sylvia Celedón Pattichis, University of New Mexico

Tucson, Arizona March 4 -6, 2010

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CEMELA-CPTM-TODOS Conference
Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Language Pitfalls and Pathways to Mathematics

Alma Ramirez
West Ed

The paper presented by Alma Ramirez appeared in the NCTM publication referenced below.

Barnett-Clarke, C., and Ramirez, A. (2004). Language Pitfalls and Pathways to Mathematics. In R. N. Rubenstein (Ed.), *Perspectives on the teaching of mathematics,* 56-66. Reston, VA: National Council of Teachers of Mathematics.

CRITICAL MATHEMATICS IN A PRECALCULUS CLASS

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This dissertation study documents the efforts of the researcher and a veteran high school teacher to create a critical mathematics learning environment in a public high school precalculus class consisting predominantly of students of color. The goals of the study were to understand what pedagogical and curricular practices the teacher employed, how the teacher negotiated the various classroom goals, and how the students engaged in or resisted such non-traditional instruction. This article describes specific critical mathematics activities implemented in the class and the reactions of the students. The author offers specific characteristics of a classroom environment that promote critical mathematics education: mathematical relevance; integration of classical, community, and critical knowledge bases; and overarching themes. Furthermore, the author suggests that future iterations of this work consider the time necessary for planning and implementation, the importance of collaborative colleagues, and the significance of personal relationships with students.

The demographics of public schools in the United States are rapidly changing and drastically different than that of previous generations (Villegas & Lucas, 2002). In response to these changing demographics and the persistent "achievement gap", the National Council of Teachers of Mathematics (NCTM) called for a new vision of mathematics pedagogy to provide more educational access and opportunity for students historically underrepresented in the field. The organization offered "The Equity Principle" as the first of their six Principles for School Mathematics and defined equity as "high expectation and strong support for all students" (NCTM, 2000). Additionally, their Standards for the Professional Development of Teachers of Mathematics recommend that all mathematics teachers know the following:

* The influences of students' linguistic, ethnic, racial, and socioeconomic backgrounds and gender on learning mathematics (Standard 3); and
* The nature of mathematics, the contributions of different cultures toward the development of mathematics, and the role of mathematics in culture and society (Standard 2).

Despite this increased focus on equity, mathematics continues to play a pivotal role as a gatekeeper to upward social mobility and access to higher education and jobs (Moses, 2001). Furthermore, access to high-quality mathematics instruction is limited or absent in schools attended by students of color and/or students living in poverty (Oakes, 2005). Rather than developing and exercising a pedagogy that values multiple worldviews, mathematics education in particular, and society as a whole, continue to perpetuate the "assumption that mathematics is a neutral, objective, abstract, culture-free discipline" (Tate, 1994, p. 56).

Of particular concern is the development of teachers to work effectively in diverse mathematics classrooms where misconceptions of mathematical ability and cultural deficits currently dominate the conversation. To exemplify, White (2002) argued that in mathematics education, "people often accept disparities in achievement across various student backgrounds as being normal, natural, inevitable, explainable, or even acceptable" (p.1). Yet teaching mathematics in ways that allow all students opportunities to succeed is a challenge that our educational system has not yet risen to meet. Gaps in achievement persist and the differences in achievement are not random, but fall along race, class and gender lines (Gutiérrez, 2007). Specifically, the schooling experiences of Latin@[1] students serve to further marginalize them as their language and community resources are seen as deficits or barriers to education rather than assets to be acknowledged and integrated into their schooling (González et al., 1995; Moll & Ruiz, 2002; Valenzuela, 1999).

With the above challenges in mind, it is my intention to contest inequity in mathematics education by investigating curricular and pedagogical innovations that aim to tackle disparities in mathematics education and promote equity. I have identified Critical Mathematics Education (see below) as one such pedagogy that seeks to have mathematics education contribute to the creation of a critical citizenry and support democratic ideals. My dissertation study aims to offer a better understanding of the design, implementation, and experience of mathematics curricula that engage students in critical investigations of their personal experiences, local community, and society as a whole. This research seeks not only to investigate how a mathematics teacher of color conceptualizes and enacts curricular activities that incorporate students' community and critical knowledge bases, but also how Latin@ students experience such non-traditional instruction. My interest in studying the experiences of Latin@s is personal (as a Latin@ immigrant myself), professional (the growing Latin@ population in U.S. schools and the continual failure to serve them demands consideration of their experiences), and theoretical (Latino Critical Race Theory).

My study is focused on the experiences of students of color in order to bring attention to the experiences of students who are often marginalized by mathematics education in particular, and society as a whole. This position is rooted in Critical Race Theory (CRT) and Latino Critical Race Theory (LatCrit) which argue that racism is endemic and deeply ingrained in American life, yet race remains under-theorized in society. Ladson-Billings and Tate (1995) offered three propositions for CRT in education that are analogous to critical legal studies: 1) race continues to be significant in the United States, 2) U.S. society is based on property rights rather than human rights, and 3) the intersection of race and property creates an analytical tool for understanding inequity (p. 48). CRT scholars further emphasize the intersection of race and property by arguing that Whiteness is the ultimate property. Therefore, Whiteness comes with rights of disposition, rights to use and enjoyment, reputation and high status, and an absolute right to exclude (p. 59-60).

Latino Critical Race Theory (LatCrit) is similar to CRT but addresses issues often overlooked by critical race theorists such as language, immigration, ethnicity, culture, identity, phenotype, and sexuality. Solorzano and Delgado Bernal (2001) explained: "LatCrit is a theory that elucidates Latinas/Latinos' multidimensional identities and can address the intersectionality of racism, sexism, classism, and other forms of oppression" (p. 312). They conceived of LatCrit as an anti-subordination and anti-essentialist project that examines how educational theory and practice are used to subordinate and marginalize Latin@ students. These authors posited five themes that form the basic perspectives, research methods, and pedagogy of a CRT and LatCrit framework in education: 1) The centrality of race and racism and intersectionality with other forms of subordination, 2) the challenge to dominant ideology, 3) the commitment to social justice, 4) the centrality of experiential knowledge, and 5) the interdisciplinary perspective (p.

[1] I choose to use Latin@ to undermine gender-normative associations with Latino or Latina.

312-314). It is these themes that drive my investigation into critical mathematics education as I employ the tools of CRT and LatCrit. These tools include: story-telling and counterstories; *testimonios*; interest convergence; and critical white studies (Gillborn, 2006; Solorzano & Yosso, 2001).

Specifically, counterstories and testimonios give voice to the experiences of students of color, a teacher of color, and a researcher of color – voices often silenced in academic literature. These tools from CRT and LatCrit aid in the examination of different forms of racial and gender discrimination experienced by people of color by "telling the story of those experiences that are not often told (i.e. those on the margins of society) and a tool for analyzing and challenging the stories of those in power and whose story is a natural part of the dominant discourse - the majoritarian story" (Solorzano & Yosso, 2001, p. 475). These methods provide a detailed portrait of the participants and their local context, describing their opinions, conversations, and actions as they "testify" to their realities and lived experiences. It is my hope that participants' critiques of racism, sexism, and other social structures will be at the core of these oppositional counterstories that challenge educational assumptions and stereotypes.

Rooted in CRT and LatCrit, my study will employ theoretical constructs relevant to *critical mathematics education*, principally *critical mathematical agency*. Below you will find my reasoning for selecting these constructs and a brief summary of the relevant theoretical and empirical work. My research questions and methodology will further delineate my study and justify my selection of critical ethnography as an appropriate methodological orientation.

CRITICAL MATHEMATICS EDUCATION AS A PATHWAY TO EQUITY

Critical mathematics education is guided by the philosophy of critical education, pioneered by the Brazilian educator, Paulo Freire. This philosophy is grounded in the beliefs that (1) the purpose of education in an unjust society is to bring about equality and justice, (2) students must play an active part in the learning process, and (3) teacher and students are both simultaneously learners and producers of knowledge (Freire, 1970). At first, mathematics seems an unlikely vehicle for liberation. Though traditionally viewed as value-free, mathematics is actually one of the most powerful, yet underutilized, venues for working toward the goals of critical pedagogy – social, political, and economic justice for all. This emerging awareness is due to how critical mathematics educators, such as Frankenstein (1981, 1983, 1990, 1995, 1997), Skovsmose (1990, 1994a, 1994b), Gutstein (2003, 2006, 2007a, 2007c; Gutstein, Lipman, & Hernandez, 1997), and Turner (2003) have applied the work of Paulo Freire. Their work aims not only to improve student access to mathematics learning, but also to transform both the discipline and instruction of mathematics by supporting the voices of those communities that are often marginalized by the formal institutions of our society.

Critical mathematics scholars consider mathematics to be a tool to understand, critique, and change the world by deconstructing power structures that marginalize certain groups. They do not see the purpose of mathematics education to be solely functional literacy and better-distributed success in the traditional system; rather, the goals go further to conceive of mathematics knowledge as the ability to use mathematics to critique and transform oppressive structures – mathematical literacy is "knowledge for liberation from oppression" (Gutstein, 2006, p. 211). They embrace Freire's notion of *conscientização* – the process by which students critically reflect, examine, and act upon their world in order to transform it. These scholars promote a problem-posing pedagogy that draws on and builds upon students' questions, interests, experiences, and mathematical understanding. In order to understand how students experience critical mathematics education, scholars have employed such constructs as identity and agency.

One such scholar, Turner (2003) identified *critical mathematical agency* as an aspect of student identity fostered and enacted through critical mathematics education. She defined critical mathematical agency as "students capacity to (a) view the world with a critical mind set and imagine how the world might become a more socially just, equitable place, and (b) identify themselves as powerful mathematical thinkers who construct rigorous mathematical understandings, and who participate in mathematics in personally and socially meaningful ways" (p. iv). This enhanced sense of agency is both *critical*, as defined above, and *mathematical* in that it focuses specifically on mathematical activity or practices. Critical mathematics education allows students to build on their own mathematical understanding in order to investigate and critique situations of importance to them and relevant to the larger society. Furthermore, students are engaged in struggles against injustice and act to transform society to be more equitable and just. Turner's work further investigated this interplay between the critical and the mathematical aspects of agency as they co-developed through critical mathematics education.

Grounded in critical mathematics education and focused on critical mathematical agency, my dissertation will add to the literature by putting these constructs to practice. There is a need in the field to conduct studies on the implementation of critical mathematics education in classrooms in order to offer teachers "practical guidance" (Sleeter & Delgado Bernal, 2004) that is grounded in theoretical concepts. Though some research exists on implementing critical mathematics in various settings (e.g., middle school classrooms, remedial high school courses, adult education, afterschool programs, pre-service teacher education, in-service professional development), there is little focused on upper-level high school content (i.e. beyond Geometry). My study will employ a relatively newly developed construct – critical mathematical agency – in an often-overlooked content area – precalculus – in order to provide insight into a teacher's implementation and students' experiences of critical mathematics education.

Research Questions

To accomplish this goal, my dissertation study aims to address the following research questions:

1) How does a teacher of color aim to create a critical mathematics learning environment in a public high school precalculus class consisting predominantly of students of color?
   a. What pedagogical and curricular practices does the teacher employ?
   b. How does the teacher negotiate various classroom goals (e.g., student mathematical understanding, development of critical agency, authentic participation in mathematics)?
   c. How do students of color experience such an environment?
      i. How do students of color enact *critical mathematical agency*?
      ii. How do students of color resist critical mathematics education?
      iii. What shifts, if any, occur in students' perspectives about mathematics, their mathematical and sociopolitical identities, their mathematical understanding, and their participation in mathematical activities?

To investigate the research questions stated above, I am collaborating with Donovan[2], a veteran high school mathematics teacher whom I have come to know through graduate coursework in mathematics education. The goal of this collaboration is to create a critical mathematics learning environment in Donovan's precalculus class. Such an environment contains well-established group norms where all members of the community (i.e., the class) feel comfortable and competent, identify the teacher as a colleague and ally, and collaboratively investigate personally and socially relevant topics. The teacher and students must co-construct what Gutstein (2007b) described as "a classroom oriented toward social justice" with the following three features:

1) "normalizing politically taboo topics" (e.g., making discussions about racism and injustice part of ongoing classroom discourse)
2) creating a "pedagogy of questioning," and
3) developing "political relationships" with students (p. 17)

Furthermore, Donovan and I aim to create, adapt, and implement critical mathematics activities in his precalculus class. These activities are fundamental to critical mathematics education in that they employ mathematics to investigate injustices relevant to students while identifying ways in which the youth can engage in action aimed at making change.

METHODOLOGY: QUALITATIVE ADVOCACY/PARTICIPATORY RESEARCH

Influenced by Freire and the theories described above, I believe that education should be directly integrated with societal change. Similarly, I believe inquiry should be intertwined with politics and a political agenda in order to advocate for action to help marginalized peoples. These claims position my study as Qualitative Advocacy/Participatory Research (Creswell, 2003). This paradigm assumes that I, the inquirer, will proceed collaboratively so as to not further marginalize the participants. I will engage the participants as active collaborators in the inquiry. That is, the inquiry is completed "with" the participants rather than "on" them. This collaboration seeks to address important issues such as empowerment, inequality, oppression, domination, suppression, alienation, collectivity, and transformation. Specifically, this study speaks to the critique of mathematics instruction, which traditionally marginalizes people of color by obstructing access to knowledge, upward mobility, and the organizational power of society. In order to advance an action agenda for change, this study will provide examples of students of color engaging in critical mathematics and demonstrate how teachers can promote such experiences. Guided by a qualitative research paradigm, my investigative process gradually makes sense of student and teacher experiences with critical mathematics education by immersing me in the everyday life of the setting, entering the participants' world, and seeking their perspectives (Creswell, 2003).

Critical Ethnographic Methods

My commitment to praxis and activism within inquiry has led me to choose critical ethnographic methodology to help me describe issues of power and oppression at the intersection of human agency and societal structures, and to offer hope for change through a critical, yet transformative perspective (Trueba & McLaren, 2000). Ethnographies study "an intact cultural group in a natural setting over a prolonged period of time by collecting primarily observational data" (Creswell, 2003, p. 14). Critical ethnographies relate the voices of participants

[2] Names of all individuals are pseudonyms.

(ethnography) to the critical analysis of power structures (Anderson, 1989), allowing for analysis of both the macro (social, historical, political) and micro (interactions, language use, and practices) contexts of learning and the interactions between the two. My study places the voices of participants at the center of data collection, analysis, and presentation with the aim of promoting political debate and discussion so that change may occur. A critical ethnographic study of the experiences of Latin@ students provides rich data by which to unveil and explain the interplay of societal structures and human agency. As a result, I will provide case studies of the students as they experience critical mathematics education. Additionally, I will illustrate, in detail, the process by which the teacher and I created and implemented the critical mathematics activities.

Data Collection Procedures

In order to understand how students enact critical mathematical agency and how the teacher fosters this enactment through critical mathematics activities, I am conducting participant observation in a precalculus classroom and other school contexts (e.g., other classrooms, lunch time, clubs), examine student work, conduct interviews and focus groups, and collaboratively reflect with the teacher. I attend the precalculus class three to four times per week in order to observe students engaging in critical mathematics activities and the classroom on a "typical" day. I videotape class sessions and focus groups, audiotape interviews, take ethnographic field notes in various classes, and collect artifacts, in order to document the students' experiences. Additionally, I work with groups of students during class and offer further assistance to individuals during lunch. It is a challenge to balance all of these roles, but my position as a participant observer is necessary to establish me as a valid member of the classroom community while also conducting my research. Because the critical mathematics activities investigate injustice and aim to work toward societal transformation, students must see me as actively participating in the activities in order to identify me as an ally in *their* action.

From the very beginning of my collaboration with Donovan, I have documented all of our interactions – audiotaping conversations, saving emails, taking notes in classrooms and meetings, and keeping a Research Journal – in order to clearly describe the entire process. Donovan and I have met regularly to discuss literature, analyze curriculum, design activities, and collaborate about the study. Additionally, I asked Donovan to keep an Instructor Journal throughout our summer planning. I posed specific questions for Donovan to consider before being overwhelmed by the daily responsibilities of the school year. Sample journal prompts included: What are you hoping to get out this collaboration? What are your goals for this particular class? How will you measure student learning and academic progress?

Though the physical learning setting does not change, my initial analysis reveals different mathematical and social practices during critical mathematics activities in comparison to "typical" days. Therefore, I focus my observations on the participants and how they engage in class discussions and activities, the kinds of questions they ask, and the context for participation. I document how they engage in the mathematics in both contexts and the nature of the mathematical activity that happens, providing me with a potential contrast to analyze. Field notes include details on the general setting, specific roles that students take on, excerpts from their dialogue, evidence of mathematical and critical understanding, enactments of critical mathematical agency (e.g., asserting intention, positioning and authoring, critique, transformational resistance, and improvisation), and my own personal reflection. These field notes from classroom observations allow me to consider participation during various activities, while not limiting me to only considering student experiences during critical mathematics activities. For example, students may participate differently during teacher-directed or textbook-centered classroom activities when compared to critical mathematics activities. I will be able to triangulate the data from my field notes, student work, video observations, and interviews to describe the complexity of critical mathematical agency, as well as its connections to different learning environments.

Data Analysis Strategies

Data analysis is an ongoing process throughout the study, integrated with data collection. As I interact with Donovan and his students, I constantly assess and adapt my assumptions, interview questions, data collection, and data organization techniques. I regularly review multiple sources of data and include participants in the ongoing analysis process. After transcribing and summarizing video and audiotapes, I revisit field notes and journal entries in order to gain a broad, yet initial, view of the data. Employing grounded theory (Charmaz, 2008), I focus on the intertwined processes of collection and analysis, create analytic memos, identify emerging themes in the data, create codes, and form conceptual categories. I code for student patterns of participation, various group roles, expression of varied knowledge bases (including language use), traditional mathematical activity, critical discussions, task characteristics, and facilitator moves. I regularly revisit the data to refine these categories. I will use qualitative analysis software to organize and analyze data. This software will aid me in identifying emerging patterns and developing code families. These codes and categories will inform further data collection, which, in turn, leads to refinement of codes and reexamination of earlier data. This process will continue to cycle until I identify the specific themes that will drive my complete analysis of the entire data set.

INITIAL FINDINGS

Planning

From the beginning of our collaboration, Donovan was clear on his general goals for the coming school year. He hoped to increase students' "consciousness" and desire for "legacy", while also decreasing attrition in the Precalculus course. He defined "consciousness" as a critical understanding of one's reality by identifying the impact of larger societal structures such as racism, sexism, and classism. This "consciousness" included a genuine interest in helping others. To exemplify, Donovan often explained that his teaching philosophy was grounded in the saying, "each one, teach one." "Legacy" referred to students achieving higher levels of education and social access than their parents in order to pass down a better existence to future generations.

With these general goals in mind, Donovan and I embarked on a summer of planning and preparation. This effort demanded a significant amount of time. Over the summer, we met a dozen times for a total of nearly 30 hours. In addition, we each spent countless hours reading, researching, obtaining resources, and emailing ideas to prepare for these meetings. Our meetings typically involved discussions of articles, curriculum, pedagogy, personal stories, teaching experiences, the educational system, social structures (e.g., racism, sexism, classism), issues of equity/inequity, mathematical concepts, and possible class activities. Though we both desired to create all relevant activities before the school year started, our meetings proved to be more conversational and less productive. Donovan accredited this need to talk to his lack of colleagues who he could turn to for such discussions. Donovan often mentioned that he could never have critical conversations with other teachers and administrators in the school. He shared comments and actions of other teachers to illustrate how "uncritical" they were. For example, he referred to teachers who leave as soon as the bell rings and to administrators who plan on reducing the number of advanced courses after students predictably drop out. This lack of critical collegiality is what originally drove Donovan to take graduate courses at the university. He desired conversation, reflection, and literature that helped him better align his instruction with his sociopolitical identity. Donovan and I had often had such conversations in common graduate courses and he appreciated having this opportunity to focus them on his classroom. However, as stated above, these conversations often overshadowed our curricular planning.

The goals of our summer planning meetings were to discuss critical mathematics education and to identify major themes, frameworks, and resources that would aid with curriculum development. Additionally, we hoped to create specific activities and projects that Donovan could implement during the first semester. As a result of our conversations and research, Donovan and I identified three central themes for the course. First, the mathematical content of the course would be developed conceptually rather than procedurally. In addition to our summer conversations about society and equity, we had often discussed the mathematical concepts to be covered during a year of high school precalculus. Though he had never taught Precalculus before, Donovan had taught the prerequisite Intermediate Algebra course and the capstone AP Calculus course for several years. These teaching experiences provided him with insight into what knowledge and skills students bring to Precalculus and what will be relevant in future calculus courses. As we reviewed the previous course syllabus, a traditional textbook, several states' standards, and various reform curricula (i.e., Interactive Mathematics Program, Core Plus, COMAP), Donovan framed the course as focused on conceptual understanding rather than the development of algebraic skills. He came to the conclusion that the previous iterations of the course had focused on the "pre" part of precalculus, dedicating the majority of the course on developing students' skills and speed with algebraic calculations and rules. Donovan considered this to be part of the weeding out process in mathematics education that resulted in so few students reaching upper-level mathematics courses. Instead, Donovan wanted his course to focus on the "calculus" concepts in precalculus. This entailed framing lessons and activities around two central themes: rate of change (derivatives) and accumulation/area under a curve (integrals). Rather than having students practice countless algebraic manipulations that may some day later help them to calculate derivatives in a calculus course, Donovan aimed to have students develop a comprehensive understanding of functions and their rates of change. This would entail lessons that emphasized multiple representations of functions and distinctions between functions. Furthermore, real world applications and models would reinforce student understanding of functions and aid students in making connections between the mathematical content and reality.

Second, Donovan and I borrowed from Gutstein (2006) in focusing curriculum development on the integration of three central components: Classical knowledge, Community knowledge, and Critical knowledge (the 3 C's). Classical knowledge relates to the skills and competencies typically seen in mathematics classrooms. Students are to master traditional texts and assessments in order to achieve mathematically, in a traditional sense. Community knowledge emphasizes that "ordinary" people have and produce knowledge about their lives, experiences, and contexts. Students develop mathematical understanding through informal and everyday experiences and instruction should access and utilize these experiences. Furthermore, students should be exposed to and understand non-Western mathematics in order to broaden their sense of the creation of knowledge. Critical knowledge draws attention to the knowledge students need to understand their sociopolitical context. Students, especially marginalized youth, have experiences with and perspectives of oppression and power. They may rarely have been asked to conceptualize or communicate these in classroom settings. Instruction should access and utilize this knowledge as it may support mathematical investigation by offering relevance, authenticity, and insight into a topic of inquiry. Conversely, mathematical knowledge can support critical investigation by providing students a mathematical lens with which to understand complex social phenomenon. Though individual class days may appear to only focus on one of these three knowledge bases, it is essential to develop lessons that integrate two or three of them. Donovan and I planned to create critical mathematics activities aimed at this integration by engaging students in critical investigations of their personal experiences, local community, and society as a whole. These activities would rely heavily on conversation, discussion, and debates that integrate personal, community, and critical knowledge into the mathematics activity.

Third, Donovan and I wanted to produce curriculum that gave students opportunities to "mathematize" the world and to humanize mathematics. Mathematization is the ability to see the role of mathematics in one's personal life, family routines, community activities, and societal structures. This allows students to better understand, analyze, and critique reality. Much of mathematics education focuses on the application of mathematical concepts to "real world" problems. In contrast, mathematization begins with a phenomenon and utilizes mathematical modeling to better understand it. That is, mathematization attempts to make sense of reality through relevant mathematical models rather than applying predetermined mathematical concepts to contrived problems. While it is important to have students understand the world mathematically, it is also essential for them to combat the decontextualization that often occurs with mathematical data. The humanization of mathematics promotes students keeping their mathematical investigations and findings in context, not forgetting the human experiences behind the data. For example, rather than simply considering the number of AIDS deaths as data in a table or graph, lessons could help students identify with AIDS patients and recognize the possibility of someone they know becoming infected.

In order to promote the above themes, Donovan and I identified two approaches to curriculum development that would allow for the integration of mathematical content and critical topics. The first approach focused on *applying* already-learned mathematical skills and knowledge to relevant investigations. For example, students would be asked to apply what they had learned about rates of change and derivatives to analyze the change in college-going rates for low-income youth over the past 50 years. As a teacher, Donovan would start with mathematical content and identify appropriate activities that allowed for the application of the mathematics. Conversely, the second approach focused on *modeling* real world phenomenon. For example, the class would be asked to consider the spread of AIDS over the past 40 years. In addition to studying the history and progress of AIDS, students would need to move beyond basic linear and quadratic functions to better represent the data. Class activities would introduce students to exponential growth and logistic functions, providing them with relevant, in-the-moment tools to advance their inquiry. In this approach, Donovan would start with a relevant topic and incorporate mathematics into a lengthy and comprehensive investigation.

To aid with curriculum development, Donovan and I employed and adjusted Varley Gutiérrez's (2009) "Feminist Critical Mathematics Framework" (see Appendix). Her framework details phases and cycles of activity to facilitate student mathematical investigations of relevant topics. These phases range from early determination of interests and concerns, to synthesizing and representing mathematical findings, and ultimately to action and reflection. Though completion of all of these phases is the ultimate goal, we decided it would first be necessary to create activities that introduced students to group norms, collaborative processes, investigative tools, and critical mathematics activities. The class would then engage in a whole-class investigation in order to offer a shared experience for all class members. Next, the class would conduct a disaggregated project where groups focused their investigations on specifics aspects of a topic and then report back their contribution to the class. In other words, we planned to start small by first creating one or two-day activities and then introducing longer projects. Ultimately, students will dedicate time, in and out of class, to complete the phases in the framework in preparation for presentation at an end-of-year "Encuentro" (a community gathering to celebrate and share student knowledge).

In order to provide foundation for the work ahead, Donovan planned for an early and recurring focus on developing collaborative and communicative norms in the classroom and in groups. This entailed activities early in the school year that helped build a sense of community in the classroom and a spirit of cooperation within groups. Donovan drew on his experience in a graduate course focused on group-worthy tasks and Complex Instruction (See Cohen & Lotan, 1997) in order to create equal-status interaction within small groups as students used each other as resources to complete challenging group tasks. Students would be asked to discuss and debate not only mathematical topics, but also themes traditionally considered taboo in a mathematics classroom (e.g., racism, poverty, social capital). Furthermore, we would provide examples of mathematization and conduct critical mathematics activities in order to demonstrate to students that the integration of classical, community, and critical knowledge bases was relevant to and appropriate for their class.

Though we identified the above major themes, collected a wealth of resources, created frameworks for curriculum development, and had a general idea of where we wanted the course to go, we did not have a detailed plan in place for the beginning of the school year. This made implementing the above ideas very difficult, as there were no ready-made models to pull from or example courses to follow. As a result, Donovan often found himself relying heavily on the traditional textbook and lessons focused on developing primarily classical knowledge. Donovan would interject conversations, activities, and labs that deviated from the traditional text, but their mathematical relevance was often unclear and students considered these activities to be disconnected from the "real" mathematics in the textbook. The following section describes the exceptional, non-traditional activities that Donovan did implement to promote critical mathematics. The section will also describe how students experienced these activities, including how they participated and/or resisted.

Implementation and Student Experiences

In an attempt to address the predetermined themes, Donovan implemented three major assignments in the first semester: Where's the Math, Local Unemployment, and AIDS Lab. These activities were not part of previous iterations of the course and did not appear in any established curriculum. Donovan considered these activities experimental and made several in-the-moment adaptations to the planned lessons. In addition to videotaping and assisting with these lessons, I conducted several focus groups to inquire into student opinions of the relevance and enjoyment of these activities.

Where's the Math was a recurring assignment that asked students to mathematize a topic in the news. This assignment was similar to Current Events assignments in other courses, but asked students to specifically tease out the relevant mathematics in an article. Students could choose any article or topic in the news that was of interest to them and were asked to consider the role of mathematics. For example, some students were interested in the budget crisis in their state and the proposed cuts to education. They researched the state's average spending per pupil and compared it to other states. They then compared scores on standardized tests and conjectured about the relationship between state spending and student achievement. By inviting students to mathematize anything they wanted, we hoped to learn what topics were of interest to students and how they saw the role of mathematics in their every day lives. Their choices for topics were to inform our decisions on future projects and investigations. This assignment aimed to integrate the 3 C's by allowing students to share the topics and mathematics they considered in their every day lives (Community), apply mathematical knowledge and skills they were already comfortable with (Classical), and develop their understanding of the role of mathematics in social issues (Critical).

Though some students enjoyed the assignment and strived to mathematize topics they cared about, other students showed little interest and did the bare minimum, if any, work. For example, some students turned in a copy of a data table from the newspaper or provided the scores from the weekend's football games. These students shared that they typically looked through the newspaper for anything with numbers and just turned that in. They complained of not seeing the relevance of the assignment to precalculus and not truly understanding what was expected for the assignment. In order to address these issues, Donovan attempted to provide more structure and direction to the assignment. For one week, he specifically asked the students to run a quadratic regression on a set of data. As a result, students dedicated a significant amount of time to learning the procedures for running regressions on their graphing calculators. Though this helped develop their Classical knowledge, the students typically did not focus on the context of the data. Their reports that week contained few conjectures relevant to the topic, instead focusing more on graphs and formulas.

Recognizing that Where's the Math was moving away from the intended goals of integrating the 3 C's, Donovan and I created a set of prompts to guide student mathematization. Student reports were to address the following four questions:

1. Summary of article in your own words (paragraph)
2. How did the <u>author</u> use mathematics in their argument?
3. How did <u>you</u> use mathematics to better understand the situation? (critique author, strengthen argument, etc.)
4. So what? Why does this information matter? What do you think of this situation? What can you do about it? How might math help?

For the first time, the resulting reports included harsh criticisms of authors and counter-arguments to conclusions. Students offered alternative interpretations and explanations of their positions. Though students were not necessarily using precalculus-level mathematics for their analyses, they were looking for and critiquing the use of mathematics (Critical). Furthermore, students shared during focus groups that the prompts helped them understand the assignment better and made them more likely to complete the assignment thoughtfully.

The second major assignment was a project on local unemployment and poverty. Donovan shared various articles and sets of data from recent reports on the national economic crisis that demonstrated how different demographic groups were affected. Students learned how the federal government defines poverty, minimum wage, and living wage. They then looked at local data to identify trends in median household income and home prices. To assist with this investigation, Donovan and I collaborated with a university professor engaged in participatory action research with local youth. The professor attended class once a week for several weeks to guide students through exercises aimed at identifying root causes to local poverty and possible steps that can be taken to address these issues. After several weeks of discussion and computer research, student groups presented posters on a specific action they think holds promise for reducing local poverty. Though most groups focused on increased funding for education, some argued for job creation through state investments in such enterprises as movie making and entertainment.

Throughout the project, several students engaged openly and honestly during discussions of inequity, poverty, and racism. During one exchange, a student wept while describing her experiences observing poverty in Haiti. During other conversations, a student openly shared racist beliefs about African Americans and Latin@s. However, most of the class did not actively participate during whole class conversations. Some students would work on other assignments, hold side conversations, or just put their head down on their desk. When it came time to do research in the computer lab or to present their posters, most groups had one or two students carry the burden of the work. During focus groups, students shared that they did not see the relevance of the project to their precalculus class. They saw days with the university professor to be breaks from "real math" and resented this distraction from their mathematical learning. Activities seldom connected to the mathematical topics discussed on other days and students were not applying any precalculus to the project. Additionally, since Donovan had not communicated a point value to participation during discussions or to the posters, students did not feel they had to actively engage in the project.

The third major assignment was the AIDS Lab (Davis, Moran, & Murphy, 1998). This lesson was taken from a laboratory manual designed to supplement a standard precalculus course by providing opportunities for students to work collaboratively on lengthy, context-based problems. The lab followed a mathematical modeling approach to understanding the spread of AIDS as logistic growth. The lab was a group assignment that culminated with individual written reports. Students had previously been assigned such labs on quadratic growth and price maximization. Donovan assigned the lab as the final activity of the semester and considered it to be 25% of the final exam.

The lab had students consider news articles and data concerning the AIDS epidemic of the 1980s. The investigation included plotting data, making estimations, and curve-fitting to develop student understanding of exponential growth, piece-wise functions, and logistic functions. The lab report asked students, among other things, to critique a quote from one of the articles on the spread of AIDS, to discuss the limitations inherent in an exponential model, and to comment on data for specific demographic groups, including African American women and South Africans.

During the three days of class that were dedicated to the AIDS Lab, student participation was very high. Most students were working diligently in their groups on the assignment and asking for assistance from the teacher when needed. Some students left their group to go work with other individuals. Though Donovan allowed for the lab to be worked on in groups, he did not communicate any directions for students to remain in their groups or to turn in a common report. As a result, the students who were typically seen to be best at mathematics were approached by several other students and asked to share their work. In the focus groups that followed the AIDS Lab, students praised the assignment and expressed their interest in doing more like it. Several students admitted that they originally only started doing the lab because it was part of their final exam grade. But once they started learning about the context and connecting the content, they appreciated the lab and thought the report was easy to complete. When asked which of the assignments over the course of the semester best integrated classical, community, and critical knowledge, the students unanimously chose the AIDS lab.

Over the course of the semester, Donovan implemented several different types of lessons. The three mentioned above were each distinctive in purpose, design, implementation, assessment, and student engagement. My classroom observations, interactions with students, and focus groups provide me with a unique perspective on teacher implementation and student experiences. At this early stage of my analysis, I am able to infer a few essential characteristics of the classroom learning environment that support critical mathematics education. In the following section, I will discuss these characteristics and offer implications for future implementation and research in this field.

DISCUSSION

As evidenced above, the goals, curriculum, and activities in a critical mathematics learning environment are quite different from those of traditional mathematics classrooms. Therefore, there can be significant resistance from students if they feel they do not see the mathematical relevance of the content. From my experience collaborating with Donovan, I have identified several classroom characteristics necessary to develop and support critical mathematics education: mathematical relevance, integration of the 3 C's, and overarching themes. These characteristics will help reduce student resistance to non-traditional instruction while promoting mathematization of personal, community, and social issues.

Mathematical Relevance

Throughout the first semester of Donovan's precalculus course, there were several instances of student resistance to activities that were not primarily focused on classical mathematics knowledge. In some instances, the resistance was blatant in the form of disengagement (e.g., putting head on desk), expressing discontent (e.g., "Why are we doing this?), or working on other assignments. During focus groups, students openly commented on the lack of relevance of such activities in their class. They considered the activities to be distractions from "real math" and questioned the validity of dedicating class time to "extra" activities. However, most students did not consider the activities to be unimportant. They valued the conversations and enjoyed learning about social topics. They just did not see why these topics were being discussed in a mathematics class. Several students commented that they thought these activities should be reserved for social studies courses or for a mathematics course particularly focused on social justice. Other students stated that they believed the conversations had mathematical relevance to precalculus, but they did not see exactly how. They expressed faith that the teacher and professor were doing valid work, but were unsure precisely how everything connected.

These student comments and resistance demonstrate the necessity of critical mathematics activities being seen as relevant by the students. The teacher should make every effort to help students see connections between mathematical content and the relevant social context. Students should believe that critical investigations are a valid use of class time and that their opinions and experiences are appropriate additions to class discussions. Creating such an atmosphere where students feel comfortable sharing, discussing, and debating while also seeing the mathematical relevance takes time and concerted effort by the teacher. Class activities focused on building community and establishing group norms should be implemented early and often in the course. Without student buy-in, curricular innovations will stand little chance of being accepted and appreciated by students. Furthermore, student resistance to alternative instruction may reinforce a belief that classical mathematics is the only "real" mathematics.

The importance of perceived relevance was evidenced in focus groups where students expressed frustration with activities not explicitly related to precalculus. Though some students enjoyed doing the Where's the Math assignments and participating in the Local Unemployment project, few believed they were appropriate activities for a precalculus class. As the semester went on, a shrinking portion of the class actually turned in the Where's the Math report. In contrast, most students appreciated the AIDS lab for obviously involving precalculus content in the investigation of a social issue. Almost all students turned in the lab and most received high scores.

Integration of the 3 C's

As stated above, Donovan had identified the goal of integrating classical, community, and critical knowledge into course activities. Though he found it easiest to implement classical lessons, the inclusion of students' personal experiences and opinion helped to develop student understanding of a particular social topic. Class discussions and projects engaged more students when they could share openly and connect their personal lives to the general topic. For example, students referred to their experiences with police response times in different part of town to humanize statistics on the city's emergency services. Also, when analyzing recent trends in college access, one boy shared about his mother going into debt to go to college. He assumed everybody could make such sacrifices. However, when another student shared that her mother worked tirelessly and they still could not access college loans and scholarships, he realized that there are multiple factors to college access.

Such opportunities to have students personally relate to the content lies at the root of critical mathematics education. The teacher must create opportunities for students to bring their experiences to the classroom and find them relevant for a true understanding of a social topic. Students have experiences and opinions that are not only relevant to the conversation but also essential for all class members to appreciate various perspectives. Identifying and accessing community and critical knowledge is a constant struggle, but one that critical mathematics teachers must pay particular attention to. They must create lessons, such as Where's the Math, which have explicit goals to address all 3 C's. However, as was evidenced in Donovan's class, implementation must help students see the relevance of such activities in order to minimize resistance, or else they will be a waste of time for both students and teacher.

Overarching Themes

With various goals for the classroom (i.e., mathematical, social justice), it is important for the teacher to provide students with overarching themes that help build connections across content, activities, and discussions. These themes can be focused on content, context, or process. For example, when his students investigated college-going trends, Donovan was able to relate the investigation to the larger mathematical concept of rates of change. Rather than being a mathematically irrelevant conversation, students saw how the topics could be examined using precalculus tools. This allowed them to see how college-going rates for different demographics changed historically and what direction they were heading in.

In the case of the AIDS Lab, students were introduced to mathematical content in order to better understand the spread of AIDS. When new concepts were introduced, Donovan was able to relate them to the AIDS investigation in order for students to make connections to previous content. For example, students first used exponential functions to consider the spread of AIDS in the 1980s. When they later learned about logistic growth functions, students were able to make sense of more recent AIDS data. The context provided a framework from which to develop student mathematical understanding.

Mathematization provides an example of an overarching theme focused on process. As Donovan facilitated conversations on social issues in class, he could make connections to mathematical concepts and demonstrate how to mathematize the topic. For example, when students raised questions about a local ballot initiative for school funding, Donovan was able to guide students through a mathematical investigation into state and local spending on education. Donovan modeled how to mathematize a social issue, while the students learned about school funding policies, trends in spending, and national rankings of state performance. With mathematization as an overarching theme to the course, countless topics can be analyzed mathematically and students can see the relevance of mathematics to aspects of their lives and community. This will also help address student resistance to non-traditional instruction, as students will develop their ability to mathematize and see mathematical relevance to activities focused on social issues. These efforts will help students see the validity of discussing social topics *in* their mathematics class and will promote student mathematization of their world *outside* of class.

Mathematical relevance, integration of the 3 C's, and overarching themes are essential attributes of a critical mathematics learning environment. The failure to implement any of these as classroom norms will result in increased student resistance to non-traditional instruction. For students to engage in learning in a classroom setting different from their previous 12 years of schooling, they must believe that the new system is valid and appropriate. They must find the activities to be relevant as they are guided into new ways of integrating their personal knowledge into their mathematics classroom. Broad themes must help them bridge concepts and connect across various content and contexts.

With the goal of offering practical guidance in mind, the following section will offer implications for future implementation of critical mathematics education in upper-level high school mathematics courses.

IMPLICATIONS

From my experience working with Donovan, it is clear that teachers attempting to reform their instruction towards critical mathematics education must commit a significant amount of time, energy, and resources to the endeavor. As mentioned before, Donovan and I dedicated weeks to preparing for the course. In addition, we spent countless hours reading, researching, planning, reflecting, and discussing. I cannot emphasize enough the importance of taking the time to thoughtfully prepare these courses and to continually make adaptations during implementation. There are no ready-made curricula for teachers to adapt for these courses. Therefore, the teacher must play the roll of curriculum developer, assessor, editor, and publisher.

In addition to the commitment of time, teachers of critical mathematics education must engage in active collaboration in order to constantly reflect on their work. As noted by Donovan, it is important for the teacher to have a true colleague with whom to discuss issues, consider curricula, and contemplate student learning. Such a relationship will also help the teacher stay focused on the difficult job of integrating the 3 C's, rather than settling for the simpler path of assigning purely classical lessons. Just as critical mathematics is a new experience for the students, it is a new challenge for the teacher. To struggle in collaboration with others will assist teachers in resisting traditional schooling practices.

Finally, Donovan's personal relationship with students has proven to be his greatest asset. It is essential that critical mathematics teachers know their students well and that the students see their teachers as confidents and allies. This relationship will provide the teacher with insights into what issues matter to students and how they may react to certain topics. They will be able to have honest and open conversations with their students and accept their critiques. Furthermore, students will place a level of faith in their teacher when they come to moments in the curriculum where they are unsure of the relevance or direction. They will trust that their teacher has their best interests in mind and is helping them build a bridge to the unknown. Without this relationship, the classroom may perpetuate banking education where the students passively accept the teacher's word as gospel or turn to outright resistance.

SUMMARY

I have positioned my work as Qualitative Participatory Research, utilizing critical ethnography to illustrate the experiences of participants in a critical mathematics learning environment. The voices of participants (all persons of color) are central to the data and analysis as I seek to make both the classroom and the research locations of praxis. Though there are various challenges and limitations to this work, I aim to demonstrate how a teacher, in collaboration with a researcher, can create a learning environment that fosters critical mathematical agency. This study may serve as "practical guidance" for teachers who aim to integrate critical mathematics into their instruction. As they embark on the endeavor, they should consider the suggestions for characteristics of the classroom that promote critical mathematics education: mathematical relevance, integration of the 3 C's, and overarching themes. Furthermore, they may consider the time necessary to prepare and implement critical mathematics activities, engage in collaboration with colleagues, and establish personal relationships with their students. In addition to informing future teachers, it is my hope that this research will inform endeavors to promote teacher education environments that support the development of these dispositions and skills in teachers in order to offer transformative educational experiences to their students.

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APPENDIX: FEMINIST CRITICAL MATHEMATICS FRAMEWORK

(Varley Gutiérrez, 2009)

Cycle of Activities	
Phase 1: Determine area of interest/concern	This might include neighborhood walks/photos, surveys or informal interviews (school, family, neighborhood, friends), individual/group brainstorm sessions, presentation of data related to student population /school/neighborhood/community, etc. [Possible sources of project focus: student discussion, current events, neighborhood changes (ex: road expansions, park design, legislation), common themes in student conversations, changes at school, student interview issues, request from school board/neighborhood association] **Note: This is also a good time to show examples of youth activism or community activism that the students can relate to.
Phase 2: Investigate the issue in depth in order to determine possible problems to address. Brainstorm possible problem.	This might involve field trips, internet or library research, etc. to investigate gaps in knowledge in order to inform problem formulation; problems should be able to be solved mathematically. **Note: This is a good time to show examples of the power of mathematics in making an argument and examples of problems that can be solved using mathematics.
Phase 3: Define problem/question. Determine a strategic plan for data collection and possible action components	Roles and groups should be formed and tasks distributed; Problem formulation should be collaborative and could be broken down into smaller questions/problems to be addressed in small groups.
Phase 4: Collect data, refine problems, begin to determine action plan	Possible data sources: interviews, measurements, internet searches, surveys (people, items: i.e. trash or graffiti), maps, etc; action plan could involve investigation of who makes decisions regarding issue or whom the issue affects.
Phase 5: Solve problems/represent mathematical investigation and collect supplemental data if necessary	Problems should be mathematical in nature and might involve supplemental mini-lessons or simplified problems on specific mathematical topics that are necessary for solving the problem; problem solution could involve generating more questions/problems, which should be noted and possibly investigated; supplemental data collection if the solution needs to be refined or expanded.

Phase 6: Synthesize your mathematical findings/Represent mathematical investigation/solution and determine presentation mode, finalize action plan	Keeping audience in mind, solutions should be refined and the mode of presentations/implementation determined and then prepared.
Phase 7: Implement action plan/ share results with appropriate audiences	Action plan could involve a presentation for elected officials, city council, school board, student body, community group, etc, organizing an information session/protest for community members, creating distributable media for peers/community, etc.
Phase 8: Reflection	Although reflection should be present in every phase of the process, this is a time to reflect through individual writing/interviews and group discussion; reflection could focus on the mathematics, the action component, the process, the collaborations, the successes and challenges, and should involve a discussion of generative themes for further projects- a re-starting of the cycle.

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Mathematics Discourse Communities: Monolingual Teachers and Issues of Teaching and Learning with Latinas/os

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Abstract
In the U.S., more and more teachers find Latinas/os in their classrooms. Yet, few are adequately prepared, both academically and socially, to address the unique strengths and needs of this growing population. This paper depicts the journeys of two middle grades teachers as they navigate through the complex reality of the mathematics teaching and learning process with linguistically diverse students. The concept of Mathematics Discourse Communities is introduced and elaborated in order to highlight the critical role of language development in the mathematics classroom, particularly how it conveys particular beliefs, values, and meanings surrounding students' ability to do and be successful with mathematics.

For roughly the past 25 years, there has been much discussion around and research conducted on the role of language in mathematics education (Schleppegrell, 2007). The interest in mathematical communication was accelerated with the inclusion of "Communication" as a standard by the National Council of Teachers of Mathematics (NCTM) in 1989 and again in 2000 (NCTM, 1989, 2000). Communication, however, has often been narrowly conceived as "talk about mathematics" (O'Connor, 1998). Recent texts addressing the mathematics teaching and learning of English Language Learners (ELLs) present the role of language in mathematics learning in relatively simple ways that do not reflect the complex social dynamics that impact language use and development (e.g., Bresser, Melanese, & Sphar, 2009; Kersaint, Thompson, & Petkova, 2008). It has become clear that new perspectives are needed on the way that language and literacy intersect with mathematics teaching and learning. Language, or mathematics discourse, can no longer be thought of merely as a mechanism to express mathematical thinking.

At the same time, discussions of Latinas/os in education inherently involve issues of language given their affiliation with Spanish, a language that holds a much lower status in the U.S. compared with English (largely due to a perceived need to unify the nation with an official language [see González & Melis, 2000]). While many Latinas/os are native English speakers, the majority is not (U.S. Census Bureau, 2005). Therefore, millions of Latinas/os enter schools with linguistic skills that are largely not valued or seen as resources by schools. In classrooms across the country, their "English learning" status is seen as an obstacle, a flaw that requires intense, compensatory attention to "fix" (Ruiz, 1984). Often times, the consequence of this position is to design learning environments that emphasize remedial skills, positioning English learners to fall further behind (Lipman, 2004). A preoccupation with learning English becomes the priority above all else (Gutiérrez, Asato, Pacheco, Moll, Olson, Horng, Ruiz, García, & McCarty, 2002).

The serious issue of underachievement in mathematics among Latina/o students calls for a re-evaluation of our orientations towards Latina/o learners, our conceptions of the intersection of mathematics and language, and the alignment of our mathematical teaching methods with these realities. A focus on how mathematics teachers create and develop Mathematics Discourse Communities (MDCs) has the potential to address all three of these elements.

Studying the development of Mathematics Discourse Communities among Latinas/os is important not only to "see" how they are socialized to and within the discipline of mathematics, but also to see how other realms of schooling intersect with the mathematics teaching and learning process. Despite common belief, mathematics is not an isolated field of study that produces an objective skill base with which students should walk away; rather, it is embedded in a complex, multi-dimensional social web of values, ideologies, and modes of operating. The teacher is the primary – though not exclusive – vehicle through which these (dominant) values, ideologies, and modes of operating are brought into classroom and transmitted to the students. However, students are agentive (not passive in the learning process) and unique (when compared to one another), and they, too, will bring unique perspectives forth in the mathematics learning process. Examining Mathematical Discourse Communities with Latinas/os will illuminate the points of contention and difference amongst students and between the students and teacher and how these contentious spaces lead to (mathematics) learning (Gutiérrez, Baquedano-López, & Tejeda, 1999) or alienation (Gee, 2004).

The primary question I aim to address is: *How do monolingual middle school teachers develop and utilize Mathematics Discourse Communities with Latina/o students?* There are three sub-questions that highlight particular foci: 1) What issues and challenges (e.g. sociopolitical, institutional, pedagogical, curricular) surround the teachers' development and utilization of Mathematics Discourse Communities? 2) What linguistic factors influence the development and utilization of mathematics Discourse Communities? 3) What ideological, knowledge, and skill factors influence the development and utilization of Mathematics Discourse Communities? Implicit in these questions are issues of the teachers' perceptions of Discourse and its relation to developing mathematically as well as the teachers' perceptions of the role of the first and second languages in mathematics Discourse and socialization.

In this paper, I will elaborate upon the concept of Mathematics Discourse Communities and include a partial list of indicators I used to recognize teachers' enactment of particular MDCs. Then, I will describe the context of the study and provide vignettes of the two teachers. The vignettes will reveal how each teacher perceives and addresses the unique strengths and needs of bilingual learners in mathematics classrooms, as well as the teachers' perceptions of the importance of Mathematics Discourse Communities when teaching Latina/o youth. An analysis of the data will follow each vignette. Finally, I will outline the implications of these findings and offer recommendations for practitioners and researchers.

Mathematics Discourse Communities

Understanding that language is unique to a particular social setting, Gee (1996) introduced the notion of Discourses (with a capital D). Discourses are

> ways of behaving, interacting, valuing, thinking, believing, speaking, and often reading and writing that are accepted as instantiations of particular roles (or 'types of people') by specific *groups of people*, whether families of a certain sort, bikers of a certain sort, business people of a certain sort, church members of a certain sort, African-Americans of a certain sort, women or men of a certain sort, and so on through a very long list. Discourses are ways of being 'people like us'. They are 'ways of being in the world'; they are 'forms of life'. They are, thus, always and everywhere *social* and products of social histories (p. viii).

Discourses, Gee argues, include much more than language and should be appreciated in its social context. Consequently, when investigating the role of language in any context, we cannot focus on language alone.

From this perspective, mathematics classrooms inherently maintain a unique Discourse community, a Mathematics Discourse Community. When combined with the work done on cultural social practices (e.g. Civil, 2007; Gee, 2004; Rogoff, 1991, 2008), the power of Discourse communities to socialize youth becomes apparent. Given that Latinas/os largely are not finding success in mathematics, examining the Mathematics Discourse Communities in which they learn becomes all the more necessary and urgent. Schieffelin and Ochs (1986) put forth that, in terms of language, there are two, concurrent socialization processes: "socialization through the use of language and socialization to use language (p. 163)." In other words, we not only learn language through social interactions, but are also socialized into particular communities of practice (Wenger, 1998) through particular language practices, or Discourses. Furthermore, membership in these particular Discourse communities is intimately connected to one's identity (Gee, 2001; Wolfram, Adler, & Christian, 1999; Wortham, 2006). Martin (2000) writes: "…it is my firm belief that detailed analyses of mathematics socialization and identity – and the multiple contexts that affect them – offer the best hope for understanding long-standing achievement and persistence problems (p. 186)."

The focus of this investigation is on the teacher's role in developing Mathematics Discourse Communities. Mathematics Discourse Communities involve ways of being, thinking, and speaking that are unique to a mathematics environment. While MDCs refer to the participants, the setting, and the interactions within the setting and between the participants, the process of being apprenticed into the specialized community can be thought of as socialization. The teacher, being the person of authority, is instrumental in the mathematics socialization process. Each teacher, as a result of their particular beliefs, values, and experiences, initiates the mathematics teaching and learning process in a unique way, and students interact with this socialization process incurring mixed results (in terms of affiliation with mathematics, for example) (Martin, 2000).

It is important, also, to be clear about the role of language in school. Social linguists and other education scholars have become increasingly articulate about the different styles of language that exist in schools and out-of-school contexts. Indeed, schools utilize and promote a specialized form of discourse (Heath, 1983; Gee, 2004, 2008), one that is often referred to as "formal language" or "academic language." Gee (1996, 2004, 2008) extends this argument to claim that each discipline (i.e. math, science, technology, etc.) or community setting (i.e. teachers, administrators, counselors, students, etc.) has its own specialized style of talk. This implies a need for teachers to be aware of, knowledgeable of, and deliberate with the ways in which they socialize students into these specialized forms of discourse (Valdés, Bunch, Snow, Lee, & Matos, 2005).

Gee (1996) also argues that classroom discourse is not a discrete feature of the classroom experience, but rather is inextricably and intimately connected to the particular ways one ought to act, think, believe, and value, with respect to a particular community. In mathematics, Franke, Kazemi & Battey (2007) contend that "students ways of being and interacting in classrooms impact not only their mathematical thinking but also their own sense of their ability to do and persist with mathematics, the way they are viewed as competent in mathematics, and their ability to perform successfully in school (p. 226)." From this perspective, language is more central to mathematics learning outcomes than previously thought. Accordingly, attention should be focused on how language interacts with other dynamics in the Mathematics Discourse Community. The need to study mathematics discourse, as it is developed in particular communities of practice (Wenger, 1998), becomes even more critical.

Potential Indicators of Mathematics Discourse Communities

While Mathematics Discourse Communities can be indexed by an infinite array of words, gestures, and actions, it is useful to articulate some of the more discrete indicators that play a major role in establishing distinct MDCs. These indicators are meant only to provide the reader a clearer picture of the way the two teachers view and convey the mathematics teaching and learning process, specifically with Latina/o learners. Table 1 outlines some of the themes that emerged repeatedly throughout the year and contributed to the respective Mathematics Discourse Communities established by each teacher. While not all of the indicators will be referenced in the vignettes, there are abundant examples of each indicator in the data set of the two teachers (for a more thorough description of the role of each indicator in the teachers' development of Mathematics Discourse Communities, see Willey, forthcoming).

Table 1. Indicators of Mathematics Discourse Communities

INDICATOR	**CLARIFYING QUESTIONS**
Tool Use in Mathematics	• Which tools do teachers explicitly or implicitly endorse when doing mathematics? • Which tools are neglected or discouraged from being used?
Utility of Mathematics	• For what reasons is mathematics deemed useful by the teacher? • How can mathematics help improve students lives?
Students Positioned as Competent	• Who can do mathematics? • Is everyone equally capable of succeeding in mathematics? • Which approaches are valued?
Participatory Patterns	• Which students are talking in whole-class discussions? • How do students engage in collaborative work?
Delivery of Mathematics Content	• Does the teacher solely hold and convey mathematical knowledge? • Is mathematics treated as a linear progression, in which certain skills and concepts must be mastered in order to proceed to more complex problem-solving situations? • Are students treated as knowledge generators? Do students have and share original mathematical ideas? • What is the medium of instruction (i.e. text, oral instructions, symbolic representations, note-taking, etc.)?

Language Development	• How is the specialized language of mathematics developed? • What opportunities do students have to read mathematics text, write mathematically, and represent mathematical situations using various methods? • How is mathematics vocabulary treated? • What kind of mathematical talk is elicited from students?
Habits of Mind	• What are the prominent mathematical habits of mind emphasized by the teacher? • How are the students taught to improve themselves mathematically? • What is the role of homework in the mathematical development process?

Context of Investigation

The two teachers, Ms. Hendrix and Mrs. Lenihan, teach at the same school, Southwest Elementary School. I have known the two teachers professionally for two years, because I have been working with them in their classrooms and have had them as students in Masters courses at the university. Our work together was initiated in the spring of 2008 when Mrs. Lenihan sought professional assistance from our research center in an effort to improve her knowledge base, mathematics curriculum, and instructional practices when teaching mathematics to Latinas/os and other students whose first language is not English. I began observing in her classroom, as well as Ms. Hendrix's, in May 2008. In June 2008, Mrs. Hendrix participated in a summer institute, designed and presented by the research center, in which district mathematics teachers learned about and collaboratively explored issues of language in the mathematics classroom. In an effort to support the teachers to continue their development around these issues, I joined the Southwest teacher team at their school at the start of the 2008-2009 school year. The data set for this investigation spans two academic years.

Our collaboration was initially centered on the complexities of integrating reform-based mathematics instruction with first and second language development. Each teacher articulated a personal instructional objective, towards which we would work together to accomplish. For example, Mrs. Hendrix expressed her desire to create lessons that would allow students more opportunities to use and develop mathematical language; Mrs. Lenihan wanted to explore various ways to develop students' ability to communicate mathematically through writing. While issues of language and mathematics were originally the centerpieces of our collaboration, group meetings eventually were taken over by teachers' preoccupations about curriculum coverage and standardized tests. It became clear that the teachers lacked a clear curricular vision for the school year. While curriculum certainly plays an important role in Discourse communities, we aimed to avoid this same divergence of attention in the 2009-2010 school year by mapping out a curriculum plan for the year in the summer months. Still, issues of curriculum coverage and pacing persisted, but we were also able to maintain our focus on the Mathematics Discourse Communities the teachers intended to implement with their students.

The data consists of fieldnotes, audio recordings of planning sessions and interviews, and video recordings of lessons. Collaborative planning sessions (the two teachers and I) took place once-per-week. This planning session eventually evolved into a sessions where we watched footage from past lessons and discuss what transpired and what might be done to improve upon lessons. In the beginning of the 2009-2010 school year, each teacher identified two classes on which they wanted to focus. For each teacher, one of those classes was the English Language Learner (ELL) class. Per administrative decision, the students scoring lowest on language proficiency assessments were grouped into one class. In 8th grade, the "ELLs" were prohibited from enrolling in the Algebra class, which is the other 8th grade section on which we focused our attention. The four classes were videotaped about twice-per-week.

For data analysis, I used two approaches to arrive at appropriate and accurate claims about the phenomenon of my investigation: grounded theory (and constant comparative method) and critical discourse analysis. Grounded theory (Glaser & Strauss, 1967; Corbin & Strauss, 2008) was utilized to isolate the most important themes that surfaced in the data. These themes resulted from an ongoing analysis of the empirical data and are developed in concert with the theoretical lens through which the data was evaluated. For example, I sought to discover themes pertaining to mathematical ideologies, the nature of student interactions, and the nature of mathematical discourse, the facilitation of mathematics language development, and the promotion of mathematical practices.

In order to place and maintain boundaries on my categories, I used cross-case analysis (Miles & Huberman, 1994). This technique entails comparing successive examples of a particular category with those originally found. By making continual comparisons, I was in a constant state of reflection; that is, I was continually evaluating whether instances fit the category or code. It facilitates the process of making sound claims based on the findings, because categories are held to well-defined terms.

With respect to critical discourse analysis, Gee (1996) argues that "language is inextricably bound up with ideology and cannot be analyzed or understood apart from it (p. ix)." This is the underlying rationale for discourse analysis. Because of the intimate relationship between language and ideology, created through sociohistorical interactions, discourse needs to be analyzed in a particular way. Critical discourse analysis aims to answer that call. Traditionally, most discourse analyses only consider the literal value of words; that is, what can be seen or heard in this moment. Gutiérrez and Stone (2000), however, argue for a critical discourse analysis that accounts for the vertical text, or the diachronic history of a text, which considers the sociohistorical context that informs and supplements what is being said.

The primary data source from which I drew my conclusions was classroom discourse. Another data source is the conversations I have with teachers. In both cases, what is said (and not said) and how it is said can help reveal how the person (teacher, student, or myself) is thinking about a particular issue. Additionally, throughout the data collection process I identified and relied on focus students to help me validate my interpretation of classroom interactions and gauge their affiliation with the instruction and curricular activities (Cobb, Gresalfi, & Hodge, 2009).

The School

Southwest Elementary School is located on the southwest side of a very large, Midwestern city. It is a unique school, because it operates two buildings a block apart from one another. Grades K-6 are in the main building, and the grades 7 and 8 are in the middle school "branch," a small building that lacks many amenities normally found in schools. For example, there is no cafeteria, specialty rooms (i.e. art, computers, music), or library. Both buildings are severely overcrowded, and the principal successfully obtained modular classrooms this year. In the main building this past year, two classes shared space in the library to conduct class.

The student body consists of nearly 1300 students. Approximately 70% of the students are Hispanic, 15% White, and 15% African American. Of the "White" demographic group, the majority is of Middle Eastern descent. 85% of students qualify for free or reduced-cost lunch, and 30% of students are enrolled in the bilingual or ESL program (though probably 75% of students are native speakers of a language other than English).

Vignettes

Mrs. Lenihan teaches mathematics to all of the 7th grade students, and Ms. Hendrix teaches mathematics to all of the 8th grade students. In 2008-2009, each of them was responsible for teaching an additional class of Language Arts (essentially, vocabulary development) to their homeroom class, and in 2009-2010, each taught an additional writing class.

<u>Mrs. Lenihan</u>

Mrs. Lenihan is in her third year at Stevenson, and her fourth year overall. Prior to Southwest, she taught at a high school in the "collar" suburbs of the city. She is a young, White, energetic, monolingual teacher who completed her teacher preparation at a well-known, local private university. She often speaks of trying to create a classroom that is "student-centered." As a result, students' desks are arranged in clusters, and on most days, the students are given an activity in which they are to work with their groups. Though she has access to traditional mathematics textbooks, she chooses not to use them with her classes. Instead, she strives to develop meaningful mathematics lessons that are project-based and grounded, as much as possible, in the lived experiences of her students.

Mrs. Lenihan often remarks about the importance of being able to communicate mathematically. For the past two years, she has attempted to develop her students' ability to write proficiently about mathematics problem-solving activities. Her motivation for this goal is largely rooted in the state's assessment program, which requires students to respond to multiple "extended response" items. Mrs. Lenihan has developed an extended response protocol (see Appendix A) and regularly draws on past test questions in order to help her students improve their mathematics writing skills.

Also, Mrs. Lenihan tries to cover the 7th grade mathematics topics through projects as much as possible. For example, she has designed a lengthy project called the Dream Home Project to connect many concepts, such as measurement, area and perimeter of polygons, surface area, scaling up, and cost formulas. Not only does this project activate students' creativity and engage students who might otherwise not fully engage in mathematical tasks, it is successful in tapping into family resources, or funds of knowledge, as students frequently rely on parents and other family members to help them through the complex process of envisioning, designing, constructing, and decorating their Dream Homes. Typically, the project is completed in pairs or groups of three, though working alone is an acceptable option.

The students reported that the Dream Home Project helped them master particular skills. For example, the students overwhelming claimed that this project allowed them to learn well how to measure with a ruler. Surprised that they had not learned this skill in the primary grades, I asked them to share their experiences with measurement in the elementary grades and explain how this measuring in the Dream Home Project was different. One student responded:

> When we were in elementary school, we really didn't measure things like we measure in 7th grade, like how do you use a quarter inch, or how do you use… We measured small things. The only time that we would measure things was on a test, or we'd just skim past it and that would be it.

Mrs. Lenihan offered this evaluation of her students' comments:

> I think what you're getting at is when we measure something now, it has to make sense; it has to all go together. Maybe in previous years or when I gave a quiz, I would just put lines on it [to measure]. You could say, "Oh, that's 1 ¾, or 2 inches, or 3 inches," but [now] it has to make sense. If it didn't [in the Dream Home Project], you knew your measurements weren't right.

This exchange uncovers an important distinction: teaching to cover topics and teaching for meaning. As it turns out, the students did have experiences measuring with a ruler. However, their experiences were likely limited to two dimensional shapes and test-like problems, activities that did not carry meaning when evaluated in light of their lived experiences. Mrs. Lenihan's Dream Home Project intended to provide a significantly different context, an authentic problem-solving situation. The measurement tasks were embedded in a larger framework and enabled the students to make meaning of measurement as it related to their lived experiences or future lives. As one student put it: "I learned how to construct walls and use creativity and imagine things as though I were a real architect."

Mrs. Lenihan developed another small project in which students identified their daily or weekly activities (in a bound unit of time) in order to examine the various representations of rational numbers (i.e., fractions, decimals, percents). The culminating activity of this project is to construct circle graphs representing the students' various daily or weekly activities. While this idea is not necessarily unique to Mrs. Lenihan's class, her theoretical approach to learning mathematics contrasts sharply from a teacher implementing reform-oriented curriculum through a traditional approach (see, for example, Brown, Pitvorec, Ditto, & Kelso, 2009).

Even though Mrs. Lenihan's lessons often operate within a larger project, some are delivered more traditionally. Below is a depiction of a lesson delivered during the rational numbers project mentioned above:

> As usual, the day begins with a warm-up. Mrs. Lenihan has worked hard to establish a routine in which students enter the room and immediately begin work on the warm-up. In order to institute this mathematics class norm, she walks around the room passing out tickets, which is part of a school-wide positive reinforcement program. At times, Mrs. Lenihan will also use tickets to reward students who have completed their homework. The tickets can later be redeemed for out-of-uniform passes, entry to holiday dances, or entered in a drawing for various prizes.
>
> Concurrently, the following warm-up appears on the board:
>
> Put the #'s in order from least to greatest: 8/3, 63/7, 0.25, 12½%

The students struggled with the meaning of 12½%. Many students argued that it equaled 1,250, apparently moving the decimal point two positions to the right. Mrs. Lenihan suggested that this equality did not logically make sense, and she asked the students to consider 12.5% "as a portion of a pizza." The moment Mrs. Lenihan was about to say "pizza," one student finished her thought with the word "whole," but his appropriate use of mathematics language went left unacknowledged. Mrs. Lenihan quickly instructed the students to "discuss in their groups why 12.5% is less than 1." After the two minutes and after she felt she had sufficiently convinced the students that 12.5% was a fractional part less than 1, Mrs. Lenihan offered the following summary:

L: So, 1,250 doesn't make sense; neither does 12.5 pizzas. The decimal has to be less than 1.

Expressing frustration with this hurried process of ordering abstract numbers, one student commented:

St: It's easier to do it on the board than talk about it.

Next, Mrs. Lenihan quickly moved on to the lesson of the day: converting fractional parts of a circle into percentage equivalents (e.g. 6/24 = ¼; ¼(360°) = 90°). She delivers this lesson in 4 or 5 minutes with the work-up from the previous class. Mrs. Lenihan is talking very fast in an effort to complete this lesson and the rest of her objectives in the short 40-minute class period. She does not conduct any checks for understanding, as there is no time.

Mrs. Lenihan quickly moves on to go over the rubric that will be used to assess the students in their construction of circle graphs. She engages a student in the interaction:

L: Chris, what does 'sum of fractions is a whole' mean? (referring to one of the criteria listed on the rubric)
C: Some of the fractions will be whole.
L: I see what you mean, like, some of my friends, s-o-m-e. In math, *sum* means "add 'em up."
C: Oh, I know what *sum* means, like the sum of the fractions add up to make a whole.

Moments later, Mrs. Lenihan works with the students to make sense of percentages greater than 100%:

L: So, what does 110% mean?

After no response from the ambiguous question, she adds:

L: …Like the expression "Give me 110% out there."

St: It means "give it all your effort, all of you, and then some that you don't even realize you have.

Capitalizing on this student's interpretation, Mrs. Lenihan masterfully facilitates a conversation to help students understand that 110% is more than 1.

Analysis

While this is certainly only a snapshot of life in Mrs. Lenihan's mathematics class, there are a number of themes that can be gleaned from these interactions. First, from dialoguing with Mrs. Lenihan and observing her class over the years, it is clear that she has wonderful intentions to maximize student engagement, provide plentiful opportunities for students to communicate (both in whole-class discussions and small group collaborations), and generally help students make meaning of mathematics concepts. However, there are institutional pressures that weigh on her (i.e. short class periods, frequent modifications of daily class schedules, obligatory external assessments, pressure to directly prepare students for what is typically on high-stakes assessments in an effort to raise test scores, etc.).

Mrs. Lenihan's good intentions and corresponding instructional practices are compromised by these pressures. For example, the implications of short class periods are observable daily. Classes at Southwest are 40 minutes, with zero minutes to pass between classes, essentially leaving 37 minutes for instruction. In the instance above, when Mrs. Lenihan was about to say "pizza" and the student finished her thought with the word "whole," Mrs. Lenihan may have missed an opportunity to capitalize on a brilliant student contribution, but given the context, it is more likely that she felt hurried to move along. Moreover, Mrs. Lenihan frequently has to end whole-class discussions abruptly, take over discussions for the purpose of making her point quickly, or carry over lessons to the following day. This results in a more disjointed flow of the curriculum (and pedagogy) and places the class behind their ideal pacing schedule.

Second, it has been my experience that ticket distribution for positive reinforcement campaigns and "character development" programs are wide-spread in urban schools. It is hard to criticize teachers' efforts to make clear behavior expectations and class norms, and acknowledge those who successfully accomplish them. However, there seems to be an underlying objective to use the program to curb non-conforming behaviors and nudge less productive students to do more. Thus far, there is no evidence that shows that not giving these students tickets motivates them to change their ways. The implementation of this program raises questions about how we perceive particular students and what ought to be done to change students' academic and behavioral patterns.

Third, the warm-up "problem" chosen by Mrs. Lenihan is decontextualized and illustrates how she has caved to pressures to familiarize students with test questions. By decontextualized, I mean that the "problem" only carries meaning to those with a firm understanding of rational numbers. Furthermore, only those students fluent in the discourse of rational numbers will be able to participate in sharing-out. With her students' "achievement" in mind, she selected the item because she is aware that there is always a problem like this on the state's standardized assessment. This type of "question", however, does not work well if Mrs. Lenihan's objective is for her students to cognitively understand the relationship between the different representations of rational numbers. The task makes it difficult to facilitate the learning of the concept, and as a result, the students struggled to make sense of the activity.

Finally, the student's aside about the difficulty of talking about complex mathematical idea speaks both to the inadequacy of the warm-up problem as well as the lack of opportunities to engage in mathematical communication. While Mrs. Lenihan regularly incorporates opportunities to problem-solve collaboratively and negotiate meaning, communicating mathematically is a difficult norm to establish in a matter of months – especially if students have experienced years of mathematics instruction centered around teacher explanations. To her credit, Mrs. Lenihan continually aims to foster productive group dynamics and mathematical responses from her students that reflect the specialized language of mathematics.

<u>Mrs. Hendrix</u>

Ms. Hendrix is in her third year at Southwest as well, and her fifth year teaching overall. Prior to Southwest, she taught for two years at an elementary school in a predominantly Latina/o neighborhood on the city's west side. She is a young, African American, relatively traditional teacher who completed her teacher preparation at a local state university. Ms. Hendrix's disposition is calm and collected, and her teaching style is consistent and traditional.

Though Ms. Hendrix speaks limited Spanish (e.g., *Tienes tu tarea?* [Do you have your homework?]), I classify her as monolingual. One of the reasons I make this classification is because she is either not capable or not comfortable speaking Spanish at length with Spanish-speakers (i.e., students, parents). For my purposes, if her limited Spanish skills serve no purpose in the school setting, they are effectively non-existent.

Ms. Hendrix's instructional style is teacher-centered, though she has ambitions to promote a more collaborative, student-centered classroom. For the first half of the 2008-2009 school year, students' desks were arranged in rows. Almost exclusively, classroom interactions follow an initiation-response-evaluation (IRE) format, meaning that Ms. Hendrix would ask a question, a student would be selected to respond, and then she would comment on the student's response and generally move on. Most questions are "known answer" questions; that is, there is a pre-determined solution, so students either know the answer or they don't, minimizing cognitive engagement. Below is an example of a typical day:

> As usual, the day begins with a Warm-up. Ms. Hendrix writes the following directions and problems on the overhead:
>
> Solve each 2-step equation.
> 1) (x/-9) – 3 = 10
> 2) 7y + 25 = -24
> 3) -8.3 = -3.5x + 13.4
> 4) (y+5)/11 = 3
>
> Ms. H: We went over the rules for solving 2-step equations. (St1 is at the overhead completing #1. After St1 completes his work, Ms. H evaluates by asking questions.)
> Ms. H: [Looking at the first step…] What do we do now?
> St1: Divide.
> Ms. H: We're already dividing.

St1: Multiply.
Ms. H: By what?
St1: Nine.
Ms. H: Positive nine?
St1: Negative nine.

Next, Ms. H engages in an aside and explains the long division process when dividing by a decimal (i.e., 21.7 divided by 3.5):

Ms. H: Move the decimal one place to the right, and do the same for the other number. Then, the decimal in the answer moves to the right. This doesn't mean that you can't have digits to the right [of the decimal]. You can keep dividing to the right.

The lesson continues with a worksheet walking the students through ratios and rational numbers. Ms. H wants the students to have a definition of rational numbers:

Ms. H: I'm going to give you the most kid-friendly definition: Rational numbers are a ratio of two integers, a fraction which can be converted into a decimal.

On the overhead, she writes "½ = 1:2; 1 out of 2" and leaves it at that.

One of the worksheet problems requires students to subtract 4.6 from 9. Ms. Hendrix explains that "the decimal points need to line up."

Predictably, the students become disengaged by the disjointed, seemingly meaningless tasks and begin to chat. One student asks, "Can we just skip this lesson?" I ask, "Why?" The student responds, "I don't get it." Another student chimes in, "It's boring." Ms. Hendrix gets tired of the chatting and declares that they will move on to "Notes." It is not lost on the students that this pedagogical decision is a form of punishment for their lack of obedience. Ms. Hendrix briefly scolds the students for "being behind where they should be."
Ms. Hendrix writes on the overhead:

NOTES: Dividing Decimals.
32.8/8.2 = 32.8 divided by 8.2. 8.2 x 4 = 32.8
32/8 = 32 8.

Sts: [Predicting the question.] Four.
Ms. H: How do you know?
Sts: Because 8 goes into 32 four times.

Ms. Hendrix writes a new problem on the overhead: 17.9/3.1

Ms. H: I want all of you, in your groups, to solve this, and then one of you from each group will come up here to present.

<u>Analysis</u>

Again, I acknowledge that this is only a snapshot of Ms. Hendrix's classroom. Still, I have chosen this episode because it is representative of daily classroom interactions, and important features of Mathematics Discourse Communities can be highlighted.

The initial portion of this lesson shows St1 reporting his work on one of the decontextualized warm-up problems. This is a common interactional patterns facilitated by Ms. Hendrix. In this scenario, she only has one student engaged, though it is her expectation that all students are following along and understanding St1's thinking. Ms. Hendrix does not appear to be worried about the other students' level of engagement, because they appear to be working and are not being disruptive.

Another example of the Mathematics Discourse Community created by Ms. Hendrix occurs when she offers an explanation of long division with decimals, but never stopped to ask why one moves the decimal point. The Algebra students don't seem to mind. But, while this approach may work for the Algebra class (the class with the highest scoring students), it is less effective with language learners. Talking through extensive procedures without attention to meaning or understanding is a dangerous routine to set and reflects Mrs. Hendrix philosophy on how mathematics ought to be delivered. For the students, mathematics remains rote and irrelevant, and they become detached.

Mrs. Hendrix regularly displays strong mathematical content knowledge. Above, when she isolates a potential hang-up on the worksheet (subtracting decimals from whole numbers), she shows that she can sufficiently predict the errors students might experience with the problem. This also shows, however, that the class becomes a collection of mathematical problem solving tips. This contrasts sharply with a classroom driven by authentic problem-solving tasks, conceptual meaning making, and opportunities to communicate.

The latter part of this episode is representative of how Ms. Hendrix incorporates group work into her lessons. Needless to say, the task does not warrant collaboration, but rather cues the students to solve it individually. There is nothing to discuss but the answer. Alternatively, Ms. Hendrix will tell students to "get with your buddy," a predetermined partner, in order to go over the answers to their homework. Clearly, Ms. Hendrix knows that there is value in having students work together, that learning is facilitated by discussing mathematical situations. However, it is also clear that she lacks an understanding of the type of problem-solving situation and the nature of interactions that ultimately advance students' understanding of mathematical concepts and help them develop meaning of mathematical concepts more generally.

In a university course in the Spring 2010 semester, I opened the course with the following questions: What do you think makes teaching bilingual learners special or different from simply good teaching? What does a teacher need to know or be able to do to instruct bilingual learners?

Ms. Hendrix had the following response:

> I think that good teaching is the key to instructing bilingual learners, so there isn't too much that is vastly different or "special" that a teacher must do. I believe that teachers do need to have a sympathetic or "open-mindedness" about their students' cultural backgrounds and how that will impact their educational experiences – this will enable bilingual learners to have a good connection with their teacher. In addition, teachers should try to gain info about their students' funds of knowledge and try to incorporate some of the cultural backgrounds of these students into the learning experience.

Ms. Hendrix is right about the importance of a solid, positive teacher-student relationship. Indeed, she treats her students respectfully and whole-heartedly cares about their education and well-being. But, it is apparent that caring is not sufficient. Being "sympathetic and open-minded" does not automatically initiate a transformative pedagogy. Similarly, while incorporating students' funds of knowledge, Ms. Hendrix does not typically attempt to activate and capitalize upon her students' unique knowledge base beyond a superficial level. Her opening belief, that "there isn't too much that is vastly different or 'special' that a teacher must do" in order to effectively facilitate bilingual students' mathematics learning, is a fair characterization of her actual teaching practices and the lack of accommodations she makes for her students, who are non-native English speakers.

Conclusions and Implications

It is well established that how we have been teaching mathematics is insufficient, especially for Latinas/os (e.g., Khisty, 1995; Khisty & Willey, 2008; Secada, 1995). Given the persistent mis-education of Latinas/os and the corresponding social realities they experience (Gándara & Contreras, 2009), there is an urgent need to evaluate classroom learning from new perspectives. No longer can we examine and view the mathematics teaching and learning process with Latinas/os solely through a cognitive lens, as if learning can be defined by what the teacher says and how well the student's brain absorbs the information. There is an emerging need to foreground particular components of the mathematics teaching and learning process, such as the role of language and the development of specialized discourse communities, the tools used to make sense of mathematics, and students' participatory patterns, among others.

Paying attention to Mathematics Discourse Communities requires us to focus simultaneously on the overall mathematics classroom environment, embedded in a school context and nestled in a sociopolitical reality, as well as the micro interactions that make up daily activities. Examining MDCs allows us to see the implicit and explicit messages conveyed to students about the discipline of mathematics and the mathematics teaching and learning process. It allows us to identify what norms and mathematical obligations the teachers intend to establish in their classrooms. It will begin to answer Cobb, Gresalfi, & Hodge's (2009) call to "understand not merely whether but why students have come to identify with their classroom obligations, are merely cooperative with the teacher, or are developing oppositional identities (p. 48)."

For teachers, reflecting on the community of practice(s) we establish makes evident to us what we believe about mathematics, how that manifests in classroom instruction and activities, and how easily our underlying beliefs and assumptions are portrayed in what we say and do. It will help us clarify what it is that we want our students to be able to do, important among them being able to seamlessly participate in sophisticated mathematics discourse communities. Utilizing the framework of Mathematics Discourse Communities will enable us to implement the strategies that promote productive micro interactions, ultimately achieving more of the results we desire. At the same time, it will help us weed out counter-productive assumptions – assumptions that contribute to the macro educational reality Latinas/os endure today.

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Math Extended Response

1. Question
What do I need to find out?

2. Facts
Write down all the information needed to solve the problem.
(You can use bullet points and use the facts to explain your answer)

3. Strategy and label
Use a strategy to solve the problem correctly. Some strategies include a t-chart, graph, picture, trial and error, and step by step mathematical process written on paper. Remember to write all mental work and calculator work on your paper. Label your work and circle the answer.

4. Explain
Explain in words how and why you did what you did to solve the problem. Use the facts from part one to explain your answer.

5. Answer
Write your answer in a complete sentence. Does it answer part 1?

6. **Check**
Does your answer make sense?

Visions from the Classroom—Focus on Teachers Discussion

The following section presents comments from the discussion that took place after the research presentations and the practitioner panel. The participants met in small groups that included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. We have captured multiple simultaneous discussions and have attempted to be as faithful as possible to the participants' comments.

The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

Whereas other working groups reflected on the connections that exist between this strand and the other strands as a separate reflection question, this group chose to respond to the connections within the context of the discussions. The following summary represents common themes identified within and across the working groups.

What do we know and what are the implications for practice and research?

Based on the research studies presented as part of the Visions from the Classroom—Focus on Teachers Strand, along with the poster sessions, and our own professional experiences, we are able to state what we believe to be true about the teaching of mathematics for English Language Learners (ELLs) and particularly with Latinos/as. For mathematics learning to improve for these students, we must as a society honestly examine these profound questions:

- Why educate Latino/a students?
- Why educate through this institution that we call "school?"
- Why are we learning mathematics? Is it to understand power structures and systems that are designed to make policy to protect white privilege?

"Why educate Latino/a students?" If we truly believe that educating "all" students will benefit our democracy and citizenship, then we must have a system that supports all teachers and students and reduces the marginalization of Latino/a students. Without a system in place, nothing will change for marginalized Latino/a students. We must redefine what it means to be a teacher and what we believe to be mathematics teaching. We know that teaching needs to be much more than merely the traditional explaining and discussing of mathematics problems at the end of each lesson. Teaching requires a context in which students engage in problem solving and discussions with their peers, and utilizes students' culture and home language as a resource.

The teaching of mathematics must be a scholarly endeavor. Teachers must understand and develop in their students an "interrelationship of community, critical and classical knowledge (the 3C's)" (Gutstein, 2006, p. 204). To be effective, teachers must not only have content and pedagogical knowledge, but also knowledge of sociopolitical, economic and cultural-historical workings of society. The complex social dynamics that impact language use and development must be recognized as a critical component in the development of mathematics discourse and "socializing" students into mathematics communities (Willey, 2010).

The current approach to mathematics education has been one of equality and not equity. "Equity does not mean that every student should receive identical instruction. Instead equity demands that respectful and appropriate accommodations be made as needed to promote equitable access, attainment, and *advancement* for all students" (Aguirre, 2009, p. 296).

In order to change the present inequitable conditions for Latino/a students, we must first recognize them. Presently, speaking a language other than English is viewed by the dominant society as a deficit and English fluency is held as a prerequisite to mathematics learning. Students are often placed in ELL classes focused on the learning of English and are denied access to classes in higher mathematics or have very limited instructional time dedicated to mathematics (Flores, 2007; Garcia, et. al. 2010; Mosqueda & Téllez, 2008; Scott, 2010). It is as if knowing English determines a student's mathematical ability or capacity to learn mathematics.

To compound the issue of language further, Spanish has not been a highly valued language in our society and those who speak it are often treated as second-class citizens. This bias and stereotyping is manifested in the classroom through low expectations of Latino/a students (Lopez-Leiva, 2010). "The teacher is the primary—though not exclusive—vehicle through which these (dominant) values, ideologies, and modes of operating are brought into classroom and transmitted to the students" (Willey, 2010, p. 2). Teachers must be aware of their own cultural biases and understand how their position of power influences their students' access to mathematics and also students' mathematical and bilingual identity. Teachers must examine how consistent their words and beliefs are to their actions.

Beginning in elementary school, student agency must be developed. Students must be aware of their rights as learners and be taught to leverage for those rights, think critically, participate as a member of a community of learners, and hold others accountable. Learning and using mathematics as a relevant tool in their lives to promote equity and social justice make apparent the value of education and are critical elements in countering student resistance in middle school.

Pressures from the current climate of high-stakes testing, benchmark assessments, language policies and institutional structures impede optimum learning opportunities for students and compromise teacher practice. Even the best-intentioned and most skilled teachers will continue to be pressured to alter their practices to conform to more rigid language and assessment policies, adhere to curriculum pacing calendars that establish the pace of lessons throughout the year, and be faced with the restructuring of instructional schedules and practices based on assessments given during the year.

There must be advocacy through coalitions of bilingual educators, mathematics educators, researchers, teachers, families, and other stakeholders to question and challenge these practices. Angela Valenzuela's work changing the assessment policies in Texas offers an example of positive changes that can happen (Valenzuela, 2005). Policy-makers, administrators, and the public need to be presented with more quantitative studies as evidence for needed changes. "Myths" often are substituted as general knowledge and the basis for policy decisions. The conversation must shift away from dropout rates and test scores to examine political, social, and economic structures that lead to the development of educational policies, curricula, and instruction.

Individual teachers working with a researcher will not have much impact. It will be necessary to develop a culture of collegiality with teachers and researchers forming collaborations working together for systemic change. Current professional learning community structures need to be examined to see if they are perpetuating the "status quo." Membership within these communities must be integrated by ethnicity and linguistic backgrounds for more expanded viewpoints and recognition of issues.

The work of teaching mathematics is complex and demanding. Teaching mathematics to ELLs requires even more specialized skills. Preservice and inservice teachers must be better prepared for this work. Being bilingual or even an English speaker does not necessarily mean that the teacher has facility with the specialized language of mathematics in that language. Careful attention must be given to this. Teachers must have foundational mathematics knowledge and their own mathematics anxiety needs to be addressed. In particular, the self-selection of teachers to elementary education who are math phobic must be addressed. Teachers have an important role in students' mathematical identity and math phobia transfers to students.

Professional development for teachers and experiences for pre-service teachers must include conversations about diversity, equity, white privilege, using language and culture as a resource, and critical mathematics knowledge. Video clips portraying diverse classrooms, service-learning projects and home visits all provide valuable experiences to develop teachers' cross-cultural understanding. These should become standard components integrated throughout the teacher preparation program, and continue during a teacher's practice.

Teachers need to be aware of cultural differences that may influence families' student expectations. Parental expectations may not be similar to those of the teachers. Knowledge of the ways families and students experience mathematics can be used as a resource and built upon in the classroom. It is important for students to recognize the mathematics their families use in their daily lives. We must redefine our traditional notions of parent involvement and address barriers that keep families from being connected with school (Civil & Andrade, 2003; Civil & Menéndez, 2011; Civil & Quintos, 2009).

Teachers of ELLs must be knowledgeable about the principles of second language acquisition and utilize strategies that support the ELL, for example honoring wait time and the "silent period" of second language acquisition while maintaining the curriculum. The use of their first language should always be a resource for the students to improve their access to the mathematics and make connections to their experiences. While there are a variety of educational tools, visuals and manipulatives that may be used with students, these practices do not ensure quality education for ELLS. "These students must contend with many more language layers than their native English-speaking peers and thus require additional scaffolds to acquire mathematical linguistic competence in English" (Barnett-Clarke & Ramirez, 2004, p. 57).

Teachers should be aware of confusions that may arise from misleading or ambiguous language. Phrases and words often used in the "mathematics register" may be misinterpreted if differences in meaning between the colloquial usage and the mathematical usage are not discussed (Barnett-Clarke & Ramirez, 2004). Students' "mistakes" need to be viewed by the teacher and students as learning opportunities.

Teachers must develop and support a "Mathematical Discourse Community" (Willey, 2010) integrating language and the students' culture as well as the classroom culture. Making learning of mathematics relevant for Latino/a students through meaningful, authentic, purposeful and intentional discourse is essential for access to powerful mathematics, developing academic language and specifically the mathematics register, and making connections. Learning mathematics is a social activity that requires two-way, meaningful, authentic, purposeful, and intentional interactions between students and teachers.

This is not a simple task. "Mathematical Discourse Communities" require expert guidance from the teacher to evolve, especially if this is a new experience for students who have been subjected to traditional classroom structures and curriculum. Resistance from students unfamiliar with non-traditional instruction may be encountered, and teachers must be prepared to reduce such resistance by addressing "mathematical relevance, integration of the 3 C's, and overarching themes" (Gutiérrez, 2010, p.17). Teachers involved in this endeavor need to be supported through professional learning communities, teacher study groups and/or collegial work with researchers. Currently, teachers are often working in isolation and have few "safety nets" to buffer them from administrative or political pressures.

Attention also needs to be given to the development of teacher leaders. Activities that support teacher leader development include: professional learning communities; involvement and partnerships with universities for research opportunities and disseminating knowledge as a result of the research; and making teacher practice public—not only an avenue for teachers to present their students and themselves as teachers to a larger audience, but also for students to see that their work is valued and that they have a voice.

Another serious concern addressed was the resistance to change in our secondary schools. Traditional structures in high schools such as departmentalization make incorporating cultural relevance into the curricula a major task. Veteran secondary teachers have historically been the most resistant group to change and often work in isolation. Also, there is an approaching shortage of secondary mathematics teachers. There will need to be systemic policy changes in secondary education and a change in the teaching culture to one of more collegiality. Secondary video clips and vignettes to be used in professional development need to be collected.

What questions do we need to research further?

We identified the following questions as needing further research and investigation:

- How do we prepare more and better bilingual mathematics teachers?
- What cognitive and affective domain strategies do effective teachers use to build positive, emotionally safe learning environments where students are engaged in conceptually challenging mathematics? How are they formed? What are some common steps that teachers take?
- What role does home language have in helping students learn mathematics?
- How do teacher beliefs about Latino/a students affect their teaching of these students? Also, what about other ethnically or racially diverse groups?
- How do we address "differentiated" cultural treatment of mathematics students?
- How do we engage a critical mass to make changes in current policies and beliefs?
- Is student resistance to "critical mathematics" and "social justice" activities generalizable to all students presented with this curricula? Are students resistant because they are concerned about their future mathematics options or are they resistant because this non-traditional curriculum does not resemble their prior mathematical instruction?
- How do we support teachers who are faced with administrative and/or political mandates that disregard equitable practices or have restrictive "English only" language policies?
- How do we develop teachers' commitments to supporting students both mathematically and culturally through using the home language as a resource?
- What kind of professional development programs can we create to address resistance by secondary mathematics education teachers?
- How do we address the upcoming shortage of qualified bilingual secondary mathematics teachers for Latino students?

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Curriculum

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Mathematics Curriculum and Latino English Language Learners: Moving the Field Forward

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Every year, large numbers of children move to the United States from other countries. Between 1979 and 2003, the number of children aged 5-17 who spoke a language other than English at home grew from 3.8 million (9% of that population) to 9.9 million (19%) [National Center for Education Statistics (NCES), 2005]. Some U.S. classrooms have one isolated English language learner (ELL), while others are composed of 100% ELLs. The U.S. has a multitude of different classroom configurations involving ELLs, but Latinos comprise the largest population of English learners in the United States (Garcia, 2001). Latino ELLs may be native-born or recent immigrants, but they come from homes and communities where two languages (Spanish and English) play a prominent role in children's lives. Consequently, we need to consider how to design and enact mathematics curriculum materials so that ELLs are supported in simultaneously learning academic content and acquiring a second language.

Recent curriculum and instructional recommendations emphasize cognitively-demanding mathematical learning tasks for all students, language-rich environments, and multiple modes of communication (NCTM, 2000; Steinbring, Bartolini Bussi, & Sierpinska, 1998). In addition, researchers in bilingual education (e.g. Mohan, 1990) argue that the development of the second language that is needed for schooling can be better achieved when students use the target language in active and dialogic communication that is purposeful, such as negotiating meanings of texts or solving a problem (Mohan & Slater, 2005). Yet, some studies indicate that curriculum materials incorporating these reforms may further disadvantage low-income, minority students, and Latinos specifically, and widen existing educational and social disparities between these students and middle-class White students (Lubienski, 2000; 2002; McCormick, 2005; Sconiers, Isaacs, Higgins, McBride, & Kelso, 2003), especially if students are silent non-participants in the classroom.

Unfortunately, Latino students, like many other linguistic minorities in the U.S., too often sit silently in mathematics classrooms (Brenner, 1998), and thus, do not have opportunities to use and experiment with language, especially the specialized language of mathematics content. They also encounter barriers as they work with curriculum materials (Doerr & Chandler-Olcott, 2009). Furthermore, their interactions with the teacher typically involve responses to low-level questions that only require simplistic language use and do not facilitate participation or significantly advance second language development (Duran, 1987; Kramsch, 1998; 2002). These ineffective teaching practices are likely due to the fact that most teachers have not participated in professional development related to teaching ELLs (Wenglinsky, 2002) and do not feel prepared to teach ELLs (Lewis et al., 1999). Thus, teachers do not have the necessary knowledge base to adapt or enact mathematics curriculum materials or to help students negotiate meanings through dialogic communication (Silva, Weinburgh, Smith, Barreto, & Gabel, 2008). These realities have facilitated one of the most disturbing and persistent patterns of underachievement for Latinos in U.S. schools, particularly in mathematics.

Changing these realities and improving the teaching and learning of mathematics for Latinos in the U.S. will require attention, research, and investment in many areas including: policy, teacher education and professional development, teaching practice, family engagement, student support programs, student assessment, and curriculum. However, for the purposes of this paper, I will focus the discussion on curriculum as it relates to Latino ELLs and other linguistically-diverse students.

FRAMING THE COMPLEXITY

The term, *curriculum*, has different interpretations (CSMC, 2005) or different phases (Stein, Remillard, & Smith, 2007). Some educators and parents assume *curriculum* means materials such as textbooks, while others assume it means the standards that identify what mathematics students should learn (Jackson, 1992; Phillips, 2009). As we discuss issues related to curriculum and Latinos, we need to consider the different interpretations of curriculum and the forces that influence the nature of these interpretations. Figure 1 shows a curriculum research framework created by researchers associated with the Center for the Study of Mathematics Curriculum (CSMC). See Tarr et al., (2008) for an extended discussion of this framework.

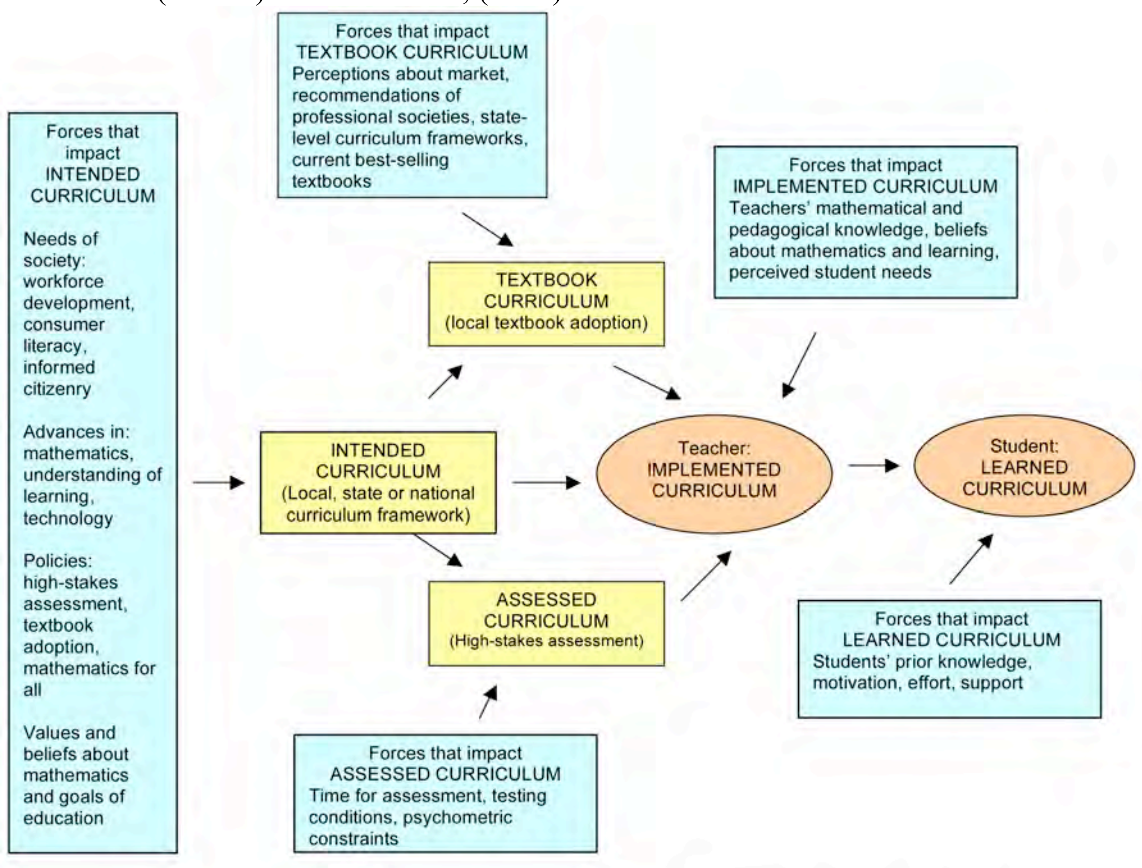

Figure 1. CSMC framework

This framework can facilitate our discussions as we work together to articulate what we currently know and what we need to know regarding different aspects of mathematics curriculum and Latinos.

Recent calls for rigorous academic standards (NCLB, 2002), researched-based curricula (Clements, 2007), mathematics curriculum materials that enhance student learning (Whitehurst, 2003), criteria for evaluating curricular effectiveness (NRC, 2004), as well as public controversy (i.e., the math wars) have elevated attention placed on mathematics curriculum. Different researchers have targeted efforts at different aspects of curriculum resulting in publications related to examining mathematics curriculum standards (Reys, 2006), the design of Standards-based curriculum materials (Hirsch, 2007), teacher's use of mathematics curriculum (Remillard, 2005; Remillard, Herbel-Eisenmann, & Lloyd, 2009), and the relation between mathematics curriculum and student achievement (Slavin & Lake, 2008; Slavin, Lake, & Groff, 2009). Yet, these publications rarely mention Latinos or other linguistically-diverse students. Moreover, Willey and Pitvorec (2009) analyzed seminal publications in mathematics education such as the NCTM *Principles and Standards* (2000), *Classics in Mathematics Education Research* (Carpenter, Dossey, & Koehler, 2004), *Lessons Learned from Research* (Sowder & Schappelle, 2002), *Research Companion to the Principles and Standards of School Mathematics* (Kilpatrick, Martin, & Shifter, 2003), *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992), and the *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007) and also found minimal or no references to language diversity.

At this point, there are a small number of studies related to mathematics curriculum and Latinos. For example, in searching the NCTM online database for articles published in the Journal for Research in Mathematics Education (JRME), I found four studies that included "Latino" or "Mexican" in the abstract (Fuson, Smith, & Lo Cicero, 1997; Gutstein, Lipman, Hernandez, & de los Reyes, 1997; Gutstein, 2003; Hufferd-Ackles, Fuson, & Sherin, 2004;), but only one author included both, "Latino" and "curriculum" in the abstract (Gutstein, 2003). In the following sections, I summarize studies involving Latinos and curriculum design, curriculum analysis, curriculum implementation, and the influence of curriculum on student achievement.

Curriculum Design

Fuson et al., (1997) conducted a year-long teaching experiment in two first-grade classrooms (one English-speaking and one Spanish-speaking) with predominantly low-income, Latino student populations. The authors created learning activities related to 2-digit quantities as tens and ones that "fit the ecology of urban, Latino classrooms" (p. 739). To inform the design of the learning materials, the authors analyzed the structure of Spanish words for two-digit numbers. The authors found that most children in the Spanish-speaking classroom demonstrated sequence-ten strategies, while more children in the English-speaking classrooms demonstrated separate-ten strategies. However, by the end of the year, students from both classrooms predominantly demonstrated tens-and-ones thinking and outperformed comparison groups. Fuson et al., document the complex process they used to design and research learning activities specifically developed for Latino first-grade students. Although, this publication narrowly targets specific mathematical concepts at one grade level, it does provide a unique glimpse into the design of learning activities for Latinos.

Silva, Weinburgh, Smith, Barreto, and Gabel (2008) briefly describe their efforts to design mathematics and science curriculum materials for recent immigrants in grades 3-5. The purpose of these materials is to help ELLs gradually transition into mainstream classrooms over a two- to three-year period. Freeman and Crawford (2008) created supplementary curriculum materials, but for an online environment. These materials titled, Help with English Language Proficiency (Help) Math, are designed for teachers to individualize instruction and remove linguistic and cultural barriers to the learning of mathematics. Both the Silva and Freeman efforts have focused on the development of supplementary curriculum materials for ELLs. Curricular resources for after-school, remedial, or home environments are needed for specific contexts and purposes; however, current resources are inadequate for ELLs (Seeley, 2005). Consequently, additional curriculum design efforts (for both supplementary and mainstream mathematics curriculum) must be prioritized to provide opportunities for Latinos and other linguistically-diverse students to successfully learn mathematics in U.S. mathematics classrooms on a daily basis. Furthermore, we know very little about the curriculum design process for existing mathematics materials that have considered ELLs during the design and testing phases. The development of future mathematics curriculum materials needs to involve research at every phase and the knowledge that is generated through this process needs to be disseminated to the field (Clements, 2007). We cannot lose sight of the users throughout this process as Clements argues:

> Thought should be given to the students who are envisioned as users and who participate in field tests; a convenience sample is often inappropriate, such as when a curriculum is designed for "all" or specifically at-risk students and yet the field testing is done in affluent schools. (p. 47).

Curriculum Analysis

Willey and Pitvorec (2009) analyzed the teacher materials for two Standards-based mathematics curriculum programs to examine the messages conveyed about ELLs. They found that the teaching suggestions targeting ELLs were reductionist in nature and did not communicate the complexity involved in teaching ELLs. They also analyzed the core features of the supplementary curriculum materials, *Finding Out/Descubrimiento*, which demonstrated significant improvements for ELLs in both development of content and language (De Avila & Duncan, 1980; Cohen, 1986). Willey and Pitvorec also compared the features of *Finding Out/ Descubrimiento* with the features of two Standards-based curriculum materials. Based on their analysis, they created an analytic framework with 15 core features for the purposes of evaluating ELL access in a mathematics curriculum. This work shows great promise as we consider how to facilitate improvement in the mathematics curriculum design process specifically in relation to teaching Latino ELLs. The field needs frameworks that will help us analyze existing curriculum materials, but also to guide the design of future curriculum materials. Overall, there have been insufficient research efforts in analyzing mathematics curriculum materials and designing mathematics curriculum materials so that Latino ELLs are successful in learning mathematics in U.S. classrooms.

Studies of the Implemented Curriculum

Some researchers have investigated teachers' use of curriculum materials—the implemented curriculum—in classrooms with Latinos. Most of these research efforts have been situated in classrooms using Standards-based curriculum materials funded by the National Science Foundation. I highlight a few examples in the following paragraphs.

Gutstein (2003) conducted a two-year study while he taught an honors-track mathematics class to Latino seventh and eighth grade students in an urban school. One of the purposes of his study was to understand the relationship between a Standards-based curriculum program, *Mathematics in Context*, and teaching for social justice. As a result of combining the *Mathematics in Context* curriculum materials and supplementary projects with his teaching approach, the students changed their attitudes and orientations toward mathematics. Gutstein argues that *Mathematics in Context* potentially helps to support social-justice pedagogy and Standards-based curriculum materials "can theoretically promote equity, but certain conditions may need to exist" (p. 37). He also reported that Latinos in his study did not always connect with the real-life contexts presented in *Mathematics in Context*. This is an important finding that I discuss below. Gutstein writes, "No single curriculum will be relevant to all students, and a real-life context is not necessarily a meaningful one" (p. 63).

Hufferd-Ackles et al., (2004) conducted a year-long case study of one third-grade classroom in an urban Catholic school. All of the students in this class were Latino and the teacher was bilingual. The teacher implemented the research-based mathematics curriculum materials, *Children's Math Worlds*, developed by Fuson and her colleagues. Based on the data collected in this classroom, the researchers created a framework that outlines the developmental trajectories in both student- and teacher-actions of a Math-Talk Learning Community. They identify four components that capture growth of the math-talk learning community over time: (a) questioning, (b) explaining math thinking, (c) source of mathematical ideas, and (d) responsibility for learning, and describe how the learning community in the third-grade classroom grew over time. The authors attributed movement from Level 0 (i.e., traditional teacher-directed classroom with brief answer responses from students) to Level 1 (i.e., Teacher beginning to pursue student mathematical thinking. Teacher plays major central role in math-talk community) in all the components partially to the use of the curriculum materials. They also found fluctuations in the upward trajectories in levels when new topics were introduced. These fluctuations were attributed to the fact that students had to learn new language and representations when they were introduced to new topics and this required more direct involvement of the teacher.

Doerr and Chandler-Olcott (2009) conducted a multi-tiered teaching experiment in a K-8 setting that was 11% Latino and 20% ELL. Five of the six middle-school mathematics teachers from the school participated in the study. The authors identified barriers, especially in relation to the writing demands placed on students that the middle-school teachers encountered in implementing the Connected Mathematics Program (CMP) (Lappan et al., 2004). Initially, the teachers reported that the readability and vocabulary presented significant challenges for the students and that the corresponding teacher materials provided little guidance. When students struggled to write productive responses to writing prompts, teachers responded to this barrier by skipping the writing prompts. However, when teachers began to view these barriers as opportunities for student learning, recognized the importance of supporting students' ability to write, and developed "writing plans," there was a significant shift in teacher practice and student written responses.

Brenner (1998) conducted a study of two first-year teachers implementing an algebra program called College Preparatory Mathematics: A Change from Within (CPM). Brenner argues, "There is also a lack of information about how innovative high school curricula, including CPM, have been implemented, particularly with regard to the needs of students with non-English speaking backgrounds" (p. 106). Brenner specifically examined patterns of large group and small group instruction during the last six weeks of the school year. The CPM curriculum program explicitly states the expectation that small group interactions should be the site for the generation of most mathematical knowledge within the class. However, one of the teachers in Brenner's study did not enact this expectation. Moreover, students avoided participation in large-group discussions and two-way communication between the teacher and students was not achieved in this classroom. As a result, the ineffective enactment of CPM hindered the students' learning of mathematics.

Studies Examining Mathematics Curriculum and Student Achievement

In recent years, some researchers have compared student achievement in classrooms using Standards-based curriculum materials and traditional materials (e.g., see Senk & Thompson, 2003; Reys, Reys, Lapan, Holliday, & Wasman, 2003). Other studies have compared achievement of student subgroups considering SES, gender, and ethnicity (e.g., Harwell et al., 2007). Yet, most of these research efforts have not examined achievement of Latino students. For example, Harwell et al., (2007) studied student achievement in five districts and compared African American, Asian, and White students, but there were only 11 Latino students across the districts so Latino results were excluded from analyses.

Other studies have compared ELLs with other groups, but have not reported the data for Latinos, specifically. For example, Heller, Curtis, Rabe-Hesketh, and Verboncoeur (2007) examined the effects of the Math Pathways and Pitfalls (MPP) program on ELL and non-ELL participants. The MPP program combines professional development and supplemental intervention lessons (available in English and Spanish) for students in grades K-8. A study using a cluster-randomized experimental design involving 99 teachers and 1,971 students was conducted in grades 2, 4, and 6 in 40 schools in 5 districts. The researchers found that student math performance in MPP classes was higher than students' performance in the comparison groups at all three grade levels. In comparing ELL and non-ELL groups, MPP impacted both groups equally in grades 2 and 4, but there were higher gains for ELLs in the grade 6 group. In another study, Resendez and Azin (2006) compared ELLs from California using *Saxon Math* with ELLs not using *Saxon Math* in elementary and middle grades. At the elementary level, ELLs using *Saxon Math* performed better on the CAT 6 than ELLs not using *Saxon*, however, ELLs not using *Saxon* outperformed *Saxon* students on the Stanford 9 and California Standards Test. At the middle school level, ELLs using *Saxon* outperformed ELLs in the comparison group on all three tests.

A small number of studies have investigated the effects of curriculum materials on Latinos as measured by achievement tests. I highlight two studies that indicated positive effects and two studies that did not. Webb (2003) published one of the few studies that examined curricular effects on Latinos at the high school level. This study compared students using the *Interactive Mathematics Program* (IMP) with students in traditional college preparatory courses. He found that a higher percentage of Latino students in the IMP sequence completed more high school mathematics courses than students in the comparison group (i.e., 80% of the IMP group and 62% of the comparison group completed 3 years while 23% of the IMP group and 11% of the comparison group completed calculus).

Riordan and Noyce (2001) examined student performance on the Massachusetts state assessment for students in grades 4 and 8. They compared performance of *Everyday Mathematics* and *Connected Mathematics* with matched peers from schools using a mix of traditional programs and curricula. They analyzed data from 13,433 students including data for 245 Latinos and 1,195 students who received free/reduced lunch. In general, students (including Latinos) who used the *Everyday Mathematics* and *Connected Mathematics* curriculum materials outperformed their respective comparative groups. However, the authors caution:

> One limitation of this study is that it addressed a population of target curriculum and comparison schools that are relatively advantaged: they are schools that have a small percentage of students eligible for free and reduced price lunch and that are predominantly White, at least when compared to schools in the rest of the state…further study should be done on the impact of these standards-based programs in schools with a larger representation of low income and minority students (p. 391).

The ARC Center Tri-State Student Achievement Study (Sconiers et al., 2003) was a large-scale study that involved 51,340 students including 3,002 Latinos who had studied either, *Everyday Mathematics*, *Math Trailblazers*, or *Investigations in Number, Data, and Space* and 49,535 students in the comparison group. The findings in this study did not indicate positive results for Latinos. The authors write:

> With the exception of probability and statistics, virtually all of the effect sizes for Asians, Blacks, and Whites are highly significant and favor the reform students within each racial subgroup. The results for Latinos, however, are quite different. None of the effect sizes for "math," "total," computation, algebra, and probability and statistics are statistically significant for Latinos. The effect sizes for measurement and geometry are both positive and significant; each however, is smaller than the corresponding effect size for Asians, Blacks, or Whites. (p. 16).

A smaller scale dissertation study (McCormick, 2005) examined student performance of 380 third graders from eight urban schools using the second edition of *Investigations* and a comparison group. In this study, the higher-SES students had higher gains than their lower-SES peers and Whites had higher gains than their African American and Latino peers. The normalized gain scores for the low SES and minority students were no different than those of their non-Investigations counterparts.

A few studies have examined curricular effectiveness determined by student achievement for Latinos and ELLs as mentioned above. The studies that have been conducted have examined different grade levels and curriculum materials using different methodologies and measures of student achievement. Different comparisons have been used including Latinos versus other ethnic groups, ELLs versus non-ELLs using a specified set of curriculum materials, and ELLs using Curriculum X versus ELLs not using Curriculum X. These studies have various limitations such as not examining Latino ELLs specifically, not including a sufficient sample of Latinos, not including a representative sample of Latinos, and not considering textbook integrity (Chval, Chavez, Reys, & Tarr, 2009). Curriculum implementation is an uneven process within and across schools (Grouws & Smith, 2000; Kilpatrick, 2003; Lambdin & Preston, 1995; NRC, 2004; Senk & Thompson, 2003; Snyder, Bolin, & Zumwalt, 1992). Kilpatrick explains,

> Two classrooms in which the same curriculum is supposedly being "implemented" may look very different; the activities of teacher and students in each room may be quite dissimilar, with different learning opportunities available, different mathematical ideas under consideration, and different outcomes achieved. (p. 473)

Therefore, studies investigating the relationship between curriculum materials and student achievement cannot ignore what actually occurs in classrooms (NRC, 2004; Chval et al., 2009). We also need to investigate the conditions that are necessary for specific mathematics curriculum to be effective for Latino and Latino ELLs. Finally, results from these studies have not been consistent suggesting that future studies must address the limitations from previous efforts.

CHALLENGES IN THE FIELD

Research such as the studies mentioned above have heightened my interest in the important role of curriculum in facilitating the learning of mathematics for Latinos in U.S. classrooms. In addition, during the past 20 years, I have had the opportunity to work in and research contexts involving both mathematics curriculum and Latinos. For example, I co-authored NSF-funded mathematics curriculum materials for students and teachers and served as an ELL consultant to write teacher materials related to teaching ELLs. I collaborated with preservice and practicing teachers to improve the teaching and learning of Latinos in teacher preparation and professional development settings. Furthermore, I studied an effective teacher of Latinos (Chval, 2001; Chval & Khisty, 2009) and ineffective curriculum enactment through the eyes of Latino ELLs using head-mounted cameras (Chval et al., under review). These studies took place in communities in urban, rural, and small-city settings in classrooms with 1-7 Latino ELLs or in one case the student population was 100% Latino. Studying these different contexts shed light on some of my ignorance as I witnessed significant differences in the mathematical experiences of an isolated Latina in an fourth-grade classroom as compared to classrooms that were primarily Latino. These experiences have also elevated my concern about the influence and role of curriculum materials in facilitating mathematical learning for Latino ELLs in different types of classroom settings.

Teachers must consider how to enhance and enact curriculum materials for ELLs as these students spend most of their time in mainstream classrooms (Anstrom, 1997). However, successfully enacting curriculum materials to facilitate the participation of Latino ELLs is challenging and teachers need support as they begin to improve this aspect of their practice. This past September, I began to work with five third-grade teachers to improve the teaching and learning of mathematics for Latino ELLs. Every week, I met with the teachers to plan lessons using their district-adopted textbooks, to videotape two lessons in each classroom, and to interview them individually about their lessons using selected video clips. I also conducted interviews with 13 Latino ELLs and their parents. As we worked together, we carefully considered how the curriculum materials hindered or facilitated student participation and learning. Throughout this process, we recognized two challenges related to curriculum:

1. The design of mathematics curriculum materials must consider Latino ELLs and other linguistically-diverse students.
2. Teachers must learn how to analyze, enhance, and enact curriculum materials to facilitate the mathematical learning of Latino ELLs.

These challenges affect large numbers of teachers and students in U.S. classrooms and must be addressed on a large scale to ensure that Latino ELLs have the necessary resources to be successful in learning mathematics in U.S. schools. In the following sections, I elaborate on each of these challenges.

Challenge 1: The design of mathematics curriculum materials must consider Latino ELLs and other linguistically-diverse students.

Mathematics textbooks and curriculum materials on the market have been designed and created using different design principles, development processes, and priorities, written by different authors, and disseminated by different publishing companies (e.g., see Hirsch, 2007). Fifty years ago, mathematics curriculum materials did not include additional resources for ELLs or their families. However, in recent years, state adoption policies for mathematics textbooks have required textbook components that address the needs of ELLs. As a result, textbook publishers have responded by investing additional resources to hire ELL consultants to create materials for teachers (e.g., notes about teaching ELLs in the teacher's guide and supplemental handbooks), parents (e.g., translated letters), and students (e.g., translated materials and additional activities). This investment has been a step in the right direction, but unfortunately, the effort to address ELLs in the curriculum development process has been "added on" after the curriculum materials have been created. In other words, after the authors have written the lessons and possibly the teacher materials, consultants have been hired to write or translate the ELL component to be added onto the "curriculum package." This approach is better than making no adaptations at all, but we need more if Latino ELLs are going to be successful learning mathematics with mainstream curriculum materials in the U.S. We must work together to discuss questions such as:

- Are current curriculum materials helping Latino ELLs learn mathematics? How do we know?
- What curricular resources would be most helpful for teachers, students, and parents?
- How would the textbook design process change if greater attention were given to ELLs?
- If we were to imagine curriculum materials that addressed the needs of ELLs, what would be the critical features or characteristics of these materials?
- As a community, how can we influence publishing companies so that future design efforts involve (a) ELL experts working along side author teams to conceptualize design principles for ELLs, (b) ELL experts working interdependently on author teams to design mathematics lessons and related materials for teachers and parents and (c) research to inform and guide the curriculum design process (Clements, 2007)?

Future curriculum design efforts must involve significant attention to language diversity. As a field, we need to articulate what this means and what it would look like. We also have to determine how to prioritize the research efforts given our constraints (i.e, human capital and financial resources). I offer one example that illustrates the complexity of this challenge. Many educators, curriculum developers, and researchers have argued that mathematics curriculum materials must include rich mathematical tasks and contexts that facilitate the participation of ELLs. For example, Diez-Palomar, Simic, and Varley (2007) write:

> We have found that to establish a mathematics learning community with students, it is equally important to provide rich mathematical contexts and to constantly make conscious efforts towards community building. Finally, it is an important aspect of incorporating students' funds of knowledge into a culturally relevant mathematics curriculum for teachers to learn more about their students' lives and experiences (pp. 28-29).

Although many authors have written about the importance of rich mathematical contexts, Gutstein (2003) cautions students often feel that they cannot relate to the contexts that are presented in mathematics curriculum materials. In addition, too often, teachers, especially at the secondary level, feel they do not have time to help students read contextual problems or build meaning for all the contexts involved in a typical mathematics lesson. Dossey (2007), while reflecting on common lessons learned by authors in the design efforts of 15 Standards-based curriculum programs wrote,

> Reading in context was a problem across the grades. Care was taken in revisions to lesson the amount of unnecessary reading while working to build the crucial reading skills that are necessary for doing mathematics. This is a continual balancing act and one that is made more difficult for students who speak English as a second language. Continued work remains to be done here. (p. 192).

As educators, developers, and researchers consider the design of future curriculum materials, we must ask important questions about contexts such as: What roles should contextual problems play in curriculum materials? How do authors select contexts when they write materials? Does the context facilitate or hinder the learning and application of mathematics, especially for Latino ELLs? How do teachers respond to and enact contexts they encounter in their curriculum materials? I use three vignettes from Mrs. Bristow's third-grade classroom this past semester to illustrate why context in the curriculum design process needs further consideration.

Mrs. Bristow was preparing to teach a 1-2 week unit from a curriculum program that used a T-shirt factory as a context. The teacher assumed that all of her students, including the Latino ELLs would benefit from discussions throughout the unit to help the children build meaning for terms used in the lessons (e.g., factory, inventory, supply, demand, and shipping). To open this discussion, she decided to show a tour of the Crayola Crayon factory filmed by CBS News and available online (http://www.cbsnews.com/video/watch/?id=5305844n). She knew that all the students used crayons and would be interested in learning how they were made. Every child in that classroom was fascinated by this video and responded throughout the clip with, "Cool!" Mrs. Bristow knew that the students would solve problems about boxes of T-shirts that would be shipped. As a result, she decided to stop the video to point out the shipping boxes in the background, and asked, "What do you think they use all these boxes for? Mrs. Bristow's introduction to the factory context was effective and helped students access the mathematics involved in the curriculum materials; however, it did involve an investment of time—an investment that some educators may consider too long.

Mrs. Bristow could not always anticipate when contexts would be problematic for her students. For example, when she introduced a new unit involving postage stamps, she brought in letters and bills that had postage stamps on them. She brought in sheets of stamps to show students what they looked like when you buy them at the post office. She showed pictures of a post office, but the students in her class had never visited a post office or used postage stamps. Her students did not understand her description so they began to ask questions about how postage stamps work such as, "Are they like food stamps?"

In addition to the curriculum units described above, Mrs. Bristow's district adopted a traditional textbook and the district pacing guide outlined the dates that specific units should be used. In examining one of the lessons in the textbook about solving multiplication "word problems," Mrs. Bristow noticed that eight different contexts were used on one page. She knew that these different contexts would be problematic for most of the students in her class, including the three Latino ELLs. These unfamiliar contexts would hinder the student from working individually or in pairs to solve the problems. She would not have sufficient time to help the children build meaning for all these contexts *and* provide opportunities for students to work to solve the problems. In other words, she thought the contexts would hinder the learning the mathematics. As a result, she made the decision to omit this lesson and write her own problems including her students' names and contexts that would be familiar to them.

Curriculum design is extremely complex and challenging work. Influencing the nature of this work for both supplementary and mainstream curriculum materials will be a daunting task. I am not suggesting that we just need to consider contextual situations in the design process. I only use this example to illustrate one issue that must be researched more carefully as we move forward.

Challenge 2: Teachers must learn how to analyze, enhance, and enact curriculum materials to facilitate the mathematical learning of Latino ELLs.

Teachers and administrators in U.S. schools work to ensure that ELLs have the support and resources necessary to be successful in U.S. classrooms. However, most teachers are unfamiliar with effective strategies for communicating with ELL parents and facilitating the participation of ELLs, especially in mathematics classrooms (Gándara et al., 2005). This is not surprising considering that U.S. teachers are not participating in professional development related to teaching ELLs (Wenglinsky, 2002) and future teachers are not prepared to teach mathematics to ELLs in teacher preparation programs. "Schools, and more generally the educational system, are not adequately prepared to respond to the rapidly changing student demographics." (August & Hakuta, 1997, p.13).

At this point, the design process for mathematics curriculum materials has not involved sufficient attention to language diversity and creating mathematical tasks and contexts that facilitate the participation of ELLs. Therefore, teachers are in a position that requires them to analyze, enhance, and enact curriculum materials to facilitate the mathematical learning of Latino ELLs. However, most teachers do not have the knowledge or competencies to make materials more accessible to ELLs (Wong Fillmore & Meyer, 1992).

I worked with five third-grade teachers this past semester who all admitted they were not prepared to enhance curriculum materials or teach them so that Latino ELLs were successful learning mathematics. They did not have strong enough knowledge bases related to mathematics or teaching ELLs. They had not read any professional literature or engaged in conversations about teaching ELLs in their teacher preparation programs or professional development settings. The teaching notes in the curriculum materials did not provide worthwhile recommendations to help them learn how to enact curriculum materials to meet the needs of ELLs. Furthermore, they did not speak Spanish and could not communicate directly with students or parents in their first language.

As I began to plan lessons with these teachers, they admitted that they did not consider questions about how to enhance curriculum materials for Latinos ELLs (Chval, Chavez, Pomerenke, & Reams, 2008) such as:

- How can I help children build meaning for the language involved in the lesson?
- How can I encourage students to use gestures or drawings to communicate their mathematical thinking?

- What representations would help the children build meaning for these important mathematical ideas and concepts?

They also acknowledged that they did not use gestures, physical objects, pictures, or videos to help ELLs build meaning during mathematical discussions. They did not require Latino ELLs to participate in classroom discussions or to write mathematically (this was typically done by the partners of the Latino ELLs) because they did not want to put these students in a position in which they would be embarrassed. Although well intentioned, in some cases, these teachers had low expectations for Latino ELLs. The following transcript of an interview with Mrs. Vaughn illustrates:

> Mrs. Vaughn: At the beginning I didn't really expect a lot from Javier just because he was so quiet and I wasn't really sure if he knew what was going on for the most part… I never imagined that he would be able to be one of the students that I am going to ask to help other students.
>
> Chval: Why?
>
> Mrs. Vaughn: I think that's because of him being Hispanic. I have never had a [Latino] student in my room be able to do that.
>
> Chval: What do you learn from this?
>
> Mrs. Vaughn: "That I am wrong."
>
> Chval: What would you look for in future years?
>
> Mrs. Vaughn: "Give it more time and look at what they can actually do and push them to be involved… Push them because they can do it, there is no reason they can't."

Mrs. Vaughn and her colleagues had never analyzed curriculum materials to determine if modifications would be needed for Latino ELLs. During a lesson planning meeting, we discussed a lesson related to sorting data about places to eat. The written task included a list of different types of restaurants and places in the home (e.g., Ice Cream Shoppe, Baskin Robbins, kitchen, and dining room). The teachers decided to alter the task so that it included places in their community the children would recognize. They also discussed which terms from the curriculum materials would be problematic and which ones could be used. They decided to eliminate "Ice Cream Shoppe" as the spelling of "Shoppe" would require additional time to explain and it had posed a "nightmare" the year before. They assumed Baskin Robbins would not pose a problem as their community did have a Baskin Robbins. The terms, "kitchen" and "dining room" did not enter the conversation. As the teachers observed the students work in pairs to sort the data, they discovered that the students did *not* know the meaning of Baskin Robbins or dining room, among other places. However, they all agreed that the addition of places close to the school, such as a local park, significantly improved the materials from the previous year. They began to see the benefits of enhancing their curriculum materials. Yet, they continued to struggle with anticipating which aspects of materials would hinder access to the mathematics for Latino ELLs. Moreover, they did not informally assess students' understandings or ask them what specific words meant during their mathematics lessons. For example, during one lesson, one Latina ELL student, Julia, was working with a partner to graph "kind of animals." I approached Julia and pointed to "kind of animal" on her graph and asked, "What does kind mean?" Julia responded with a smile, "Nice." Julia's teacher had not anticipated that "kind of animal" may lead to alternative interpretations.

Teachers need to provide opportunities for ELL students to participate in purposeful and contextual conversations with others to support both language and conceptual development (Moschkovich 1999). Yet, too many teachers are not prepared to accomplish this challenging task.

CONCLUSION

Typically, the development and implementation of mathematics curriculum policy, curriculum standards, curriculum materials, and related assessments have not considered the needs of Latino ELLs and other linguistically-diverse students as well as their teachers. Furthermore, research efforts related to mathematics curriculum and Latinos have been insufficient and the current U.S. teaching workforce has not had sufficient preparation for teaching mathematics to Latino ELLs. Changing these realities and achieving excellence and equity for Latino ELLs in K-12 mathematics classrooms in the U.S. will require attention, research, and investment. As we move forward, we must identify research priorities for the different interpretations of curriculum identified in Figure 1 and influence the forces such as textbook publishing companies, policy makers, curriculum developers, and professional developers. I look forward to continuing the conversation, but more importantly to identifying solutions and future directions with this community.

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Paper by
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Math in a Cultural Context
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Working Title
Applying the lessons learned from a culture specific group to mainstream mathematics education

Abstract

Culturally based education has been on the defensive for almost a century, struggling to include indigenous and minority knowledge and everyday practices in schooling. The larger mathematics community looks for "solutions" to improving mathematics education everywhere but from our nation's first people. Remnants of colonialism still persist. This presentation argues that the lessons learned from working with Yup'ik elders and applied in the Math in a Cultural Context (MCC) program in Alaska, may well have applicability to mathematics education in the wider mainstream context.

While writing this paper I started to confront an inner unease about the field of culturally-based/culturally relevant education and how the field has been on the defensive for almost a century. There are many valid reasons why we should feel this way. One example from Alaska underscores why many of us have been fighting for "cultural inclusion" in education and how that very act places us in a defensive posture--arguing for inclusion. Dora Andrew-Ihrke, a former bilingual coordinator and Alaska teacher of the year, puts this struggle into perspective. She stated that when she was a school-aged youngster she was "told to wipe her feet at the school door, wiping out her culture and language" (personal communication, 2009). Policies, practices, and personal attitudes excluded indigenous and minority cultures and languages in the processes and content of schooling.

Many of us have been fighting for the inclusion of cultural, linguistic, and everyday knowledge in the schooling of indigenous and minority students, and more specifically, in its application to mathematics education. This struggle has been going on for a long-time. The 1928 Meriam report represents a marker for how long federal reports and educators have been calling for the inclusion of American Indian/Alaska Native (AI/AN) knowledge in the schooling of indigenous students. The report urged not only the inclusion of indigenous knowledge and language in the processes and content of schooling but also indicated the need for local communities to be active participants in school policies and practices. This report was strongly anti-assimilationist and fought against so-called genetic arguments concerning the educability of AI/AN students (a battle lasting another 50 years).

Deyhle & Swisher (1997) chronicled a shift in the educational models away from assimilation and deficit-oriented models to local indigenous knowledge as a strength and asset. Others such as Hollins (1996) envisioned basing schooling on the extant cultural knowledge in indigenous and minority communities. Hollins observed this challenge as an opportunity to redesign the educational system for indigenous and minority communities. Her theoretical perspective consisted of ideas from psychology and anthropology resulting in "authentic culturally mediated" instruction. She noted schooling should be an extension of the enculturation processes of the home and community. Hollins (1996) contends that deep cultural knowledge of a minority and/or indigenous group should be part of schooling. Still, the field continues to argue for the inclusion of local and cultural knowledge in the processes and content of schooling.

There is a considerable body of cross-cultural literature from psychology and anthropology (e.g., Berry, 1966; Cole & Scribner, 1974: Dasen, 1973) which addresses relative cognitive strengths and abilities of indigenous people. .This and similar research shows how cognitive abilities develop in and out of school (e.g., Brenner, 1998; Carraher, Carraher, & Schliemann, 1985; Lave, 1977 and 1982; Saxe, 1988a, 1988b). Furthermore, a small but growing number of programs have connected out of school learning to in school learning: from Harris in Australia (1991), Lancy and others in Papua New Guinea (1981). In the U.S. there are a number of projects that have effectively connected in and out of school learning such as: Moses and Cobb (2001) in rethinking education for African-Americans through the Algebra Project, Moll, L., Amanti, C., Neff, D., & González, N. (1992) and Civil (2002) with the Funds of Knowledge project, and Lipka, et. al. 2005, Lipka, et. al. 2009.and work with Math in a Cultural Context program in Alaska. This body of research provides insights on how to link everyday practice to schooling in indigenous and minority contexts.

More recently, the work of Nasir et. al. (2008) views the blurring of boundaries between in and out of school learning, or between indigenous and minority knowledge and in school knowledge, as further indication of the increasing legitimacy of out of school learning. Although the field has evolved since the days of the Meriam report, our work is not completed, and resistance to the inclusion of indigenous and minority cultural/linguistic knowledge persists.

In Alaska, it is my observation that No Child Left Behind (2002) and it's over emphasis on high-stakes testing once again threatens the inclusion of indigenous language and culture (Castagno & Brayboy, 2008) and it may inadvertently place undue pressure on LEP serving schools (Abedi, 2004). Many rural indigenous school districts are considered to be "failing" based on the high-stakes tests. Caught in this high-stakes testing cycle, districts increase their efforts on test taking, ushering in frequent assessments, and devoting enormous chunks of time so that schools will be taken off the dreaded "Not Making Adequate Yearly Progress" list. These threats tend to marginalize culturally and linguistically based programs. This is occurring in school districts that have adopted policies and practices to include culturally relevant curriculum, including MCC's program. Now, these districts find themselves caught between what they wish they could do and what they feel forced to do. Again, indigenous and minority culture and language may be what is left behind. Those of us who do this work continue to fight and stand for the inclusion of such approaches.

Even as we are taking this stand, we are faced with additional arguments that further place us on our collective heels. Are "culturally relevant" approaches valid? Where is the evidence? The literature on "culturally-based" curriculum and pedagogy is contentious with arguments about its validity (Castagno & Brayboy, 2008); yet these arguments lack empirical evidences on its efficacy (Demmert and Towner, 2003). Some researchers have even defended the face validity of culturally based education (Adam, Alangui, & Barton,(2003) against such critiques (Rowlands and Carons, 2002).

In part, we need to continue to make the argument beyond the face validity of culturally based education because in a review of over 8,000 studies, Demmert and Towner (2003) found only a handful of "culturally-based" efforts that meet their criteria of rigorous research methods along with findings of statistical significance. MCC is one of the few programs cited by Demmert and Towner. MCC is a long-term supplemental mathematics curriculum and pedagogical reform-oriented approach to improve schooling in rural Alaska, particularly in indigenous school communities. MCC has conducted over twenty distinct trials, testing its curriculum and pedagogical approach using both qualitative and quantitative methods to understand its potential impact. Since 2003 we have published multiple articles on the potential efficacy of MCC (see Lipka et. al, 2005 and Lipka et. al, 2009). The project's research designs have included quasi and experimental designs. The tables below highlight one of MCC's supplemental math modules that have been studied repeatedly. The Picking Berries module is a 2nd grade module that emphasizes the mathematical concepts of measuring, representation, and data collection and organization.

Table 1: Impact of the Picking Berries Module Randomized Controlled Trial

Randomized controlled trial	Number of students	Unadjusted Average: Pre-test % correct	Unadjusted Average: Post-test % correct	Unadjusted Average: % gain	Pre-test standard deviation (SD)	Standard-ized gain	% gain adjusted for clustering	Adjusted effect size
2005-2006 2nd grade Berries students (treatment group)								
Total test score	233	32.31	54.23	21.92	13.71	1.60	10.35	0.82 ***
2005-2006 2nd grade Berries students (control group)								
Total test score	461	35.88	45.51	9.62	12.56	0.77		

*** p<.000 ** p<.01 * p<.05
(a) The impacts were estimated using two-level hierarchical linear models (HLM) that controlled for the blocking variables at the school level and student demographic characteristics at the student level. The effect sizes were calculated using the standard deviation of the control group's pre-test scores in the denominator.

Subsequent descriptive studies of the Picking Berries module implemented from 2007-2009 with second grade students (See Table 2) have demonstrated standardized gains from pre-test to post-test that are similar to, or greater than, the gain made by the treatment group in the previous randomized controlled trial. Furthermore, the average gain of the treatment group exceeded that of the control group, providing evidence that use of the Picking Berries module may have enhanced the math learning of students in the subsequent studies.

Table 2: Descriptive Studies of the Picking Berries Module

Descriptive studies	Number of students	Unadjusted Average: Pre-test % correct	Unadjusted Average: Post-test % correct	% point gain	Pre-test standard deviation (SD)	Standard-ized gain
2007-2008 2nd grade Berries students						
Total test score	63	36.98	67.42	30.44	22.44	1.36
2008-2009 2nd grade Berries students						
Total test score	84	38.81	77.32	38.51	14.27	2.70

It is also worth noting that on the subscales of measurement, the treatment group, those using the Picking Berries module, had an effect size of 1.41, 1.35, and 1.46 respectively during trials conducted in 2008-2009, 2007-2008, and 2005-2006 respectively. In the subscale of representing data, MCC treatment students showed an effect size of 3.40, 1.36, and 1.88 respectively during trials conducted in 2008-2009, 2007-2008, and 2005-2006 respectively. These rather large effect sizes as well as repeated findings in favor of the culturally-based MCC

Picking Berries module lends credence to this project's efficacy. Yet, MCC continues to struggle for inclusion even though the program meets the criteria of Demmert and Towner (2003) and, in general, meets the criteria of the U.S. Department of Education, Institute of Educational Science.

Even meeting the supposed "gold standard" for educational research does not necessarily pave the way for culturally based education. I am suggesting not resignation but new approaches to its inclusion. The following finding from MCC's data provides an insight into reconceptualizing our argument for inclusion. It is worth noting that when we analyzed MCC data in this and other studies by location, i. e. urban vs. rural, we observed that both urban and rural treatment students improved performance in relatively similar ways. Although the curriculum materials were developed with Yup'ik elders and teachers in conjunction with math educators, these materials had positive impacts on mainstream students as well as on rural mostly indigenous students. Could it be that lessons learned from Yup'ik elders can be applied to schooling more widely?

I find arguments against culturally based education to be specious and misleading. All curricula are culturally-based; the question is whose culture is it based on? Why is one set of standards used for "culturally based" education and another for mainstream education? For example, "culturally based" education needs to justify its underlying assumptions from how communication in the classroom is organized, to the inclusion of familiar contextual background,, and the inclusion of mathematically-related knowledge from everyday contexts. From a mainstream perspective, many of these "issues" are supposedly culturally-free assumptions about learning, ways of organizing classroom communication, what mathematical knowledge counts as legitimate, and what approaches to teaching mathematics are considered appropriate. Of course, the curriculum in place is considered the "standard" because it reflects the mainstream cultural norms. Rather, these arguments are about whose knowledge, power, and identity counts. My argument is that not only can indigenous, minority, and everyday knowledge be of obvious importance to the "targeted" cultural groups but also that lessons from these "peripheral" groups may well benefit teachers' and students' education in general.

Indigenous and minority everyday knowledge may well hold lessons for teaching mathematics, to all students. Further, it is clear from various international studies on mathematics that U.S. students lag behind Western European and highly developed Asian nations (TIMMS, 1995, 2003, & 2007; Pisa, 2003--http://www.oecd.org/dataoecd/58/41/33917867.pdf). In a search for solutions, mathematics educators have searched the globe for possible ways of improving mathematics education. Two powerful and potentially useful international "solutions" include Singapore Math (What Works Clearinghouse, 2009) and Lesson Study (Lewis, Perry, & Hurd, 2009). Of course, I support such efforts; yet, I wonder why we don't also look more seriously at the lessons learned working in indigenous, minority, and everyday contexts as possible ways of improving mathematics education.

Thus, the rest of this paper/talk will present a few exemplars from MCC on lessons learned from Yup'ik elders and expert Yup'ik teachers and how these lessons may be applied more broadly. Because of the length of this paper/talk I will make a cursory presentation on the ways Yup'ik elders and expert Yup'ik teachers organize social interactions and communication. More pertinent for this presentation, I will emphasize some of the unique ways Yupiaq perceive the world. These perceptions, based on their cosmology and epistemology, shape the way they solve everyday problems. Some ways of solving everyday problems offer an alternative perspective on thinking about geometry, measuring, patterns, and numbers. Furthermore, these ways of engaging in mathematically oriented problems relate well to NCTM's (2002) process standards of problem solving, reasoning and proof, communication, representations, and, maybe most of all, connections to mathematics strands. In addition, the mathematics embedded in everyday problems appears to respond to the call by Foundation for Success (2008) for a coherent mathematics curriculum which, in part, develops proficiency in fractions, geometry, and measurement. More specifically related to mathematics, this talk/paper will illustrate that the knowledge contributed by Yup'ik elders represents an elegant, systematic way to teach multiple math strands through a number of powerful, interrelated mathematical concepts/practices.

When we were first beginning our work with Yup'ik elders and teachers years ago in Akiakchak, Alaska we invited two scientists from the Jet Propulsion Laboratory to attend an elders' meeting. The theme for the weekend was on weather observation and prediction. As soon as the scientists arrived in Akiachak the elders and teachers were eager to learn from the visitors; however, they also made sure that the scientists understood that this meeting needed to be about mutual respect and mutual learning. To even the playing field, they asked the scientists to go outside with them and predict the weather without instrumentation; only using one's power of observation. Needless to say they couldn't predict the weather. Then the elders went about informing and identifying various signs in the clouds, wind, and other phenomena related to weather predictions. Elders at most meetings sought mutuality and two-way exchanges. Elders came to such meetings to help the next generation become thinkers. They also came to such meetings to learn from each other; they enjoyed learning. I believe learning from each other represents a form of respect and equality, as does considering ways in which the first peoples' knowledge may be applicable to the wider U.S. educational scene.

Background to MCC

The argument for culturally based curriculum will be made from long-term ethnographically-oriented work situated in everyday practices of a group of Yup'ik elders and expert Yup'ik teachers. Some of us have been working together for almost three decades (Lipka, 1998) and through this ongoing collaboration/partnership we have come to appreciate underlying embedded mathematical processes.

Math in a Cultural Context (MCC) began in 1995 with funding from the National Science Foundation's Instructional Materials Development program. MCC's approach was a direct and deliberate response to include indigenous culture and language in the processes and products of schooling.

Lessons Learned and How We Learned Them

The original curriculum development paradigm of MCC was based on Yup'ik Eskimo elders' subsistence life cycle. Themes of collecting or gathering resources included measuring, sorting, storing, using, and navigating. Bishop (1988) noted counting, locating, measuring, designing, playing, and explaining as six fundamental mathematical activities that appear to be carried out by many cultural groups. To develop this curriculum, we worked in situ with Yup'ik elders, expert and novice Yup'ik teachers, other teachers, mathematicians, mathematics educators, and educational researchers. We explored and analyzed subsistence related activities and developed school-based mathematics curriculum (see Lipka et al., 1998). This was and continues to be difficult, arduous, but ultimately rewarding work.

Within subsistence activities of gathering and collecting, picking berries, catching and processing fish, and star navigating, we continually noticed that Yup'ik elders' mathematics incorporated geometry and spatial relations. For example, the module on Star Navigation (Adams, Kagle, & George, 2007) is linked to the mathematical concepts of angles and measuring. This is not surprising since scholars have long identified spatial abilities and manipulation as a relative cognitive strength of many indigenous groups, particularly Yup'ik and Inuit (Berry, Poortinga, Segall, & Dasen, 2002). Cognitive preference develops through a process involving the emphasis of a particular set of skills and abilities within a culture as part of its response to the ecological demands of the social and physical environment it occupies (Berry, 1976).

Mindful of the linkages between culture and cognition (Cole & Scribner, 1974; White, 1959; Bruner, 1964), our curriculum development work included school based activities that incorporated spatial abilities and manipulation with connections to geometry. We worked with elders who instructed us in star navigating, how to build a fish rack, and other activities. As they did so, they used a pedagogical model of expert-apprentice instruction (Lipka, 1998). Additionally, while observing in some experienced Yup'ik teachers' classrooms, we began to notice the effective use of expert-apprentice modeling, joint productive activity, and cognitive apprenticeship (see Lipka & Yanez, 1998; Lipka, Sharp, Brenner, Yanez, & Sharp, 2005). These findings were supported by the work of Rogoff et al. (2003), Lee (1995), and Doherty et al. (2002). We learned and observed from expert Yup'ik teachers how they accommodated their classroom instructional norms from the typical Instruction Response Evaluation (IRE) (Cazden, 1998), to allow for multiple speakers and speaking out of turn, and encouraged peer to peer communication and assistance (Lipka & Yanez, 1998); similar to work done in Hawaii through project K.E.E.P. (Au, 1980 ; Brenner 1995).

Pedagogically, the MCC activities are developed to promote expert-apprentice modeling as a way to integrate out of school and in school learning, a form of curricular and pedagogical hybridity (Gutierrez et al.,1999 and Lipka ,1994). Expert-apprentice modeling is found in Western culture and in many "educational" settings from skilled trade workers (apprenticing carpenters, sheet metal workers, or plumbers) to doctors-in-residence learning to become surgeons. We identified a practice that exists in both Western and Yup'ik contexts, expert-apprentice modeling, but is more dominant in Yup'ik culture. In this way, we are able to bridge a more dominant cultural/educational practice from one setting to another. There are some pedagogical advantages to expert-apprentice modeling, joint productive activity (teachers and students working on the same project), and cognitive apprenticeship (expert/teacher makes internal thought processes accessible to students/apprentices); it is particularly powerful when the expert faces a problem or makes an error then tries to figure out a solution strategy. From classroom observations we have noticed that the teachers and students are more "level", creating a sense of harmony in the classroom (Lipka, J., Sharp, N., Adams, B., & Sharp, F. 2007).

We find these pedagogical practices to have traction with teachers within the Yup'ik context but also with teachers across Alaska's diverse geographical and cultural settings. Further, when presenting videotape footage of effective Yup'ik teachers to teacher groups from Israel to Australia and from Sweden to Guam we find consistent interest in the methods that Yup'ik teachers applied in their classrooms. These teachers wanted to adapt these Yup'ik classroom strategies for their own use

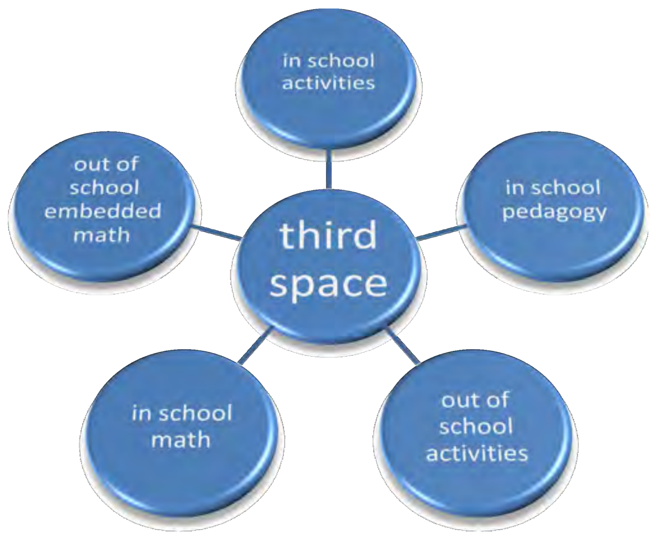


Figure 1: Bridging out of and in school practices

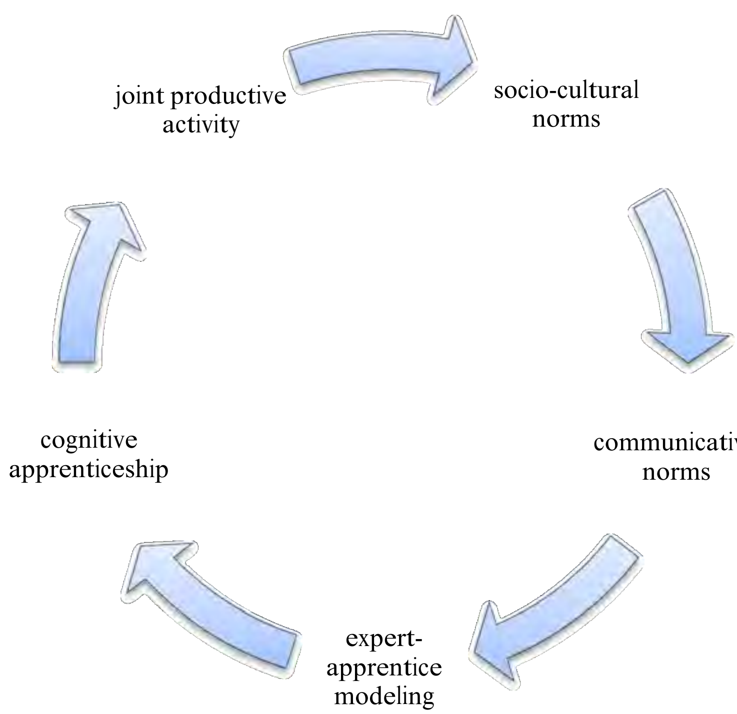


Figure 2: Developing curriculum and pedagogy on a model of instruction informed by out of school practice

The Power of Symmetry, Proportional Measuring—Connections from Geometry to Numeration

MCC's current work rests on long-term and ongoing collaborative relationships with Yup'ik elders. Collaborative work provided insights into an understanding of underlying epistemological and cosmological views of Yup'ik people, as embedded in everyday activities and values, and how these relate to mathematics (similarly noted by Gerdes, 2004; Schoenfeld, 1992). Although I was aware of the importance of particular Yup'ik values and rules governing how patterns are placed on a strip and the need for the core design to have balance between black and white, and aware of the way Yup'ik language structures numbers and the way individuals count in Yup'ik (base 20 and sub base 5), viewing the top half (digits on both hands) and the bottom half (digits on both feet), we did not see the underlying symmetry. We did not see the connection between these events, which appeared to be somewhat discrete and independent to the author. We observed elders building fish racks and how they used the center point to ensure that their construction was balanced; we observed similarly the importance of center point and balance in Yup'ik designs and in how a kayak is built. We observed symmetry in Yup'ik dance.

However, it took a fortuitous event occurred a few years ago when Dora Andrew-Ihrke mentioned to a group of MCC staff that she thought that everybody knew how to make an Eskimo "yo yo" (two sphere-like objects connected by a string) from a rectangle, which was not the case. This event connected the pieces into an integrated whole.

The Square—the center of Yup'ik constructions

Dora Andrew-Ihrke demonstrated step-by-step, how to make a yo-yo. First, she began by creating a square out of uneven material. Because natural materials do not come with right angles, she folded a crease on uneven material--for the demonstration she used uneven paper—to create a straight line, then cut along the straight line.

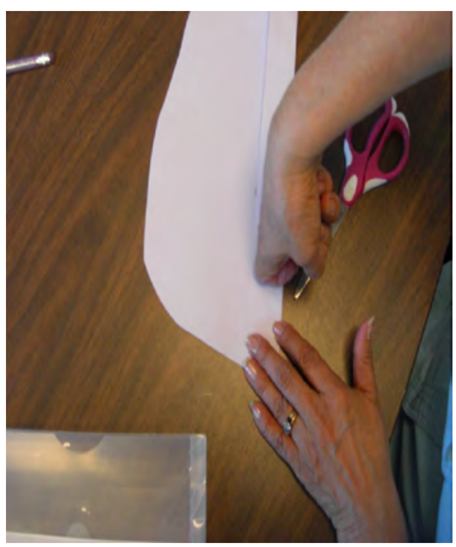


Figure 3: Making a square from uneven material

An essential element of the Yup'ik process of constructing a square is the use of body proportional measuring (a specific body measure related to the end product, e.g., knuckle length).

Dora adopted this practice in the construction of the square. She measured one knuckle length from the cut edge of the material and marked it.

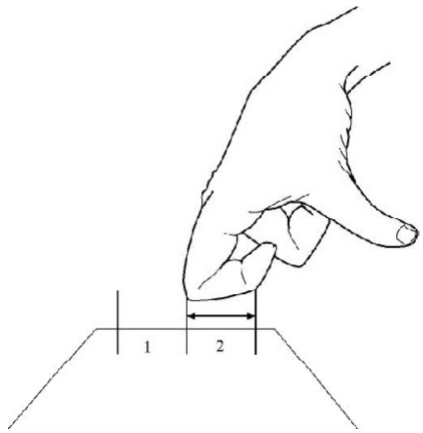


Figure 4: Using body proportional measures to create a custom made square

She then measured another knuckle length from the previous point and marked the new length. She repeated the measures at two other points along the line. Each point in the set was equidistant from the straight line. These three points established what would become the opposite side or edge of the square from the straight line. Then, she folded the straight line [cut] onto the three dots and using straight line edge, and drew in the line representing the opposite edge of the square and cut along the line. Then, she folded the paper in half perpendicular to the previous fold. This created a temporary center point. She repeated the knuckle measurement process from the center point in both directions. This established an outline of the square; she envisioned the square from the center point out, not from the outside corners. Then, she cut the outline of the square.

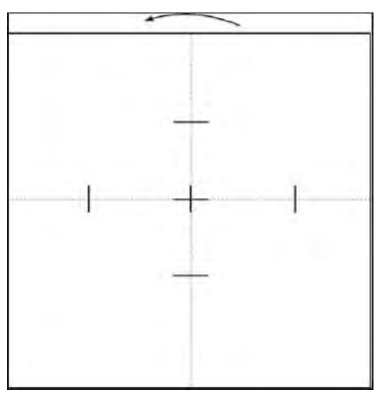


Figure 5: Finding the center and verifying a square

She folded along the four lines of symmetry, verifying that the halves of the square were congruent. This simple and elegant process or folding algorithm is the basic building block for constructing other geometrical shapes (including three-dimensional shapes) for creating pattern sets that adorn clothing, measuring, and for partitioning areas into fractional parts.

The square is the foundation of Yup'ik geometrical constructions. Elders have reinforced the importance of center points and lines of symmetry. Symmetry and measuring create geometrical centers, where plane shapes transform into one another such as squares to circles, circles into stars, and stars into flowers. The figures below show a square transforming into a circle.

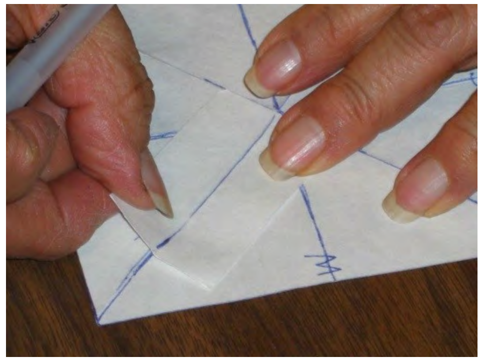
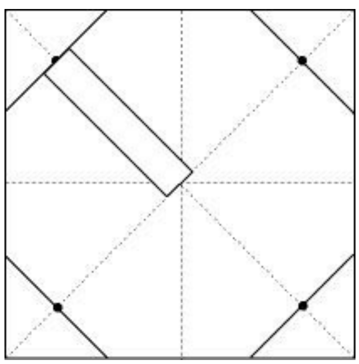


Figure 6: Dora creates an instrument to measure from the center point to the four cardinal directions

Dora continues the folding process until she has created a sufficient number of folds, usually 16, allowing her to cut out the "embedded" circle. See below.



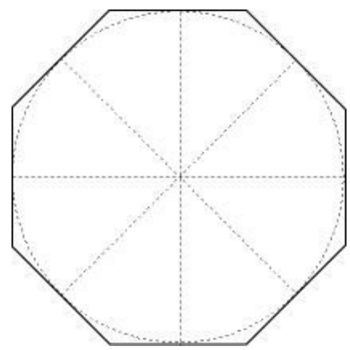


Figure 7: Square transforming into a circle

Figure 8: Square becoming an octagon

Further, squares can be transformed into rectangles, and rectangles into rhombi. What lessons can be taken from these examples to the general population? Foremost, elders have consistently shown us the importance of a center point, balance, and harmony in designs and constructions. For teachers of elementary school mathematics and for students, these processes—dynamic measuring, the use of transformational geometry (symmetry and congruence)--shift the perceptual field from the corners and external sides to the center. Teachers in our professional development workshops are quick to see how certain geometrical plane shapes are related. This approach provides teachers and students with flexible thinking and moving away from crystallized knowledge into more adaptive thinking. As will be noted later, Yup'ik solutions to everyday problems contain multiple mathematical strands, and in a school context, provide teachers and students with an integrated approach to learning mathematics. These processes are also the underpinning of MCC's most recent work.

From Geometry to Numbers and Number Relations

Typically, in Yup'ik culture, patterns are created from a square. In one method, Dora Andrew Ihrke folds a square piece of paper to simulate the way her mother folded material to create rectangular strips. These were used to make the smaller, intricate, and pleasing geometrically shaped pieces that are decoratively placed on clothing. See figure 9 below for an example. In another demonstration, she made strips from a square the way her mother did, using symmetry to verify the precision of each piece.

Figure 9: Practical everyday work reveals shapes within shapes, the relationship between constructing patterns and fractions

Dora Andrew –Ihrke partitioned the strips into 1 to 12 parts. She then arranged the strips into two piles—strips with partitions of 2, 4, and 8 parts and all other partitions. This demonstrates the halving process, which is considered a fundamental folding algorithm and which relies on symmetry as an action (Ball, 1990 Tzur & Simon, 2005).

Subsequently, she demonstrated how to construct fractions that cannot be created by repeated halving, using estimation in the construction process. For example, to construct a fifth, Dora observed the one-fourth piece and estimated a smaller amount for the one-fifth piece. With the rest of the strip, she used the halving algorithm to create four pieces folded on top of one another. Stated differently, she subtracted a one-fifth piece by folding it under, leaving a smaller strip which is folded using the halving algorithm. The final step reverts back to verifying. Is this piece (the folded under piece) equal to the other four pieces? Are all pieces of equal size? Once verified, you have five equal 1/5 pieces.

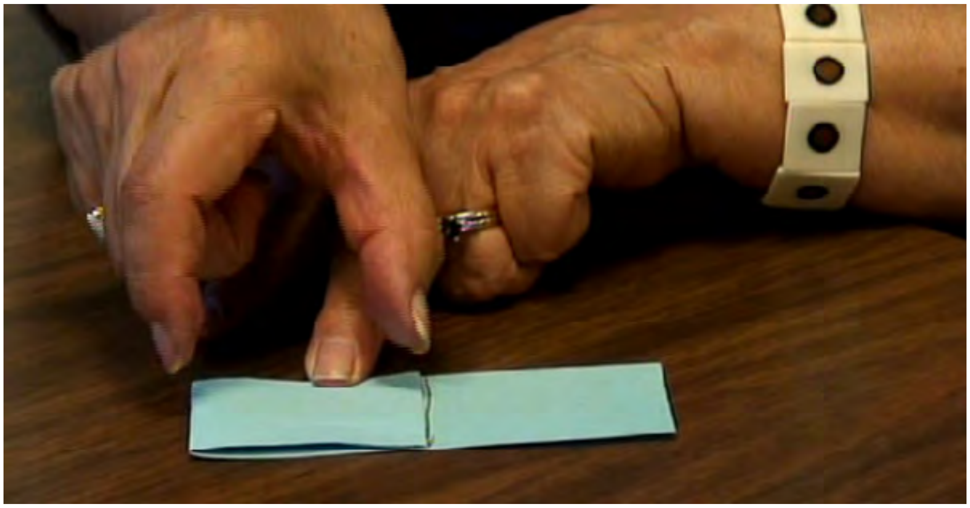


Figure 10: Using transformational geometry to verify that all pieces are equal--creating a fifth

Thus, she used the symmetry and congruence process to check if all four pieces were equal to the estimated piece. After checking for congruence, she pinched and folded the strip to create the five equal partitions. Then, Dora said, "once you know these folds, you can make any number of folds" (personal communication, 2009). You can use this method to fold any prime within the limitations of the material being used and the person's facility, using analogous reasoning. This is another intersection between out of school and in school learning, where everyday Yup'ik processes meet and extend school mathematics. When presenting this challenge to teachers, can you make a set of strips from 1/2 to 1/12? Many teachers find making the 1/5 and 1/7 particularly problematic. Thus, these have been highly instructive exercises for teachers.

To further illustrate how Yup'ik everyday knowledge can impact the teaching of fractions, the next example illustrates insights into number relations/number theory. These strategies in a Yup'ik context occur when some Yup'ik elders create patterns (taking a strip and creating fractional pieces for a design). At professional development education workshops, Dora Andrew-Ihrke or one of the MCC faculty members will create two piles of strips. One will have 1/2, 1/4, and 1/8. The other pile will have 1/3, 1/5, 1/6, 1/7, 1/9, 1/10, etc. Teachers will be asked, "How were these strips organized?" What is the organizing principle? Usually this exercise and the previous one will be very engaging for teachers. Eventually, teachers will see that the principle is not odd and even, but those strips that can be made by squaring/halving process in the first pile and all others in the other pile. We have found that mathematics educators, mathematics textbooks for math educators, and teachers in general have a difficult time determining how to create a 1/5. One textbook (Brumbaugh, Rock, Brumbach, & Rock, 2003, p.97 & SM. 14) suggests making a 1/8 and cutting off three sub units to create a 1/5!

The methods described in making fraction strips include the same fundamental cognitive and mathematical strategies employed in constructing a square. These repeating and integrated strategies begin with notions of estimation, dynamic measuring, the importance of the center point, and the notion of verifying—is this side equal to the opposite side. Figure 11 below summarizes this integrated approach to teaching mathematics. These strategies have applicability to fractions, composite, and prime numbers. This is a potentially powerful way to teach fractions. Presently, MCC has two proposals in review that build on these concepts.

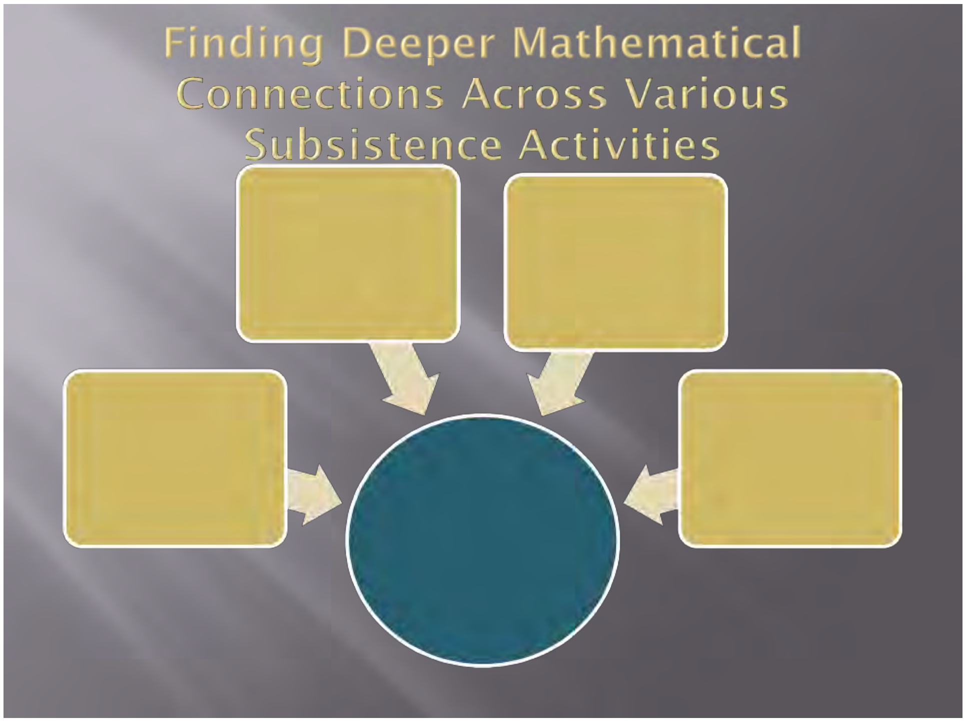


Figure 11: Transformational geometry connects geometry, measuring, patterns, and numbers

Work under development and its potential for impacting the teaching of fractions

We believe that the Yup'ik people as "original constructivists," and their expert-apprentice model can translate successfully into the elementary school classroom. Thus, MCC's new work advances our previous work in a number of important ways. First, the elegance of everyday Yup'ik activity, with its emphasis on symmetry, center point, and partitioning becomes a new component in the curriculum development process. Not only does paper folding (used instead of the raw materials elders work with but similar to their paper templates) support students' understanding of equivalent fractions (a/b x n/n = a/b), also, as Kieren (1995) observed, paper folding can be used to model fraction operators. Importantly, paper folding embodies a many-for-one (Dienes, 1967) multiplicative structure (Empson and Turner 2006) which is essential for understanding fractions (Thompson & Saldanha, 2003). Similar to Empson and Turner, we envision sets of tasks and assessments in which the use of prime factorization using half-folds and third-folds gives rise to composite numbers. Clearly, these methods have applicability beyond the Yup'ik context.

Teaching students the folding algorithm to create fifths or sevenths, or even elevenths as performed by the Yup'ik, is a rich learning task. The process of students constructing their own fraction strip sets will both challenge and further develop their multiplicative thinking. This process results in products (the fraction strips can be used as culturally mediating math tools) that will further support students' understanding of number relations and multiplication. This process also supports partitioning and measurement/sharing division. As students create fraction strip tools, they will represent units and investigate the relationship of subdividing the iterated unit (Bright, Behr, Post & Wachsmuth, 1988).

In our preliminary work with teachers in workshops, we have used the fraction constructions described above as a part of a larger process that uses these tools to teach fraction operations. In fact, the very process of creating the fraction strips embeds multiplicative thinking and provides a solid example for teachers to distinguish between additive and multiplicative thinking. For example, asking students to make a 1/8 piece can be used to distinguish students who think additively with those who think multiplicatively. As Empson and Turner's (2006) findings suggest, "Given a task structure that provides multiple opportunities to test and revise ideas, shifts from additive to emergent multiplicative reasoning are likely". Empson (1999) noted that pre-partitioned representational tools did not facilitate students' understanding of fraction equivalence, thus supporting the folding operations used by the Yup'ik.

Further, we have explored these tools particularly for the teaching of division of fractions. At first, we observed how Dora divided fractions during everyday tasks such as working with material and or making pattern strips. Dora demonstrated using what she learned from her mom and other elders and how it might apply to teaching division of fractions in a school context. We realized that she was using alternative algorithms to invert and multiply—an algorithm more in line with Euclid's subtractive theorem as well as a common denominator approach to dividing fractions. The same fundamental processes involved in constructing a square were now applied in division of fractions. We recognized that when she and other elders accomplished these tasks, they were also using transformational geometry as a method for constructing everyday artifacts. This elegant and integrated approach appears to appeal to teachers across a variety of contexts.

Thus, these examples from MCC's work begin to shift the argument about culturally based/responsive education from the margins to mainstream. Here are strategies that can be applied widely and effectively. Here are ways of engaging teachers in professional development workshops that can be engaging and challenging. Further, elders have also shown an elegant and systematic way to approach the teaching of mathematics that integrates multiple mathematical strands and ways of communicating, teaching/learning, and verifying. Expert-apprentice modeling assists in building knowledge in novices.Therefore, why not argue that lessons learned from the margins may help improve mainstream mathematics education? Can the mainstream afford not to listen?

Summary and Conclusion

The vision of this work rests on a deep appreciation for the lessons that we have learned from Yup'ik elders over many years. The examples provided in this paper show the power of symmetry and measuring, and how mathematical approaches to solving everyday problems have applicability in the larger school context beyond Yup'ik communities.

Significantly, based on Dora Andrew-Ihrke's demonstrations, her unschooled mother and other elders intuitively understood the structure of numbers—recognizing the power of halving, primes, and composites. They accomplished this in the context of making clothing and pattern pieces while developing methods to create equal partitions. From this model, we observe how Yup'ik actions, culturally preferred ways of perceiving and manipulating objects in space, result in a set of procedures resting on the importance of symmetry and balance, center point, and connecting measuring with proportionality (similar to Davydov as noted by Kilpatrick et al., 2001, p. 257).

As Dora Andrew-Ihrke and Yup'ik elders have instructed us about the importance of balance and center point, this paper seeks to support the aforementioned practices in becoming a seamless part of schooling, taking advantage of some of the unique perspectives that Yup'ik people have to offer to instruction within school settings. Our current work will ensure that these processes become an integral part of schooling, a part that accommodates students whose culture and language have been previously excluded. Yet, the approaches to teaching mathematics in this paper provide symmetry, symmetry between small indigenous groups and the contributions that they may make to the larger mainstream culture. This symmetry is one of power and respect, potentially creating more space for the contributions of others who are currently perceived as marginalized.

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Curriculum and Latinas/os: Toward Guiding Principles for Equity and Quality in Mathematics Instruction

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In the coming years, the statistics predict that most teachers in the U.S. will be teaching Latino children with little or no training to address the unique learning needs of these students. In this paper, we explore the design principles and characteristics of one curriculum that has demonstrated a positive effect on the development of both language and mathematics concepts for Latino students. We describe how we derived a framework from the principles and characteristics. We discuss how the framework might serve as a guide for both mathematics curriculum developers and for teachers as curriculum designers in their efforts to improve mathematic instruction for Latino students.[1]

Introduction

Latinas/os now comprise the nation's largest minority group, making up about one-in-five of the country's public school student population (Fry & Gonzales, 2008). According to U.S. government statistics, Latinas/os are the majority student population in the one hundred largest elementary and secondary school districts in the country (Garofano & Sable, 2008), and they are present in geographical areas and rural schools where they once were not. Over 40% of the nation's teachers have language minority students (LMS)[2]—the majority of whom are Latinos/as—in their classrooms, but only 12.5 % of the teachers have had more than eight hours of training to prepare them to teach these students (NCTM, 2004). Given these demographics, it is no exaggeration to claim that most teachers in this country will be teaching Latino children, and thus, will need to be able to address the unique learning needs of bilingual students.

Over the past twenty years, a focus on improving the achievement of bilingual learners has intensified. Evidence of this increasing focus exists in documents such as the early *NCTM Standards* (1989),[3] which call for improved mathematics instruction and high expectations for *all* students. This early document goes farther in its plea for developing equity in classrooms saying, "the social injustices of past schooling practices can no longer be tolerated (NCTM, 1989, p. 4)." As the *Standards* have evolved, they have come to include an equity principle (NCTM, 2000) that first, highlights the need for high expectations and strong support for all students so that all students will have opportunities to learn quality mathematics and second, outlines a vision of equity that challenges "a pervasive societal belief in North America that only some students are capable of learning mathematics (p. 12)."

The *Standards* documents make recommendations for changing perspectives on and practices for achieving equity in the teaching and learning of K-12 mathematics in the U.S. Often these recommendations invite teachers to teach in ways they have not experienced as learners (Ball, 1988). Researchers have noted the challenges of acquiring new teaching skills and practices considering that "by the time teachers begin professional education, they have already clocked more than 2,000 hours in a specialized 'apprenticeship of observation' (Lortie, 1975, p. 61), which not only has instilled traditional images of teaching and learning, but also has shaped

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[1] A revised version of this paper appears in a chapter of *Mapping Equity and Quality in Mathematics Education* (Pitvorec, Willey, & Khisty, 2010).

[2] We have chosen to use interchangeably the terms "language minority student" (LMS), Latinas/os, and "bilingual student". LMS notes Latinas/os' political status and "bilingual" forefronts their language roots and experiences.

[3] We use *Standards* to refer to the *NCTM Standards* from 1989 and 2000 (NCTM, 1989, 2000).

their understanding of mathematics (Ball, 1988)" (Ball, Lubienski, & Mewborn, 2001, p. 437). Some researchers suggest that teachers' early experiences as learners may have resulted in teachers developing a cultural script that guides the instructional decisions they make (Jacobs & Morita, 2002; Stigler & Hiebert, 1999).

One way to support teachers in making what in some cases may be drastic changes in teaching practice that will result in providing more equity for bilingual students is to provide them with curriculum models. The *Standards* state that "a school mathematics curriculum is a strong determinant of what students have an opportunity to learn and what they do learn (NCTM, 2000, p. 14)." Over the last twenty years, the National Science Foundation has attempted to promote changes in teaching skills and practices in accordance with the recommendations of *Standards* documents by investing over $100 million in "the development of mathematics instructional materials that reflected the recommendations of the NCTM *Standards*…[one reason being] the realization that teachers could not implement the recommendations of the *Standards* without curriculum models" (Hirsch, 2007, p. IX). School districts have then adopted the new *Standards*-based curricula in their efforts to "regulate mathematics teaching practices" with the goal of improving student achievement (Remillard, 2005).

As research has shown, however, adoption and use of a curriculum does not, by itself, determine what opportunities students will have to learn mathematics (S. A. Brown, Pitvorec, Ditto, & Kelso, 2009; Chval, Chavez, Reys, & Tarr, 2008). The curriculum, the teacher, and the students all contribute to the lesson that is enacted in a mathematics classroom (Stein, Smith, Henningsen, & Silver, 2000). Assuming that the authors of a *Standards*-based curriculum intend for all students to have opportunities to learn quality mathematics that is consistent with recommendations in the *Standards*,[4] one factor that may influence the fidelity of an implementation to the authors' intentions is the transparency of the materials (Stein & Kim, 2008)—that is, how clear and accessible the mathematical ideas, perspectives, and agendas are in the materials. Following the work of Remillard (2005) and Remillard and Bryan (2004), we acknowledge, that teachers also "interact with, draw on, refer to, and are influenced by material resources (Remillard, 2005, p. 212)" in a variety of ways as they respond to factors including their past professional experiences (Sowder, 2007), beliefs about the teaching and learning of mathematics (Ball, 2001; Philipp, 2007); professional development experiences; and their mathematical content knowledge (Ball, 2001; Ma, 1999).

M.W. Brown (2008) has proposed a way to consider both the variations in the design of a curriculum and in teachers' use of a curriculum by presenting a perspective of the teacher as an agent who designs curriculum and curriculum materials as a tool for mediating teachers' and students' mathematical activity in the classroom.[5] We believe that this is a productive perspective for addressing the challenge of improving opportunities for bilingual students to engage in quality mathematics. It makes sense to unpack the design principles and characteristics of a written curriculum that, based on improved LMS outcomes from using the materials, might be predicted to be a tool that can support teachers in developing or improving teaching skills and practices related to working with bilingual students. Developing a framework based on these

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[4] See S. A. Brown et al. (2009) for a description of the authors' intended curriculum in the context of one *Standards*-based curriculum.

[5] Fully describing M. W. Brown's perspective of teachers as agents and curriculum materials as mediating tools is beyond the scope of this paper. See M. W. Brown (2008) for more information.

principles and characteristics might support teachers' design activities and provide a foundation for guiding the interaction between teachers and the curriculum materials they use with the goal of improving opportunities for bilingual students to engage in quality mathematics.

Following M. W. Brown's perspective of teachers as agents who "design" a mathematics curriculum by interpreting curriculum materials—adding, modifying, or omitting parts according to their perspectives, experiences, and so on—and curriculum materials as tools for achieving goals that teachers and students could not have achieved without the curriculum (M. W. Brown, 2008), we propose to investigate how the design principles and characteristics of a curriculum that has been proven successful with Latinas/os might support improving instruction for bilingual students. In the first section of this paper, we explore a curriculum explicitly developed for bilingual students, which has demonstrated improved student outcomes. In the next section, we describe a process for determining characteristics of the curriculum that we believe support improved student achievement for Latinos/as. We outline features of a framework we developed based on the identified characteristics. Finally, we discuss how the framework might suggest recommendations for curriculum design—both at the level of written curricula and at the level of considering teachers as designers of curricula.

Description of a Bilingual Mathematics Curriculum

One of the authors of this paper became familiar with the curriculum *Finding Out*/*Descubrimiento* (*FO/D*) when it was developed in the late 1970's. *FO/D* was intended to serve students in grades first through fifth and was not designed to be a full curriculum but a supplement that could be used once or twice a week or as often as a teacher desired. It was originally published as materials for bilingual instruction-meant to better support—both linguistically and academically—the thousands of children of migrant workers in California (De Avila & Duncan, 1980).

FO/D is rooted in a sociocultural activity perspective of learning (Engeström, 1999; Vygotsky, 1978), one that would capitalize on students' four levels of resources: the object itself, pictures, language, and peers (De Avila, personal communication, December 1, 2008). Sociocultural activity theory plays a significant role in understanding second language development (e.g., Lantolf & Thorne, 2006) and a growing role in understanding mathematics development (e.g., Lerman, 2001). Because of this, we will briefly consider how this theory is reflected in a curriculum that meets the objectives for both Latinas/os and mathematics.[6] Many key aspects of sociocultural activity are found in *FO/D* including the following: activity, interactional spaces, and mediation through multimodal semiotic tools, including language. First, human development, fundamentally social in nature, derives from concrete communicative activity (Engeström, 1999; Vygotsky, 1978) where the conditions of social interactions and cooperation are found. Meaning resides in the activity, its actions, and the language attached to the activity. "It is through activity that new forms of reality are created, including the transformation of self (Lantolf & Thorne, 2006, p. 215)." The mediational resources for development are activity, tools, and social interactions. De Avila (personal communication, December 1, 2008) noted, "When kids are arranged as they are [in a traditional arrangement of

[6] It is beyond the scope of this paper to provide a deeper discussion of sociocultural activity theory. We suggest the reader examine some of the references we have provided for any further explanation.

rows of desks], we eliminate these resources." However, students are positioned to solve problems collaboratively and make meaning of the content—two crucial tenets of learning and development—when these resources are made available and students are socialized to use them.[7] Second, interactional spaces are a key component of activity, and in *FO/D,* these spaces are carefully constructed to socialize students into the norms of rights, roles, and responsibilities for helping one another complete a goal-directed task. Students collaborate in what Wenger (1998) calls a community of practice where they are responsible for their own and each other's learning. Third, use of tools (in *FO/D*: concrete objects, pictures, and oral and written communication) mediates thinking and development. One sign of development can be found in the changes in routine of how language tools are used (Sfard, 2008). Consistent with current thinking in reform mathematics approaches this idea deemphasizes learning procedures without connections and emphasizes problem solving (NCTM, 1989, 2000).

Identifying Key Characteristics

In this section, we present the process for our analysis of the design principles of *FO/D*, a curriculum that had been proven effective for Latina/o students (Cohen, Lotan, Abram, Scarloss, & Schultz, 2002). In this analysis we had the opportunity to identify key characteristics of a curriculum that demonstrated it is possible to support the simultaneous development of mathematical concepts, language for bilingual students, and academic progress. Our examination of *FO/D* proceeded with a two step process. First we compared the curriculum to current research in Bilingual Education in order to distill the characteristics relevant to Latina/o students. Second, to refine our identification and description of characteristics to ensure that they would be meaningful in a broader context than *FO/D*, we randomly selected one elementary school and one middle school *Standards*-based curriculum whose characteristics we could contrast with those in *FO/D*.[8] In order to determine where correlations between characteristics in *FO/D* and the *Standards*-based curricula might exist, we considered not only obvious characteristics but also the purpose of the characteristics. For example, both *FO/D* and a *Standards*-based curriculum may have visual images on student pages (see Figures 1 and 2).

However, only focusing on the existence of visual images in a curriculum can be misleading. The rationale for the visual images included in *FO/D* is significantly different. In *FO/D,* we interpret the inclusion of visual images as a strategy for minimizing the influence of any language proficiency factors and as a deliberate means of engaging students in making meaning in the context of the activity. The visual cues of the activity cards open up possibilities of student interaction and dialogue as they negotiate and build consensus on both what the card means and what they should do to complete the activity (see Figure 1). On the other hand, we interpret the visual image in the *Standards*-based curriculum as more of a reference for technical

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[7] As explained in *FO/D* teacher resource materials (DeAvila, Duncan, & Navarrete, 1987) and teacher trainings (Lotan, personal communication, December 2, 2009), a fundamental design principle of *FO/D*, which was developed in the 1970's, was to challenge traditional classroom organizational structures of sitting and working in isolation. It is worth noting that the traditional pattern still dominates the classrooms of poor and minority students (see Oakes, 1990).

[8] We use the term *Standards*-based curriculum to refer to curricula that are aligned with the National Council of Teachers of Mathematics *Standards* from 1989 and 2000 (NCTM, 1989, 2000). The two curricula we used for comparisons to *FO/D* were *Everyday Mathematics* (Pre-K through 6th grade) and *Connected Mathematics* (6th-8th grade).

definitions of key words in the activity (see Figure 2 as an example). To construct our framework, we considered one of the main purposes for the activity card images in *FO/D*—to provide students with access to instructions for the activity—and searched for a corollary in the *Standards*-based curricula. In the *Standards*-based curricula, students know what to do primarily because the teacher or the text provides verbal or written instructions.

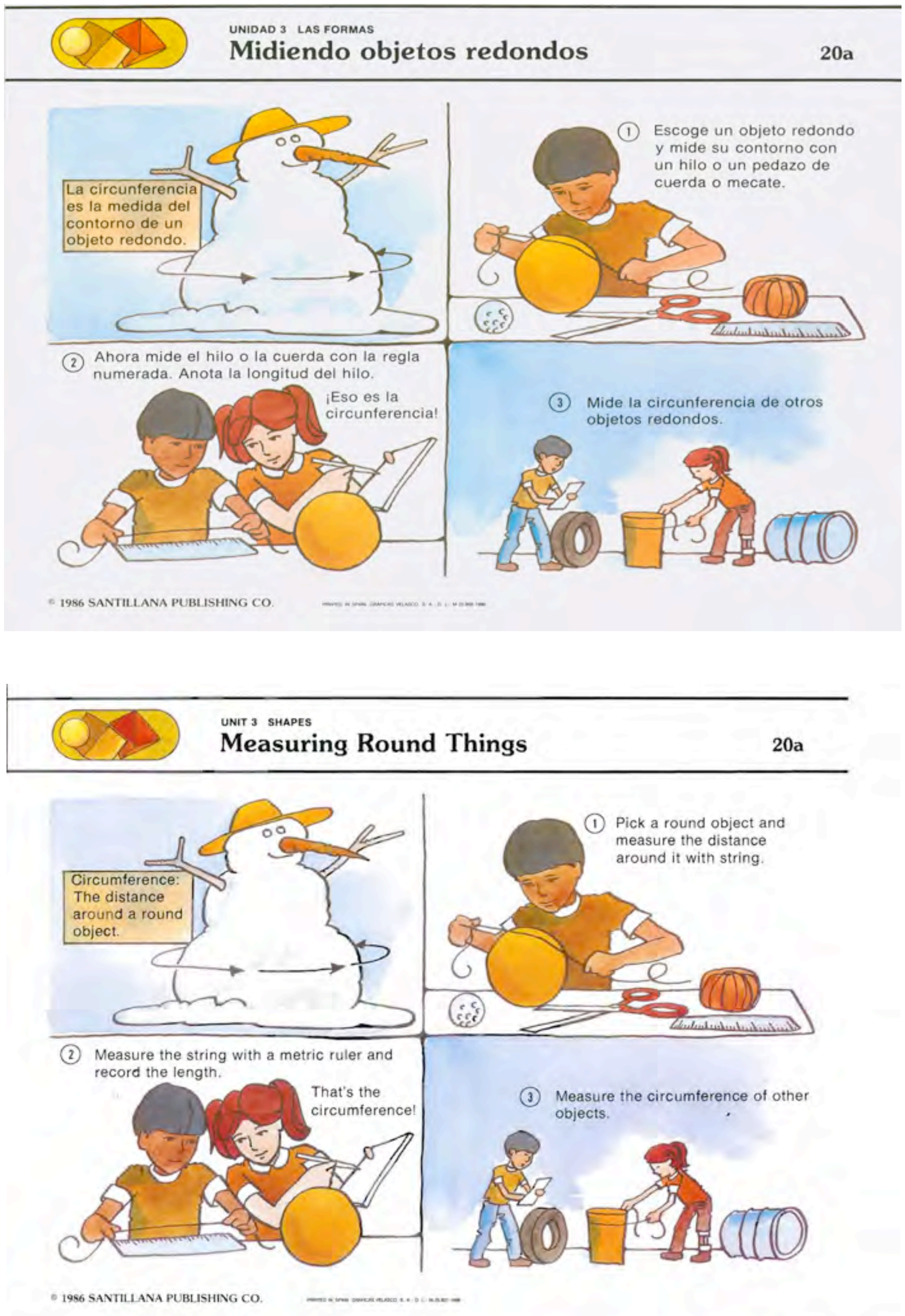
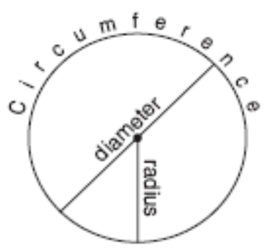


Figure 1. Student Activity Cards from *FO/D* in Spanish and English



You are going to explore the relationship between the circumference and the diameter of a circle.

1. Using a metric tape measure, carefully measure the circumference and diameter of a variety of round objects. Measure to the nearest millimeter (one-tenth of a centimeter).

2. Record your data in the first three columns of the table below.

3. In the fourth column, write the ratio of the circumference to the diameter as a fraction.

Figure 2. Instructions from a *Standards*-based Curriculum (p. 361, Bell, et al., 2007)

The *FO/D* student activity card pictured in Figure 1 illustrates how we believe these cards serve an additional function of supporting student discussion. The images invite discussion because there are sufficient visual cues to allow students to interpret what they are going to do (find and compare the circumference and diameter of objects); to figure out how they are going to do it (the method used in the illustration is to wrap a string around a round object, then to stretch the string out to compare it to a ruler in order to measure the length of the string and, therefore, the circumference); and to interpret or define various aspects of the activity (e.g., the meaning of *circumference*, how to use the tools, the process of recording measurements, and the expectation that more than one object will be measured)—all processes of critical thinking. The images themselves provide the prompts for student discussions. Student meaning making is, therefore, not reliant on the teacher's facilitation. In a *Standards*-based curriculum, on the other hand, student discussions often revolve around teacher or textual prompts or instructions. This places the burden on bilingual students' comprehension of oral or written communications, most likely in a second language, which may not be their academic strength in a second language (Khisty, 1997).

Developing a Framework for Equity and Quality in Mathematics

By making these types of comparisons, we began to develop a list of guiding questions for understanding how engaging students in *FO/D* activities contributed to providing both equity and quality mathematics. How do students know what to do, how to approach problem situations, and how to engage in doing mathematics? What roles do students and teachers have in the problem-solving process? What purpose do mathematical activities have and in what way are activities structured to promote engagement in meaningful mathematical activity? How is literacy defined and developed during mathematical problem solving?

Attending to key questions like these allowed us to construct feature descriptions for a Bilingual Mathematics Framework (BMF) that would have meaning across curricula. The BMF begins by setting forth an overarching ideology that emerged from *FO/D*, one that respects students and the resources they bring to the classroom and to learning. Learning is seen as occurring in interactional spaces. The teacher and the text are not positioned as the authorities who hold the mathematical knowledge; instead it is the learning community members (teacher and students) working together to make meaning who generate mathematical knowledge. The text, the problem context, and the mathematics needed to solve problems are all defined as tools for mediating student learning. This ideology is manifest in three broad categories of features—features related to learning communities, features related to the curriculum materials, and features related to language and communication. We argue that these three categories together with the overarching ideology form the foundation of support for bilingual students' mathematics learning (see Figure 3). In the following sections, we describe these three categories.

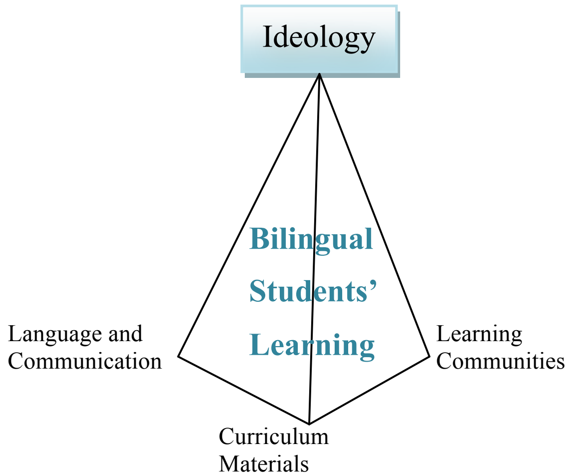


Figure 3. Pyramid of Success for Bilingual Students.

Learning Communities. A characteristic of *FO/D* is its emphasis on students engaged in what Dalton & Tharp (2002) refer to as joint productive activity. Similar to Wenger's (1998) descriptions of learning in communities of practice, effective instruction for Latinas/os is generated from the perspective that learning mathematics occurs in the interactional spaces of a community where the mathematical authority in the community is distributed productively among teacher, curriculum materials, and students,[9] and where the norms for engaging in the meaningful mathematical activity of the community are negotiated by teachers and students.[10]

Although the teacher participates as the more expert member of the community, modeling a repertoire of appropriate behaviors and setting the context for the mathematical activity, it is the students who do the intellectual work involved in problem-solving situations and who contribute to generating the mathematical knowledge in the community, exercising what Cobb et al. (2009) refer to as *conceptual agency*. The teacher supports students'

[9] We conceptualize *authority* and various distributions of authority following the research of Herbel-Eisenmann (2008). In her work, she describes six possible configurations for the distribution of authority: teacher, text, student, teacher-text, student-text, teacher-student. She suggests that there is not necessarily a best configuration but that different purposes may require different configurations.

[10] For a description of norms—specifically social versus sociomathematical—and the development of norms in the classroom, see Yackel & Cobb (1996).

engagement in mathematical explorations by facilitating the negotiation of classroom norms that call for mutual respect, participation, communication, and so on. The teacher encourages fuller participation by asking thoughtful questions that facilitate students' work of meaning making, by probing students to make their thinking visible to others in the groups, and by highlighting students' strengths and successes so that all students realize the importance of their contributions in completing the mathematical activities (Cohen, 1994). When students' mathematical ideas and contributions are made public, they become mathematical objects themselves, providing the community with further opportunities to deepen their mathematical explorations through reflective discourse (Cobb, Boufi, McClain, & Whitenack, 1997). As students participate in and take increasing responsibility for solving mathematical problems, they come to view themselves as doers of mathematics and, therefore, as more expert members of the community.

Identifying themselves as contributing members of the classroom mathematics community (or as having agency) is a critical factor for bilingual students who are often subject to negative and/or deficit beliefs in traditional school settings (Gándara & Contreras, 2009; Valencia, 2002). These ideas of community are also reflected in current principles of effective instruction for English language learners (Valdéz, Bunch, Snow, Lee, & Matos, 2005). Communities offer activity structures that support students' oral language use to address higher order questions, students teaching students, and distributed expertise around content. Unfortunately, the dominant classroom structure for Latinas/os is that of teacher-centered instruction with controlled and limited language use among students, worksheets, and individual work (Brenner, 1998; Lee, 2007). Given this, much more work needs to be put into helping teachers implement the kinds of activity structures that create communities of learners (Valdéz et al., 2005), along with supporting teachers' development of skills for setting norms for equal participation among students, both of which are aims of this project.

Curriculum Materials. Certain design features allow curriculum materials to serve as a tool for mediating learning in a community that includes bilingual students, and primary among them are problem-solving situations that facilitate critical thinking (Lee, 2007), a construct that parallels Stein, Smith, and Henningsen's (2000) idea of "doing mathematics." These are open-ended problems, providing students with a variety of entry points and allowing for multi-modal engagement (e.g., visual, kinesthetic, tactile, auditory, and so on). The situations provide contexts in which students exercise conceptual agency as they make decisions together about what the relevant mathematical questions are and which tools or strategies to use. Furthermore, the problem-solving situations require students' mutual engagement in identifying the mathematical problem embedded in the situation, in determining possible solution paths, and in establishing the reasonableness of proposed solutions, all of which results in a negotiation of what counts as mathematically legitimate in the community. Not only do the contexts of the situations help achieve mathematical goals, but they also serve the critical purpose of providing bilingual students with cues for negotiating meanings where the mathematics content and home or second language intersect. Unfortunately, Latinas/os typically do not get mathematics curricula that emphasize these problem-solving situations; they typically get a narrower curriculum that focuses largely on basic skills (Lipman, 2004; Oakes, 1990). Even though classrooms may utilize reform-based curriculum, it may not be implemented as intended (S. A. Brown, et al., 2009).

In addition to considering design features of particular tasks in curriculum materials, the overall organization of those tasks has implications for a community that includes bilingual learners. Problem-solving situations are organized thematically (Garcia, 2005) so that students can identify common mathematical structures and the relationships among those structures, and they maximize connections to students' background knowledge and experiences (Lee, 2007). This further facilitates the integration of conceptual and linguistic development.

Language and Communication. A key feature of instruction with bilingual learners is that it should capitalize on students' knowledge, including language, as resources for learning (Moll, Amanti, Neff, & González, 2005; Nocon & Cole, 2009). This can be achieved through active communication, which increases the participation of bilingual students (Lantolf & Thorne, 2006). Communication also maximizes opportunities to develop the specialized school language related to mathematics (i.e. academic discourses) (Gee, 1996) and mathematical content knowledge. In a recent study, Lotan (2007) made a strong case for simultaneously developing academic language and content knowledge in bilingual classrooms. Literacy, which involves reading, writing, listening, and discussing, is best developed through a functional approach (Mohan & Slater, 2005) by way of engaging in the mathematical activities of the community. As bilingual learners engage in what Hufferd-Ackles, Fuson, and Sherin (2004) describe as a high-level Math-Talk Community, they also learn through the modeling, guidance, and facilitation of the teacher to ask questions about explanations, follow points made in class discussions, and explain and articulate their ideas (Valdéz et al., 2005). In addition, as part of students' negotiations around the mathematical meaning embedded in problem-solving situations, they also have to respectfully challenge each other, build arguments for and justify their thinking, and build on each other's ideas (Moschkovich, 1999). Therefore, a problem-solving community structure can potentially promote authentic talk about mathematical content, and thus, provide a context in which bilingual learners can further develop mathematical literacy. As they participate more fully in the mathematical activities and discourse of the community, changes in their use of language tools will signal development (Sfard, 2008).

In related research, Cohen et al. (1997) demonstrated that English literacy for bilingual students can develop together with competency in mathematics when students' home language and English are used in learning and have equal status in the classroom. This is another form of capitalizing on students' language. As much as possible, materials, explanations, vocabulary, and conversation about the mathematical problem solving should be available in both languages (Cohen, et al., 1997). As students explore the problem situations, they are encouraged to use the language (i.e. Spanish, English, or a hybrid form of language) (Gutiérrez, Baquedano-López, & Tejeda, 1999) with which they can best express their meanings and ideas. In addition to discussions in both languages, students should have opportunities to read and write in both languages. Students should be expected to process their experiences in the learning community through recording (again, in either language) their individual ideas, explanations, questions, observations, and arguments. Although more traditional classroom community norms may not include developing English through encouraging meaningful content-related talking, listening, reading, and writing in a second language, the results from Cohen et al.'s research suggest that this may be the best way to lay the foundation for the development of more advanced academic English. However, the preparation of teachers, especially monolingual teachers, is still lacking to be able to fully incorporate Latinas/os' language as a resource.

Discussion and Conclusion

We began our exploration by uncovering the design principles and characteristics of a curriculum explicitly developed for bilingual students that has demonstrated success in providing these students with opportunities to engage in quality mathematics. We found that Latinas/os/bilingual students do need a different sort of mathematics curriculum, but not one that is easier. They need one that is designed with certain characteristics, characteristics based on creating different kinds of learning spaces, based on capitalizing on students' resources—in fact, defining Latinas/os' individual traits as learning resources—and based on a principled approach to curriculum and instruction—but principles derived from research that has focused on bilingual learners. After describing the principles and characteristics of the curriculum, we then outlined the process we used to generate the Bilingual Mathematics Framework (BMF) based on the identified principles and characteristics, and we described the features of the framework. Our development of the BMF led us to various conclusions about how to support curriculum developers and teachers as designers of curriculum.

Curriculum Development. First, it is not enough to insure that Latinas/os and other bilingual students simply will have *access* to quality mathematics programs. The BMF suggests that curriculum development must consider the needs of bilingual students from the beginning in the design of a curriculum. In the case of the two reform curricula with which we compared *FO/D*, access for bilingual students, narrowly construed as access to vocabulary and instructions, was added on after the development of the original materials.[11] Second, more attention must be given to being explicit about the ideology in which a curriculum is embedded. The ideology for non-dominant students should clearly respect and support the agency of students and see students and their home language as resources for developing mathematical proficiency; this ideology must influence the way mathematical activities are designed, organized, and presented. Third, when learning is seen as occurring in interactional spaces, as described in the BMF, activities must be designed to support the existence and use of those spaces. Language ought to not be conceived of as an entity that needs to be taught one technical word at a time, but rather as a natural tool that is continually being refined from students' everyday words to a specialized style of language related to the mathematics discipline (Gee, 1996; Lantolf & Thorne, 2006). Fourth, where the text, the problem context, and the mathematics needed to solve problems are all viewed as tools for mediating student learning, mathematical activities must be constructed to provide students with opportunities to make use of these tools. These opportunities should not be prescriptive but should invite exploration by students with a follow-up that requires an explanation and comparison of student ideas about the problem. While some current mathematics curricula may strive to do this, in their implementation the effort sometimes falls short (S. A. Brown et al., 2009) and instead results in eliminating critical resources for language minority students.

What *FO/D* has demonstrated is that none of the features of a curriculum that we have described can be an afterthought. They must be part of the fabric and structure of the curriculum and corresponding instruction from the beginning. Instead of only focusing on content first, we advocate a curriculum development process that reflects current sociopolitical and cognitive

[11] Note that one author of this chapter was part of the author team of *Everyday Mathematics* and so is familiar with how the curriculum addresses the needs of bilingual students. In the case of both *Everyday Mathematics* and *Connected Mathematics*, the latest editions include appended notes for addressing the needs of bilingual students that did not exist in previous editions, and in the case of *Everyday Mathematics* several new activities added as lesson options aimed at primarily supporting vocabulary development.

research on bilingual learners and that demonstrates a shift towards mathematics equity in the form of student-centeredness and community-centeredness as described in *How People Learn* (Bransford, Brown & Cocking, 2000). The features that we derived from *FO/D* and identified and described in the BMF provide us with a starting point for transforming existing mathematics lessons so that implementation of those lessons provides both quality and equity for Latinas/os and all bilingual students and their dominant classmates together.

Teachers as Curriculum Designers. It is unlikely that the development of new written mathematics curricula will provide a means of improving instruction for bilingual students in any timely fashion. In recognizing this and the fact that teachers have considerable agency in designing the mathematics curriculum that they will implement in their classrooms, we believe that the BMF provides a foundation on which teachers can build instruction that will better meet the needs of their bilingual students. The BMF outlines how teachers and students would be expected to engage in mathematical situations and presents recommendations for the role of teacher and students. With emphasis on student agency, students as knowledge generators, students and language as resources for solving problems, and communication, it is clear that a mathematics curriculum cannot be implemented in a stand-and-deliver format. Instead, bilingual students are the heart of the mathematical activity as they discuss what mathematical problem or problems they can identify in a situation, how they will approach the solution process, what to try, and whether what they tried seems to have worked. As the students and their mathematical ideas are at the heart of the mathematical activity, the teacher has moved more to the periphery, which offers greater opportunities to observe students' use of tools and to listen to student thinking *in situ*. Although still taking responsibility for setting up the mathematical situation, the teacher becomes more of a coach and facilitator, encouraging, prompting, supporting, extending, and probing students, helping them build off of each other's ideas. At the same time the teacher's talk becomes a model for both the second language and mathematics discourse (Lena Licón Khisty & Chval, 2002).

Clearly, many teachers will need support in their design activities as they attempt to incorporate features described in the BMF into the planning and implementation of their existing mathematics curriculum materials. We anticipate many challenges as they evaluate their curriculum materials and decide what to modify, add, and omit, so that they might improve their instruction as it relates to providing bilingual students with opportunities to access and engage in quality mathematics. Professional development activities designed to involve teachers in authentic mathematical experiences incorporating some of the features of the BMF and providing them with the space to unpack those experiences with colleagues might provide a starting point for developing teachers' facility with including features of the BMF in their mathematics curriculum.

Summary. The philosophy of teaching and learning threaded throughout the BMF leads us to focus on interesting questions for which the framework features contain some recommendations. For example, how can we establish learning communities that support bilingual students? Are norms established for such a community to operate productively, given the unequal social and linguistic statuses of students (Cohen, 1994)? How can the curriculum materials be used as a resource, enhanced by the resources bilingual students bring to the classroom, to support the learning and engagement of the students? What does it mean that multiple languages (including the dominant language) have equal status? Are appropriate steps taken to ensure that the home language is a resource in the classroom? How does a teacher recognize the overlaps between teaching to meet mathematics goals and meeting bilingual students' sociopolitical and linguistic needs? How does a teacher engage students in multiple modes of communication and how does the communication become an integral part of the mathematics activity? Each of these questions along with the related recommendations from the BMF offer us opportunities to further explore the relationships among the three dimensions of the BMF—learning communities, curriculum materials, and language and communication—and bilingual student achievement.

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Curriculum-Reactor: Sharon Nelson-Barber

WHAT: CEMELA-CPTM-TODOS Conference

WHO: Sharon Nelson-Barber (WestEd)

PLACE: Tucson, AZ

DATE: March 5, 2010

TRANSCRIPTION

Sharon Nelson-Barber

Not only do I face the challenge of speaking to you right before lunch, I also have to follow the wonderful testimonies of the previous three speakers. I want to provide additional emphasis to the very weighty issues they raise.

I also want to say that I am so pleased to be here. I want to thank Marta and CEMELA for the kind invitation. In addition, I want to acknowledge the Tohono O'odham people--the original custodians of the lands on which we're meeting. My career began in the nearby Akimel O'odham community 20+ years ago and educating across cultures has been the focus of my work ever since. We work side-by-side with diverse communities, looking inward, capitalizing on local intellectual and cultural heritage and bringing this knowledge to bear on the critical and enduring needs of our diverse school communities. We certainly don't have enough research in this arena. As a result we don't have access to images and roadmaps for the best solutions. That's the beauty of the three stories presented today.

(Note: PowerPoint slide reads "Ecological Focus, Relationships, Contexts, Languages, Tools and Practices, Community Knowledge Base") They offer an ecological focus that finds central importance in aspects of learning that have gone unrecognized such as relationships, contexts, languages, tools and practices based on community knowledge. Acknowledgement that these complex elements in mathematics learning demand innovative approaches offering great potential for transformative change in the development of curriculum. There's so much that goes untapped in many of our communities where research is centered more on models of under-performance and disadvantage rather than the strengths and competencies that students have developed in their own context and that can serve as bridges to learning.

(Note: PowerPoint slide reads "Teachers do not have the necessary knowledge base to adapt or enact mathematics curriculum materials for English Learners or to help these students negotiate meanings through dialogic communications. (Chval)") Both Chval and Pitvorec discuss approaches to designing and adapting mathematics curriculum so that English learners are supported in simultaneously learning academic content and acquiring a second language. They contend that the mantra in the literature since the 90s is to emphasize cognitively demanding mathematical learning tasks, language rich environments and multiple modes of communication using the target language in active and dialogic communication that's purposeful. Yet teachers still do not have the necessary knowledge base to adapt or enact mathematics curriculum materials for English learners or to help these students negotiate needs through dialogic communication. Chval also addresses relationships between mathematics curriculum and student achievement, but this literature does not address linguistically diverse students. Overall, she concludes there's insufficient research for analyzing and designing mathematics curriculum materials so that English learners successfully learn mathematics in U.S. classrooms.

(Note: PowerPoint slide reads "Opportunities to learn mathematics occur in the complex ecologies of peoples' lives.") However, another body of literature demonstrates ways in which opportunities to learn mathematics occur in the complex ecologies of people's lives, with some of the best examples coming from indigenous communities. Because these communities maintain their traditional subsistence lifestyles, from an early age students begin to acquire deep local, ecological knowledge that models problem solving and ways of knowing. Activities implicit in their life ways include number and operations, patterns, functions and algebra, geometry and spatial sense, measurement, data analysis, statistics, probability, reasoning and proof, representation, and on and on. Their lifestyle is optimal for connecting school math with their own lived experiences, which Lipka shows to be essential in efforts to bring formal mathematics education for all students. Math in a Cultural Context focuses on everyday Yup'ik knowledge related to mathematical thinking directly linked to student's cultural experiences, which necessarily involves familiarity with local values and traditions, but also requires some understanding of culturally defined preferences for thinking and interacting. In other words, teacher's instructional approaches are rooted in Yup'ik learning and systems of problem solving. The approach is essentially constructivist, but as a number of the authors mentioned, it seems that the current interpretations of constructivism in curricular reforms often ignore the sociocultural dimension of student knowledge.

(Note: PowerPoint slide reads "Sociocultural Approach - Knowledge construction is not simply an individual act but something that occurs within a social and historical context.") Knowledge construction is not simply an individual act, but something that occurs within a social and historical context. A social constructivist stance leads to concerns not only about curriculum content, but about the organizational structures, interpersonal communication approaches, and instructional tools and how these reflect world views and values of the school and the students that it intends to serve. Not only must teachers and curriculum developers become aware of analyses of mathematics and cultural social issues, there's a need for more of these analyses especially from within the cultural perspectives of the communities and the groups affected.

(Note: PowerPoint slide reads "Mathematics occurs in the interactional spaces of a community where the mathematical authority in the community is distributed productively (Pitvorec)") Pitvorec argues that learning mathematics occurs in the interactional spaces of the community where the mathematical authority in the community is distributed productively among teacher, curriculum materials, and students and where the norms for engaging in the meaningful mathematical activity of the community are negotiated by teachers and students.

(Note: PowerPoint slide reads "Teachers must... 1. Recognize that students have their own ways of mathematizing the world. 2. Discover ways to bring those models to the surface for discussion") If teachers are going to help students make these connections, they need to 1) recognize that students have their own ways of mathematizing the world, and 2), discover ways to bring these models to the surface for discussion. Which is precisely what Math in Cultural Context has done. Math in a Cultural Context recognizes that teachers need to have concrete knowledge of the kinds of mathematical understandings indigenous students bring to school from their own experience outside of school in order to begin to determine how to develop them further and connect them through formal mathematics. The Yup'ik culture is considered in terms of which ethno-mathematical activities offer natural opportunities for mathematization so that students can develop these processes within a context that's meaningful to them. But it's not just the students' cultures that teachers need to understand.

(Note: PowerPoint slide reads "Teachers must... 1. Know about the values inherent in the subject they are teaching and about the cultural history of that subject. 2. Consider their own values and those of their students' cultures in relation to how and what they are teaching") Teachers need to know about the values inherent in the subject they're teaching and about the cultural history of that subject. They need to consider their own values and those of their students' cultures in relation to how and what they're teaching. These elements must be made exclusive or conscious if mathematics pedagogy is to be culturally responsive. Still, teachers cannot be expected to develop the requisite knowledge or understanding in a vacuum. We must ask, does the mathematical education most teachers receive include an examination of values and perspectives or any history of mathematics from which they might develop an appreciation of mathematics as a product of culture? This should include specific knowledge could be connected to indigenous, ethno-mathematics in productive ways. Does a teacher's education or personal experience include knowledge of the kinds of cultural experiences or worldviews students have that will form the core of their mathematical understanding? Both research and teacher development have critical roles to play in creating an improved mathematics pedagogy for disenfranchised students. For indigenous students, topics that cannot be left out of discussion are the historical processes of education such as colonization and assimilative teaching practices that have resulted in language and culture loss, and community alienation from schooling. And I think we've heard some fine examples of that today. It may be fine to draw on cultural knowledge to make school more inclusive, but students are astute in their understanding that even when presented with particular experiences or perspectives, many educators are unwilling to lend them credence if they fall outside the parameters of what is accepted as good or progressive practice in current educational literature.

(Note: PowerPoint slide reads "Mathematics -- a set of cultural assumptions and activities") So as we continue to construct knowledge about widely diverse settings, we must ensure that the ad-hoc knowledge developed by individuals within diverse settings gains the security and recognition of public domain knowledge. Math in a Cultural Context demonstrates that the incorporation of culture-based perspectives in the daily life of the classroom can be accomplished when teachers develop knowledge of their own and students' approaches to mathematics (and ways of knowing in general). Mathematics, like all school subjects, needs to be seen by teachers and students as the set of cultural assumptions and activities it is. When mathematics is taught or learned in defined cultural contexts, students have increased opportunity to relate to it and find it meaningful. Students of all backgrounds excel when exposed to Math in a Cultural Context. It exemplifies drawing upon students' own experiences that have elements of mathematics in them, and linking classroom learning to the practices of a particular local community. It is engaging and an empowering education for any student.

(Note: PowerPoint slide reads "Complex ecologies can be the driver in the mix of innovations being sought through national initiatives for educational change.") Following the work of Carol Lee, such examples of repertoires that people develop in routine settings of their lives can be used to support complex learning. There is not only high demand but high receptivity to these kinds of transformative solutions, which is what I'm finding across the diverse communities where I work. Complex ecologies must be recognized as a driver in a mix of innovations being sought through national initiatives for educational change. And I believe we've seen some excellent examples of these directions being taken. Thank you all.

Curriculum Discussion

Following the Curriculum research presentations, the practitioner panel, the reactor, and the participants met in small groups of six to eight for discussions. The groups included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The connections question became embedded in the discussions of the other questions. This summary represents common themes identified within and across the working groups.

What do we know?

Based on the research studies presented as a part of the Curriculum Strand, along with poster sessions, and our own professional experiences, we are able to state what we believe to be true about mathematics curricula with English Language Learners (ELLs) and particularly with Latinos.

Evidence from our country's assessment data indicates our current mathematics curricula provided to Latino/as is not meeting their needs in terms of learning mathematics. This is particularly distressing since our Latino student population comprises our nation's largest and fastest growing minority group and represents approximately 20 percent of our total K-12 student population. (Chval, 2010; Pitvorec, Willey, Khisty, 2010). Many complex factors attribute to their lack of success.

The majority of published mathematics curricula currently in use have addressed the language and cultural needs of students as add-ons to the main text and they are often superficial at best in their approach. The texts are written and then "adaptations" or "extra supports" are added to address the needs of ELLs and students from minority populations. Language complexity is often reduced to an introduction of mathematics vocabulary and visual support focuses around technical terminology. We believe that contexts presented in texts are from a middle class urban culture and exclude Latino/as and other minorities. Latino/as do not identify or make connections with the majority of the current curriculum materials. Teachers do not have the freedom to tap into the students' culture or the culture of the community because of the prescribed curriculum.

Publishers also have license to change texts from the original authors' work in succeeding editions and materials may no longer reflect authors' intentions. New textbook programs are adopted every five to six years and "fidelity" to the program is required, often resulting in teachers' losing autonomy to make decisions based on what is best for their students. There is a great incentive for publishers to encourage this adoption practice because of the huge profits they receive from new adoptions. However, we know that no program is perfect and "one size fits all" does not work.

Our country's current high stakes assessment policy implemented due to the No Child Left Behind Act (NCLB) has had negative consequences for ELL students. Assessment pressures have resulted in pacing guides and benchmark testing requiring teachers to "cover" the curriculum. However, students' mathematical understanding is not developed. Effective teaching practices (cooperative learning, bilingual education, etc.) have been abandoned for fear of not making Adequate Yearly Progress (AYP) and resources ELL and minority students bring to the classroom are no longer seen as resources, but are now considered deficits. The assessments in use today do not reflect cultural diversity, problem solving and making connections as described in the NCTM Standards. ELLs are often tracked into classrooms with low achieving students and the curriculum they receive typically emphasizes basic skills and not problem solving situations (Pitvorec, Willey & Khisty, 2010).

NCLB has also had negative effects on teachers. Teachers are reluctant to try out innovative pedagogy when there is the threat of job loss due to not making AYP. There has been insufficient support for teachers in learning new effective pedagogy for working with ELLs and minority students. It is unrealistic to expect teachers with minimal ELL training to modify existing curriculums to meet the language demands of ELLs. The reformed curriculums now being judged as having not met the needs of ELLs cannot be implemented completely by teachers without on-going support. Even the best curriculum materials may fail miserably if teachers are not trained. Curriculum cannot be separated from implementation. Teachers need to "own" the curriculum for it to be successful and see it as a "living thing." Static, scripted curricula cannot be responsive to students' needs and teacher's ownership.

"Knowledge construction is not simply an individual act but something that occurs within a social and a historical context" (Nelson-Barber, 2010). Teachers need to have concrete knowledge of the mathematics understandings that students bring to school in order to determine how to develop a program and connect them to important mathematics. Mathematics occurs in the interactional spaces of a community where the mathematical authority is distributed productively (Pitvorec, Willey & Khisty, 2010). Cognitive abilities develop in and out of school but there still remains resistance to the inclusion of indigenous and minority cultural/linguistic knowledge (Lipka, 2010).

We must acknowledge that all curriculum is "culturally based," and we need to examine how our classroom communication is organized, what counts as legitimate mathematical knowledge, and what are considered appropriate teaching practices (Lipka, 2010). Teachers need to have an understanding of a student's and community's culture and values, and their teaching practice needs to reflect that understanding. "…learning from each other represents a form of respect and equality, as does considering ways in which the first peoples' knowledge may be applicable to the wider U.S. educational scene" (Lipka, 2010, p. 6). Often teachers who are not culturally responsive see students' culture as irrelevant, or at worst as a problem. Minority students are being excluded and sometimes even disciplined due to interpreting cultural values as misbehavior. We know that subtractive bilingualism does not work (Valenzuela, 2002).

What are the implications for practice and research?

First we must first recognize that in order to improve mathematics education for ELL and minority students there needs to be systemic changes in which curriculum is just one part. Changes must also be made in education policies, assessment, teacher preservice programs, professional development and family and community relationships. Based on this research, we recommend the following implications for curriculum development, teaching, research and home/school connections.

The good news is there are some curricula that can offer a framework, or guidelines, for future curriculum development. The supplementary curriculum *Finding Out/Decubrimiento (FO/D)* published in the late 1970's is rooted in sociocultural activity perspective of learning and capitalizes on students' four levels of resources: the object itself, pictures, language, and peers (Pitvorec, Willey, & Khisty, 2010). "When these resources are made available and students are socialized to use them, they are positioned to solve problems collaboratively and make meaning of the content—two crucial tenets of learning and development" (Pitvorec, Willey, & Khisty, 2010, p. 5).

The ideology and philosophy of reforming math and culturally responsive curricula are not currently reflected in our schools. Ethnomathematics should be how we do mathematics all over, not just for minority, indigenous, or Latino/a populations. Currently, ethnomathematics has no place in the curriculum or assessments and is considered second-class mathematics. However, there are a number of projects that have effectively connected out of school learning to in school learning for minority and indigenous students. Some of these include the Algebra Project, Funds of Knowledge, and Math in a Cultural Context (Lipka, 2010). All curricula are culturally based, but we currently accept mainstream perspectives and norms as being culturally free. However, "not only can indigenous, minority, and everyday knowledge be of obvious importance to the 'targeted' cultural groups but also that lessons from these 'peripheral' groups may well benefit teachers' and students' education in general" (Lipka, 2010, p. 5).

Teams of curricula developers need to be from diverse cultural groups and need to include all stakeholders (teachers, administrators, educators, community members, etc.) in order to include different perspectives. Curriculum materials need to be relevant for all children with ELL supports built in, not as "added on" extras that are easily dismissed. Resources for teachers need to include cultural markers where teachers are given information to make connections between school and out of school learning, and information on different cultures' ideas and mathematical practices, algorithms and use of mathematical vocabulary. Materials also need to include tools for teachers that help develop students' enculturation into mathematical communication such as discussion starter posters found in *Math Pathways & Pitfalls.* Curriculum materials need to be field tested with diverse populations of students and teachers. For teachers to have "fidelity" to the program they need to have ownership in it and also know that it is not static and must see it as a living thing, with teaching decisions being based on sound mathematical pedagogy. Curriculum standards and materials need to be focused on depth of mathematical understanding rather than breadth, and the trend to "push down" the mathematics to younger grade levels must be resisted. The enormous volume of published curriculum materials for a grade level also needs to be addressed and reduced.

All stakeholders must develop a deeper understanding of culture and what it means to respect culture. We must recognize that ethnicity does not determine culture and that culture is dynamic and not static. Administrators and teachers need to be aware that they bring their own culture to the classroom and school. How they see curriculum is based on their cultural lens. Just because administrators and teachers are of the same ethnicity as their students, it does not mean that they have the same culture. To respect culture, they must understand and validate culture. They need to know about their students' community and cultures in order to value and respect the students. The enacted curriculum needs to be a negotiation of who the teacher "is" and who the students' "are." Teachers need to know the kinds of cultural experiences or world-views that students have that might form the core of their mathematical understanding. In urban settings there may be many different cultures and teachers and administrators must gain information about all of them in order to respect and validate everyone's culture.

Strong connections and communications with families and schools need to be established. Schools need to view and use the community's funds of knowledge as a resource. Activities that are helpful in gaining this information are: making home visits; holding parent workshops and meetings; providing a welcoming atmosphere in the school; and teachers and administrators attending community events. In order to make a difference in student performance, family engagement is crucial and parents need to be empowered partners in their children's education. There needs to be a bridge between classroom curriculum and the home. Parents need to be a part of the curriculum by engaging in activities in the home that support the curriculum. Parent workshops with supplied materials for projects that are doable for parents should be considered. Considerations for supporting family projects with funding should be a priority in school budgets.

There is a belief that language is irrelevant in mathematics and therefore, mathematics is a subject more easily taught without special considerations for ELL students. However, this is not true. Mathematical communication requires fluency with academic language that often contains complex sentence structures and specialized vocabulary. Teachers need to be aware of differences in the social and mathematics register, specifically with words that have multiple meanings dependent upon the register. Recognizing that sentence structures are different in Spanish and English is helpful in understanding the kind of scaffolding required for students to handle the language complexity that is an integral part of mathematics. However, this does not mean that teachers should offer students more simplistic problems or sentence structures, but support students in understanding the academic language.

As students learn to communicate mathematically, teachers need to focus on meaning and content while socializing the academic language without marginalizing students. ELLs need to be afforded use of multiple modes of communication, focusing on their strengths and competencies. Student engagement needs to be clearly defined in terms of the following identifiable behaviors:

- Using various modes of communication to make contributions to the class discussion such as verbal, written, related question, picture, gesture, calculation;
- Trying to make meaning from other's explanation and being open to others' ideas;
- Explaining their thinking and/or using manipulatives or other symbol systems to access thinking when working individually, with a partner or a group.

Superficial behaviors cannot be used as a measure of student engagement.

Research shows that students learn by talking with each other, and it does not matter what language it is or that their grammar, etc. is not perfect (Lotan, 2008). Written explanations and proof should also be a part of the curriculum and should include occasions for more than one draft writing.

As teachers introduce mathematical problems, attention needs to be given to the context of the problem and support given to make those problems meaningful to the students. During the preview part of the lesson, understanding context should be the focus and not introducing mathematical vocabulary in isolation with definitions. Mathematical vocabulary should be taught as part of the content in the lesson in a meaningful context (Moschkovich, 2008). Students need to have the academic vocabulary to be successful and teachers should use the correct mathematical vocabulary in the context of the lesson.

Meaningful contexts for lessons can readily be found when students' lives are used as a resource. Teachers must have concrete understandings of what the students bring from their world, about how they mathematize it, and teachers need to bring these models to the surface for discussion in the classroom. Connecting students' life experiences with school mathematics is essential to improving mathematics education for all students.

All of these recommendations will require teachers and preservice teachers to receive support. They will need to have inservice that includes models of effective practices. This can be accomplished through the use of video clips that could be provided by TODOS. Attention needs to be given to the values inherent in the subject and the historical perspectives that might develop an appreciation of mathematics as a product of culture during preservice and professional development. Teachers need to have better articulation of equity with regards to cultural awareness. They also need to have a framework to evaluate existing mathematics curricula.

Professional development needs to include time for teachers to reflect on their own practice. For professional development to be effective it needs to be part of the teachers' workday, not during after school time after a day of teaching. Follow up support needs to be given through the use of mathematics support teachers or coaches. These instructional support teachers need to be knowledgeable about mathematical content and pedagogy, as well as how to scaffold and support language development. They must also have skills for working with adults. Administrators will also need inservice to understand and support mathematics and language pedagogy.

Preservice teachers will need focused support. Mentor teachers and time for observation in other exemplary teachers' classrooms need to become part of the beginning teachers' experience. Team teaching with a mentor teacher is another possible model for beginning teacher support.

What questions do we need to research further?

We identified the following questions as needing further research and investigation:

- A framework is needed for analyzing and designing materials for ELL learners to have success in mathematics.
- Does the inclusion of cultural markers and tools improve instruction and in what ways?
- How well does curricula written for specific groups translate to the larger population?
- What principles can be identified in successful curriculums and can they be applied to other curricula?
- What are the effects of pacing guides on ELL and minority students?
- When researching curriculum effectiveness, data on Latinos needs to be disaggregated into more specific categories: Latinos that are ELLs, first generation immigrants, non-ELL Latinos, etc. Also, information on the level of English proficiency for ELLs needs to be identified when evaluating curriculum effectiveness based on state tests (in reference to Seattle court decision involving *Investigations* curriculum).
- What are the "costs" of losing a language and having one dominant language?
- How do we approach the variance in the EL population?

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CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Family Engagement

Section 5 of 9

Chair: Marta Civil, University of Arizona

Tucson, Arizona March 4 -6, 2010

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IMAGINANDO Y PROMULGANDO ESPACIOS DIALÓGICOS CON FAMILIAS Y ESTUDIANTES BILINGÜES Y SUS MAESTROS PARA ENSEÑAR Y APRENDER LAS MATEMÁTICAS

IMAGINING AND ENACTING DIALOGIC SPACES FOR TEACHING AND LEARNING MATHEMATICS WITH BILINGUAL PARENTS, STUDENTS AND TEACHERS

Martha Allexsaht-Snider

University of Georgia

I am interested in exploring with all of you this evening what we have learned together over the last five years in the work with CEMELA. I am interested, in particular, in considering what we have learned about expanding our own and other educators' and researchers', as well as parents' and students', ideas about the possibilities for engaging together to create language-rich mathematics teaching and learning experiences that draw on bilingual families' and students' funds of knowledge. How can we use what we have experienced and learned together to fire the imaginations of students, parents, teachers, administrators and policymakers about new ways to enhance mathematics learning in bilingual settings?

I am going to pose some questions for different groups to think about as I talk, and I hope to hear some of your thinking as we encounter each other in formal and informal settings tomorrow at the conference.

For those who have taken part in research and projects with CEMELA that have engaged parents, teachers and students together, what do you have to share with your CEMELA colleagues who focused on research with teachers and students in classrooms, about important findings related to engaging bilingual families in mathematics learning?

For those of you who have taken part in research and projects with CEMELA that have examined how bilingual language resources and discourse have played a part in mathematics teaching and learning in classrooms, what do you have to share with CEMELA colleagues who have focused on research and projects with families and after school programs?

For those of you who are parents, students, educators, or researchers and have NOT participated in the CEMELA research-what questions do you have about mathematics teaching and learning with bilingual students and families that you would like to pose to the CEMELA researchers?

I have recently used what I learned as a researcher and educator from my work with the earlier MAPPS project, and more recently with the work with CEMELA faculty and fellows, to implement and conduct research with a science learning project with middle school Latino students, their parents, and their science and ESOL teachers. My colleague Cory Buxton at the University of Georgia and I took a theoretical framework that Marta Civil and others drew from the MAPPS research and some of you have built on in the CEMELA work to help us imagine interactive, dialogic activities for bilingual, Latino students, their parents and their teachers. We wanted to create workshops to support students and parents' learning of science inquiry processes, academic language, and physical science content and concepts. The project is titled: *Steps to College: Burney- Harris-Lyons to UGA through Science* and is supported by the Hispanic Scholarship Fund. I am going to use a discussion of what we have learned and applied from the MAPPS and CEMELA projects in our science project for Latino families and students to get us thinking about how what we have learned could be applied by other mathematics educators interested in working with Latino students and families.

In Georgia, we designed a series of four three-hour bilingual Saturday workshops that each included a science inquiry lesson, a visit to a science lab on campus, and a discussion of issues and resources related to studying science, success in middle and high school, and attending college. The decision to conduct the lessons bilingually and engage parents and students together in challenging inquiry about physical science concepts (Physical and Chemical Changes, Force and Motion, Forms of Energy, and Sound and Waves were the topics) built on my experience as an evaluator with the MAPPS project. Over the four years of MAPPS in four urban school districts across the southwest, we saw parents and students successfully engaging together in bilingual inquiry about challenging math concepts in the areas of number, geometry, algebra and data analysis.

The research with MAPPS showed that bilingual parents in the project had opportunities to develop in multiple roles- roles as adult learners of mathematics, roles as parents seeking to understand and support their children's mathematics learning, and roles as facilitators of mathematics learning for other parents and children. Even with the much smaller scale of our science project in Georgia, we wanted to design ways for bilingual parents to engage in roles both as adult learners of physical science and as parents seeking to understand and support their middle school children's science learning. We didn't feel that we had the time or resources to support parents' roles as facilitators of learning with other parents and students, but we did feel that we could create spaces where the parents' roles as adult learners of science and supporters of their children's learning might be expanded.

As we continued to plan our BHL to UGA through Science project in Georgia, we took inspiration from some of the research with after-school programs in CEMELA that have explored new and creative ways to support bilingual mothers in integrating their community-based knowledge of mathematics in everyday contexts with development of mathematics learning activities for their children in the after-school context. We had to work with a very limited timeframe, but decided that we could stimulate dialogue between parents and children about parents' everyday experiences with and interests in science by designing and recording interviews that middle school students conducted with their parents. The conducting of interviews and reflections on them were built into the workshop framework, and students conducted a second interview with their parent at the end of the four workshops.

Our goal was for students, parents themselves, and teachers to see parents as intellectual resources for science learning. Preliminary analysis of the interviews, and presentations that two parent-student pairs made at a recent conference on Latinos and immigration, indicate that in a few cases, both parents and students came to recognize some of the parents' everyday experiences with science as something they could draw on in making sense of science concepts with their children. We are about to begin a new set of two workshops, and expect new insights from another interview that students will conduct with parents, and also one that parents will conduct with their children.

Research in all four of the CEMELA sites, both in and out of school, has demonstrated how key both the provision and valuing of bilingual language resources are to the meaningful engagement of bilingual Latino/a students and parents in mathematics learning. In the CEMELA project as a whole, opportunities to draw on both Spanish and English as resources for participating in rich discourse about mathematical ideas and problem solving has been shown to enhance student learning in the classroom and support parents' participation in activities for family learning of mathematics. Cory Buxton and I took inspiration from the commitment to bilingual learning settings and resources for Latino families in MAPPS and CEMELA, and created a bilingual science inquiry activity packet to accompany workshop activities and a resource kit for home science inquiry activities.

We are exploring a new area this year as we work to enhance the bilingual pedagogical approach and the materials we used in the first year of our Steps to College through Science project. We are considering how we might help parents and students together understand the ways that families' bilingual language and biliteracy resources could be used to help students develop academic language in English that is essential for participating in oral and written inquiry science discourse. I feel that CEMELA colleagues who have investigated the development of academic language and discourse in mathematics classrooms might have some insights to guide our thinking in this area and I look forward to any suggestions you might have.

In addition to providing bilingual materials, in Georgia we found bilingual graduate and undergraduate students and school personnel to collaborate with us in conducting all workshop activities bilingually and modeling the valuing of bilingual and biliteracy skills. This is another approach that has been utilized in CEMELA projects, and I am interested in reviewing any of the research that has explicitly focused on the interaction of bilingual university students and bilingual parents. What have you learned about how to support university students in working with parents and children in mathematics? What have you learned about how to facilitate opportunities for students and parents to learn from the experiences of Latino university students?

Parents in our project reported that they found it very valuable to hear from Latino, bilingual immigrant students who had successfully negotiated the challenges of studying science in middle and high school educational settings and were completing college. They particularly valued the opportunity to talk openly and frankly with a student who was an undocumented US high school graduate and hear his story of persisting in seeking ways to finance and complete his college education. Where do these kinds of practical conversations about "steps to college" fit systematically into programs to enhance mathematics learning for Latino students in middle and high school classrooms and where do these conversations fit into after school programs and programs for families in mathematics?

One more area that was an integral part of work in MAPPS, and has been explored in some of the CEMELA work, is the potential for learning for teachers who participate in a dialogic manner with bilingual parents and students in the context of mathematics learning activities. We included science and ESOL teachers in our "Steps to College" project last year, but as we plan for our new set of workshops, and as we wrote a new NSF proposal integrating professional development for teachers, and work in science classrooms with workshops for middle school parents and students and teachers together, we have recognized the need to develop a more explicit focus on the teachers' roles in interactive work with families. I am going to look to those of you who have worked with students, parents and teachers together in CEMELA for inspiration in this area.

So, I think you who have worked these five years in CEMELA have much to contribute to our continuing efforts to imagine and enact dialogic spaces for bilingual students, parents, and teachers for teaching and learning mathematics-and for teaching and learning with bilingual students and families in other content areas like science. To summarize, I have outlined the following areas in which I have found your work to be inspirational and helpful for conceptualizing my research and work with families and teachers. I know there are many more areas of contribution that I have overlooked, am unaware of, or have not had the time to outline here.

(1) Engaging bilingual parents and students together in inquiry about challenging mathematical content and concepts;
(2) Providing opportunities for parents to develop in multiple roles-such as roles as adult learners of mathematics and roles as parents seeking to understand and support their children's mathematics learning;
(3) Facilitating new and creative ways for bilingual parents to integrate their community-based knowledge of mathematics drawn from everyday contexts mathematics learning activities for their children in after-school contexts;

(4) Creating occasions for students, parents themselves, and teachers to see parents as intellectual resources for mathematics learning;
(5) Providing opportunities to draw on both Spanish and English as resources for participating in rich discourse about mathematical ideas and problem solving, and valuing families' bilingual/biliteracy skills enhances student learning in the classroom and supports parents' participation in activities for family learning of mathematics;
(6) Fostering meaningful interaction between bilingual university students and bilingual parents to enhance parents' roles in supporting their children's success in mathematics and to promote important conversations about the practical challenges for immigrant families seeking postsecondary education for their children;
(7) Facilitating teacher learning through participating in a dialogic manner with bilingual parents and students in the context of mathematics learning activities.

Several questions for us to ponder for another day are:

What are the theoretically-grounded understandings emerging from the research with bilingual students, parents and teachers with CEMELA and how can we use those understandings to design and conduct research in powerful dialogic learning spaces for bilingual children and parents?

How might CEMELA research inform the broader fields of general research with family involvement and research with new immigrants in education, in addition to informing our own community of researchers concerned with equity in mathematics education?

I look forward to continuing conversations with many of you over the next day.

Contributions to the research on mathematics family education. The debate on participation.

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Abstract

Family education is a re-emerging field of research both in Spain, and in the rest of Europe. While in other regions of the world it has a wide research trajectory, here in Catalonia (and in other areas of Europe) it is right now becoming a topic of debate. In this article I will first present the research lines on family education and math education at the center of our work, the main contributions used until now, in our opinion, and the elements that may contribute from our work the development of this field of research. For that purpose I will highlight several examples that allow us to discuss both evidences on barriers that difficult families participation in the centers and, the aspects that contribute to promote their participation.

STATE OF THE ART

In Spain the inclusion of the family in the school is an old debate. In 1970 the Ley General de Educación (LGE)[1] [General Law on Education] opened the possibility for students' families to create associations with the aim to support and complement the task of school. It was the creation of Parents' Students Associations (AMPAs) and schools of fathers and mothers that even today are the only ways for families to participate in the schools.

The Constitution (1978) in its article 27.7 establishes that "The teachers, parents and in its case the students will take part of the Management and control of the centers supported by the administration with public funding, in the terms that the law defines". Over the decade of the 80s, the participation of families in the school was regulated especially by the AMPAs. In 1980, the Organic Law 5/80 on the School Centers Statute (LOECE), included an article (el 18.1) that stated "in each teaching center there will be a Parents of Students Association (...) by means of which they will convey their participation in the collegiate organs of the center". The AMPAs were established as the legal channels for the families to intervene in the centers.

The development of these associations to manage the participation of families in the educational centers has not happened without controversy. In 1987, a known Journal on educational topics such as *Cuadernos de Pedagogía* published in its section "Topic of the month" a full article dedicated to parent involvement. In this article we can read:

> "It has been said that APAs were obsolete entities, reduced to the organization of after school activities. It was also said that the LOECE[2] first, and the LODE[3] later were the "death" for these associations, when those were marginalized from the School Council. Despite of it, we think that a non-restrictive Reading of the APAs functions, and above all taking into account that "the option for a modern educational system, in which an active and responsible school community is a co-leading actor of its own educational action" as stated in the Preamble of the LODE opens new possibilities to the action of the APAs". (*Cuadernos de Pedagogía*, 147, p. 17).

This quote illustrates the debate that has been taking place in Spain for years, and over which there are opposing opinions: those stating that the participation of the families is used (and reduced) by the AMPAs (that is, those who blame these associations for institutionalizing and reducing the spontaneity of parent involvement); and those who assert that establishing legal channels has served to facilitate such participation and at least, guarantee that families have a representation in the centers and stable structure for participating when they desire it.

On the other hand, this debate has to be situated in the globality of the international community, to understand as well that it has to be viewed in the context in which we have developed the two research projects that we present in this article. In the 1970s

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[1] The LGE substituted the Moyano Law of 1857. This law ruled the Spanish schooling for more than a Century. In its time it was a great advance, because it meant the establishment of compulsory education from six to nine years old. The educational outline designed then was maintained until the last third of the 20th century.

[2] Ley Orgánica de Educación que regula los Centros Escolares (LOECE) [Organic Law of Education to organize the School Centers].

[3] Ley Orgánica de regulación del Derecho a la Educación (LODE) [Organic Law to organize the Right to receive Education].

numerous research studies appeared demonstrating the positive effects on academic success from the parent involvement in the centers and school homework. Smith (1968) was one of the pioneers in United States. Álvarez (1987) states, he started a project named *School and Home* in which about 1.000 children and families were involved. This researcher states "children took home books from school with the label "Please, read me", among other planned activities for the parents to participate in the school curriculum. The results, measured by reading tests posed to the children and the parents' attitudes on the program, were highly positive". (Álvarez, 1987, p. 10)

During the 1980s these initiatives started to run into some obstacles. From some spheres some started to say that parent involvement was a potential barrier for the learning of mathematics. During those years in United States, the world point of reference was developing the reform known as *mathematics reform*. The NCTM (*National Council of Teachers of Mathematics*), the organization responsible for teacher training in the U.S., developed the mathematics standards from the point of view of the curriculum. The reform involved changes in the content (especially with regards to the sequence), as well as in some education strategies used to teach the content.

In this context some families found problems and difficulties in understanding the new methods used by the teachers. These methods were unknown to the families, who tended to use with their children at home the same methods and strategies that they had learnt at school. This kind of practice incorporating different pedagogical methods at home and at school was viewed as confusing the children. For this reason, several research papers blamed the families the lack of success of mathematics reform.

In Spain mathematics reform has not had as clear debate as happened in the United States. Things have been changed, and the influence of modern mathematics can be observed through the textbooks, its backward step, and the advancement of the constructivist approach in the form of teaching mathematics. Anyhow, the changes produced in the teaching of mathematics were not accompanied by a reflection on the work with families. Similarly to other areas of the world, in Spain the educational action policies developed to address the difficulties of learning mathematics have consigned the families to a secondary role: they have to accept the changes, without getting involved in them. Teaching was considered to be under the control of the faculty at the school. This view of parents as teachers' role in children mathematics learning is widely present today in the majority of the teaching staff (Díez-Palomar, 2009).

In this context, the impact of the research in the field of parent involvement has been limited in these last twenty years in the European context. There has not been a great number of studies with regards to parent involvement in mathematics education of their children until recent years. Those conducted, were to analyze the ideology of the meaning of learning mathematics from the point of view of the families, or to investigate on the methodologies that families use with their children.

However, in other places there is a long tradition of working with the community. In this case the research shows that the families that get linked to the teaching and learning of mathematics can better encourage their children to learn mathematics (Civil, Guevara, & Allexsaht-Snider, 2002; Ginsburg, 2006; Martin, 2006). It is when the families do not understand the mathematics taught in the educational center, or when they can not understand how the teachers teach a content that is familiar to them, then the feelings of frustration appear, making difficult on the one hand the involvement of the parents in the teaching of mathematics, and on the other hand any positive feeling from the children to the subject. Then the students observe that their parents teach them in a different manner, or that they have doubts about the questions being posed to them, creating a feeling of insecurity or confusion in the children. Only with the existence of family involvement programs, those feelings change, and not only the feelings, but also the academic results of the students, that improves as a result of the parents' involvement.

Family mathematics involvement helps or contributes directly to re-signify the meaning of identity: the families acquire more security because they understand and know more mathematics, and that gives them cause for more participation in an active manner in the mathematics education of their children, because they feel that they can really help them given that they understand what they are doing (Civil & Bernier, 2006; Díez Palomar, 2009a; Díez-Palomar, 2009b).

In this article I discuss several of the elements that explain the participation of families in the initiatives of training. For that purpose I will use data from two research projects funded by the Agencia Catalana de Gestión de Ayudas Universitarias y de Investigación (AGAUR) [Catalan Agency for Management of University and Research Grants], *Formació de professorat per a una educació de familiars en contextos multiculturals* (2007/ARIE/00026) [*Teacher training for parent involvement in multicultural contexts*] i *Formació de familiars per a una escola inclusiva* (2008/ARIE/00011) [*Parent involvement for an inclusive school*]. In the following sections first there is a discussion of the conditions that explain the participation of families in the center. Then, several examples are presented that illustrate the barriers that makes difficult this participation in the case of the two centers that participated in the research analyzed, and the elements that suggest that it works to promote successful parent involvement. The theoretical framework, as well as the lens that I'm using during the whole paper, were developed drawing on what I learned working in CEMELA between 2005 and 2007.

PARENT INVOLVEMENT IN THE CENTER

In Spain parent involvement in the educational centers is done mainly through the associations of parents of students (AMPAs), or through the schools for parents. These kinds of initiatives are regulated by the current Framework of the law, and are spaces of institutional participation (this means, that those are established within the structure of the schooling system). In addition, over the last years there has been a progressive "colonization"[4] of the institutional spaces by individuals that from their own initiative get together and try to find ways of participation. The projects of social and educational transformation such as the *Learning Communities*[5], which are strongly rooted in different regions of Spain, and involves the participation of the entire community in the educational center, are examples of this approach. On the other hand, there are other local actions in the same vein, such as the courses of adult education from the School La Verneda, the mathematics workshops for families developed in some of the primary and secondary Catalan schools, among other initiatives.

[4] I use the term in the meaning of "colonizing the lifeworld" that uses A. Schütchz in his work *Las estructuras del mundo de la vida cotidiana*, in which he does a depth analysis of the consequences of modernity, and proposes the concept of "colonization" as a means of respelling to overcome the losing of meaning that points out Weber as a consequence of the process of bureaucratization to which are objects modern societies.

[5] "*Learning Communities* is a global project of educational and social transformation that involves many agents, sistems and social processes". (Díez-Palomar, Flecha, 2010, p. 25).

Starting from this context, the questions that I will discuss here are: What do we consider as parent involvement?, and what are some of the key elements in the debates about parent involvement?

Taking into account the prior literature, we can find different attempts to define parent involvement. In the two projects in which I have based this article[6] I have observed that there are "prior ideas" on the idea of participation that directly connect with the claims of both families and teachers on this topic. These ideas draw a map of "social representations" that exist regarding the activity of parent involvement. This map somehow mediates or influences the forms in which this participation takes place and the tensions that appear in real life when fathers, mothers and different family members attend the center with the goal of finding support to better help their children with the school.

Remillard & Jackson (2006) and Jackson & Remillard (2005) distinguish between diverse ways of participation that can be observed among families that get involved in their children's education. These authors state on the one hand that the ways in which the families get organized to support their children, beyond the children's learning model; then they point out the activities developed to support their children in their school (children's schooling); and finally distinguish the voluntary forms of participation in the same educational center (children's school).

Other authors, such as Cai, Moyer, & Wang (1999), Cai (2003), and Civil & Bernier (2006) distinguish among different profiles of participation. Cai and collaborators stated the different roles that a parent can take when trying to support their children with school. For that, they distinguish among roles as motivator, resource provider, monitor, mathematics content advisors, and mathematics learning counselor.

Civil presents another view of the role that the families develop. From this point of view, Civil presents the parents as learners, facilitators, leaders, and as parents by oneself (Civil & Bernier, 2006). By doing that, Civil introduces a new element of reflection in parent involvement, that is the self-concept that the parents have of themselves, as adult persons that also want to learn for themselves (not only to have resources to be able to help their children with mathematics). In a similar view, several authors introduce the debate about the self-education and the prior experience in mathematics that the members of the families have in order to help the children with this subject. This idea is consistent with CEMELA's evidence drawing from work with Latino families. There are a plethora of quotes from parents involved in the research process that confirm the importance of their prior experiences with mathematics. A bad experience with it has a negative impact on their self-perception, so this may become a barrier to their involvement as a "math facilitators" at home. The same is true for the reverse. In addition, drawing on their past experiences, parents expect a particular way to teach mathematics; many times they feel unsure about teachers' methods in the schools.

However, a key element in prior research projects is the importance of parent involvement in the center, which aligns with evidence collected in the two studies that I present in this paper. Mónica[7], one of the mothers that has participated in the

[6] *Formació de professorat per a una educació de familiars en contextos multiculturals* (2007/ARIE/00026) i *Formació de familiars per a una escola inclusiva* (2008/ARIE/00011).

[7] All the first names of people and places have been changed, to project the integrity and anonymity of the people who have participated in this research. In the case that appears the name of a place that coincides

Workshops of mathematics that we have been developing in a secondary school since 2007, states:

> "Of course, but this is it... , let's see, I tell them… that is, in the school, for me, we are, let's see, we are three people: the student that wants to learn, the teacher on the one hand, and on the other hand the family". (Mónica)[8]

The above mentioned literature reveals that from the stand point of the teachers sometimes the involvement of the families becomes invisible and certain kind of families are branded as people with no interest for their children at school (Roma families, immigrants, etc.). The problem with regards to parent involvement is that sometimes the centers only make visible the families that somehow participate inside the center, but not those that use other channels of participation.

Remillard & Jackson (2006) and Jackson & Remillard (2005) question the conclusion that families are not interested in their children's learning. Families are interested in their children education, but sometimes they do not do it in the way that the centers expect. However, in the popular media it is easy to find this debate, which it is sometimes at the center of the tension between the teachers and the families.

Ánder, a High School teacher of one of the centers participating in the research here discussed, states:

> "In this High School there are families, many parents that barely participate in the process of their (…), because every time they are reprimanded, either they do not attend, or any other commitment that is agreed between the tutor and the family is not fulfilled". (Ánder)[9]

Ánder is a doctor of math education, and has more than thirty years of experience as a teacher in several Catalan High Schools. He has a wide experience working with families and he has found himself in centers in which the treatment towards families is very different. As he states:

> "A school of parents is a project which involves the families and the AMPA, that usually organizes… there is a teachers' commission and part of the AMPA involved in what? Each semester they do different activities: presentations by experts… usually they conclude the activities with a sort of educational play in which there is interaction. What is a school for parents? Well, it is a way for the families (...) of participating. Here we do not have one. In Manlleu, were I used to work, they have one school for parents. They used to do lectures each semester, and at the end they finished with a play. Here there are the basic channels, the minimum that they could have, and despite everything it does not always work." (Ánder)[10]

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with a real city or town, it has been chosen at random, meaning that it never corresponds with the information orivuded by the interviewee, in order to avoid possible identification.

[8] "Claro, pero por eso tiene que ser..., a ver, que yo le digo... es que, a ver, en la escuela, yo, para a mí, es que somos, a ver, somos tres personas: es el alumno que quiere prender, el profesorado por un lado, y luego la familia". (Mónica)

[9] "En aquest institut hi ha famílies, moltes famílies que no participen gaire del procés del seu (...), perquè cada vegada que se'ls crida l'atenció, o no venen, o qualsevol compromís que moltes vegades s'explica entre el tutor i la família l'acaben no complint". (Ánder).

[10] "Una escola de pares és un projecte en el què teòricament estan la família i l'AMPA, que habitualment o bé organitza... hi ha una comissió d'uns profes i la part de l'AMPA implicada en què per trimestres doncs fan diferents activitats: xerrades d'especialistes que venen... habitualment s'acaba amb una mena de obra de teatre didàctic en el que hi ha una mena d'interacció. Què és una escola de pares? Doncs una

The situation, that Ánder reports is a common situation, easy to find in other centers in Catalonia. The data that we have provides much evidence about the existence of a debate regarding parents' seeming lack of involvement. Mónica, who is one of the mothers who has always participated in the Workshops of mathematics, also states:

> "The problem that I observe, I don't know if it is only in this high school, that… I don't understand that if there are 400 students, how come in the meetings there are few parents? Without taking into account the first year families. Some because they can't, others because… I don't know, I don't know." (Mónica)[11]

The above-mentioned literature in adult education already advises that the participation of parents is in many occasions determined by the availability. The families have to face working hours schedules, family responsibilities, daily life organization; sometimes attending the training workshops of their children at the school of their children may be a complex task. Alberto, another of the participating parent in our mathematics workshops, travels from a neighboring town to attend the workshop. The occasions in which he could attend were because of changes in his working schedule. Therefore the challenge is to find flexible ways of organization in order to overcome all those difficulties. In many occasions the established institutional channels do not have enough resources to face the needs for flexibility and therefore working parents and others remain underrepresented.

I can also state that in those locations in which there is parent involvement, the context is very different. The power of the "discourse" as a "reality builder" (Searle, 1997) is obvious. In the *Learning Communities* (like the primary school that has participated in our two research projects) the existence of a discourse based on high expectations creates an atmosphere of optimism that promotes participation. When mothers and fathers of the neighborhood hear that some of their neighbors are attending the school center to learn, and that this is yielding positive results, then the "pull effect" results in an increase in participation. This adds to the desire of the families to participate and get involved in the education of their children. Again, Mónica offers a key perspective in her interview:

> "Well, anything that is helping the children, let's see, if my schedule allows me, I come. Everything for them, of course, to improve (…) Well, I, in my family, they have always instilled in us that studying is very important, my father, my mother… and I am instilling it into my children, then, any meeting that the school does, whatever it is, I come, because it is important for me for my children's sake and I want to know how it works, and to collaborate for them to be in a good track, from their part and from our part". (Mónica)[12]

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escola de pares és una manera de participar (...) els pares en (...). Aquí no hi ha. A Manlleu, on jo hi era, havia escola de pares. Es feien dues xerrades per trimestre, i al final s'acabava amb aquesta obra. Aquí hi ha els canals bàsics, mínims, que pot haver-hi, i malgrat tot no funciona moltes vegades." (Ánder)

[11] "Es que el problema que veo, yo no sé si es este instituto, que aquí con… a ver, yo no que no entiendo es que con 400 alumnos, cómo que las reuniones, a no ser que sean de primer curso de la ESO, vienen muy pocos padres. Unos porque no pueden, otros porque… no lo sé, no lo sé." (Mónica)

[12] "Hombre, todo lo que sea ayudar a los hijos, a ver, si puedo por horario, sí vengo. Todo para ellos, claro, para mejorar (…) A ver, yo, en mi familia, siempre nos han inculcado que los estudios son muy importantes, mi padre, mi madre… y yo se lo estoy inculcando a mis hijos, entonces, cualquier reunión que hacen en el colegio, del tipo que sea, yo vengo, porque me importa que mis hijos vayan bien y quiero

DISCUSSION

Parent involvement in the school centers is a controversial issue which no clear consensus among educators. We know that as far as the whole community has as main objective the improvement of the students level of education, then families get involved and the discourses around this participation are extremely positives and optimistic. We also know that when there is not a project of social and educative transformation, then contradictory discourses appear. These discourses can put parents and children in conflict.

Having in mind all these considerations, our question is what contributes to promote parent involvement, and what contribute to make it difficult? To answer this question we use quotes from the families as well as teachers involved in the two research studies discussed in this paper.

a) Exclusionary elements

Data collected during the last three years suggest that there is no agreement on parent involvement. Some teachers think that parent involvement should be promoted, but they complain that parents themselves are the ones that do not want to get involved in the educative centers' activities. Ander claims that involvement is something related only to the families:

> "Ander: I think that this is just an issue concerning the families. I think that we cannot do anything else in addition to what we already do to promote parent involvement. This is the case. I guess that they have no time or...
>
> Researcher: So that's another question that I was expecting to ask you, so what do you think that the difficulties are?
>
> Ander: The difficulties? Well... Regardless of the time available [to attend activities in the school], some of them [parents] do not want, do not want... do not want to get involved. To many parents this [the school] is like a parking lot for their children".[13]

However, from the parents' view, the discurss is the opposite one. Monica says that teachers do not provide parents spaces to get involved except the ones which are the official ones. If parents want to ask for more information, sometimes they are met with unfriendly attitudes, because according to Monica some teachers feel as if parents were questionning their work.

> "Researcher: And it was no possibility to talk with the teacher to...
>
> Monica: Uff, yes... well, this is another issue, she got angry with me.
>
> Researcher: Why? Oh, my good...

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saber cómo funciona, y colaborar para que ellos vayan bien, por parte de ellos y por parte de nosotros". (Mónica)

[13] "Ánder: Pues jo crec que és una qüestió única i exclusivament de les famílies. Nosaltres jo crec que no podem fer res més del que fem perquè participin les famílies. És una qüestió d'això. Suposo que els hi falta temps, o... Investigador: És que això era l'altra pregunta que el volia fer, llavors ¿on veus tu les dificultats? Ánder: Les dificultats? Bueno, independentment del temps que moltes vegades es té, hi ha com és que no volen, no volen... no volen implicar-se, si no que per moltes famílies això és una mena de pàrking moltes vegades, dels nens". (Original quote).

> Monica: I have no idea. I was coming nicely to talk with him and he told me... I do not know, perhaps he was angry that day, I have no idea, he asked me if I was questioning his work, and I was telling him that I was not questioning anything, I was just trying to share with him my viewpoints drawing on what I was observing in my daughter, who was not understanding mathematics, and she used to get good grades, and now she was getting bad scores, and... so I would like... to share... and he was very angry with me and he told me "look, no... I'm going to another meeting", and he left the meeting like that, and I was very upset, I don't know". [14]

After that, Monica added:

> "Monica: (...) but well, what I'm not understanding is the fear that some teachers have when the children are not doing so well, so you want to ask for an appointment, because it looks like if you are coming to the teacher to ask for an explanation (...) and I'm not coming to question anybody, what I want is to know why my children are not doing well [at school]. Or what [my child] is not doing well (...) to see if I can contribute with something extra, I do not know, for the goodness of the child, to overcome that [problem]: so, look, he fails here or there... Maybe they explain it to him, and he does not understand it, but well, she is thirteen, and from time to time he is getting older, but last year he was like not so well, but, I think that (...) teachers that say (...) say: "oh! A mother is coming! See if she is going to make arguments with me". Because there are also parents that "eih!, my child, why..." Well, I think that if a child must fail then he/she fails, but this is something to talk about, isn't it? To say: OK, well, what do you think? Where can he improve? Where does need more work? What do we need to do?..."[15]

These kinds of discourses, fights, and conflicts, produce an environment that is not conducive to promoting parents' involvement into the centers. In addition, this type of discourse also promotes a common social representation focused on the questioning of the teacher's role within the classroom. From our point of view this kind of discourse are what we call as *institutional barrier*, because it emerges both from the school context itself and the way in which the school structure is organized. These barriers are elements coming from the protectivism that sometimes we find in the schools, when

[14] "Investigador: Y no hubo posibilidad de hablar con el profe para… Mónica: Uff. Sí, bueno, ese es otro tema, que se enfadó conmigo. Investigador: ¿Por qué? Ay, por favor… Mónica: Mira, no sé. Yo venía en plan bueno a hablar con él y me dijo que… no sé, a lo mejor estaba aquel día enfadado, no sé, me dijo que si le estaba cuestionando su metodología de trabajo, le dije que no estaba cuestionando su trabajo, sólo quería hablar los puntos de vista que había visto a partir de mi hija, que no entendía muy bien las matemáticas, que siempre había sacado muy buenas notas y ahora había bajado mucho, y que quería pues… compaginar… y estaba muy enfadado y me dijo "mira, no… me voy a una reunión" y dejaba el tema así, y me quedé como muy chafada, no sé".

[15] "Mónica: (…) pero bueno, yo lo que no encuentro es el miedo que tiene algún profesorado de que si el niño va mal le convoques una reunión, porque da la sensación de que tú le vayas a pedir explicaciones (…) y yo no voy a cuestionar a nadie, yo lo que quiero saber es por qué falla. O en qué falla (…) pasa mal, pues yo a ver si puedo aportar algo extra, no sé, para el bien del nene o de la nena, para enfocar eso: pues mira, falla aquí, falla allá… A lo mejor a ella se lo explican y no lo acaba de ver, pero claro, son trece, cada vez es más mayor, pero el año pasado estaba un poco más así, pero bueno, yo encuentro que (…) profesores que dicen (…) dicen "uy, que viene una madre, a ver si es que me va a echar la bronca". Porque claro, dicen que como hay padres también que "a ver mi niño, por qué…". Hombre, yo creo que si tiene que suspender pues suspende, pero bueno, también hay que hablarlo, ¿no? decir "bueno, ¿usted qué ve? ¿Qué… dónde podría mejorar, dónde hay que apretar, qué hay que hacer?"..."

teachers defend the classroom as their own space, unconnected to other educative agents. This protectivism is a consequence of a historical process of the institutionalization of education as well as the ways to promote/organize the involvement into the school centers in Spain. The inflexibility of both the educative stratums and the democratic spaces for participation means that many people would not be able to attend the center for a number of reasons (work schedules, family responsibilities, etc.). Several people also decide not to go to the school because they just do not feel confident themselves: the image of the teacher still intimidates many parents (this is also a historical inheritance). Martina's quote is a great example of this:

> "(...) teachers, and people to whom they may have more meaningful connections or may not, but when there is a problem and you perceive that nobody works as much as they can, then you have no idea what to do, because you say: let's see, if I go to make a complaint, maybe [the teacher] will dislike my daughter, or [he/she] will say why she is explaining such things [to us]. I do not want to go to the high school to create problems, but I want that class [mathematics] to work nicely".[16] (Martina)

There are other kinds of situations, which have nothing to do with the center and the structure of the educative system, but they also make it more difficult for parents to be involved in their children's mathematics education. They are what we call *educative barriers*. These type of difficulties appears when there is a gap (most of the times because differences of age) between parents' memories of methods used by their teachers to solve mathematical problems, and the kinds of methods used nowadays by teachers in the schools. This observation is consistent with our literature review. Monica words are a good example:

> "Monica: Last year she didn't fit with her teacher... I do not know, she was not getting it [understanding]... there were... I do not know, there were many ways to explain it and the child was not able to get it, and then it was this mess as he was referring to it, that he [the teacher] was doing... he was doing a kind of mathematics regarding the equations that was not in the book (...), that that we were talking about, right? And I was not able to help him, because, as it was a kind of mathematics specific, by him, where do was I supposed to look for help? I was doing it in my way, and it was the same answer, but as the teacher did not want it in that way, and she was not getting it, I was not either, so poor thing, she was desperate".[17]

However is not always the same situation. Monica explains that the feeling of being lost depends on the level of mathematics already presented in the classroom. When her daughter was in elementary school, mathematics was easy enough so Monica was feeling that she did not need any extra help to support her daughter

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[16] "(…) los profesores, y gente con la que se entenderán más y gente menos, pero yo cuando ha habido un problema en que ves que no se trabaja lo suficiente, entonces no sabes qué hacer, porque dices, a ver, si voy yo a quejarme a lo mejor le coge manía a mi hija o dirán que por qué explica estas cosas. Yo no quiero ir al instituto a crear problemas, pero si que quiero que esa asignatura funcione". (Martina)

[17] "Mónica: Es que el año pasado el profesor que tenía como no compaginaba… no sé, ella no captaba… había… no sé, había muchas maneras de explicar y el niño no captaba, y luego tenía el jaleo este que decía él que había hecho un… hacía un tipo de matemáticas en las ecuaciones que no estaba en el libro (…), aquello que estuvimos comentando, ¿no? y yo como no le podía ayudar, pues claro, como era un tipo de matemáticas específico, de él, a ver dónde lo preguntabas. Yo lo hacía a mi manera y salía lo mismo, pero como el profesor no lo quería así, lo quería a su manera, ella no lo cogía, yo tampoco y claro, la pobre estaba desesperada".

with her homework. But, when her daughter moved to the middle school, mathematics turned more difficult, hence it was at that moment that the necessity of support to understand the new methodologies used by the teacher appeared. This is one of the reasons to explain why the mathematics workshops that we conducted for the last three years have more attendance in the middle and high school rather than in the elementary school.

> "Monica: Well, to me to teach mathematics... is what we were talking about the other day, like, something that you are not doing everyday... we are talking about basic mathematics (...) so, they are more easy, you remember them. But of course, somebody like me that it has been 30 years from the last time I was doing algebra... well, I remember something, but well, I remember them in my way. So then it is what I was explaining to you, when [my child] was doing first grade from the ESO [secondary education] I had no orientations from the teacher. My daughter was passing the tests, but she was not getting good scores because she was saying: "mom, my teacher wants the equations in this way", then I was trying to look at how he was explaining it, but [the equations] were explained in some other way in the textbook... so after that I discovered that when you explain this topic, you take away in one side and put it into the other side to compensate it, but if you are not understanding [this process], I was asking: "How do you get this?" So I went to the library and I did everything to understand... And everybody was saying the same thing... the way... but no... You are able to explain it if you are into the discussion... But for people like me, whom are way from this kind of discussion for a long period of time... we need an extra explanation, right? And then, because I saw this, my daughter has been quite improving her scores, because I'm getting it, she is getting it, and then she and I together..."[18]

Finally, another aspect emerging from the last quote, also into the category of "exclusionary elements" is the difficulty that textbooks have for parents that want to help their children at home. In this quote Monica is also talking about her difficulties understanding the examples from the textbook. In Catalonia, as well as in the rest of Spain, every single student has his own textbook of mathematics. However, sometimes (and this is true in higher grades) the examples used in the book either are not evident, nor a helpful resource to understand the concepts. To the contrast, these examples make it more difficult to understand the concepts. Monica complains about this. To not understand the textbook makes it difficult for her to help her child at home. In fact, many times during the last three years, Monica came to the workshop with her daughter's books to ask questions to be able to understand the books.

[18] "Mónica: Hombre, para mi enseñar matemáticas... es lo que estábamos hablando el otro día, a ver, una cosa que no tocas cada día... estamos hablando de lo que es las matemáticas básicas (…) entonces, como son más facilitas te acuerdas, pero claro, una persona como yo que hace 30 años que no tocaba tema de las ecuaciones mismo, a ver, me acuerdo de algo, pasa que claro, yo me acuerdo del método mío. Entonces es lo que te estaba explicando, yo cuando hizo primero de la ESO yo no tenía ninguna indicación del profesor, la niña iba aprobando, pero tampoco iba tan bien, porque claro, ella decía "mamá, el profesor quiere las ecuaciones de esta manera", entonces yo las miraba cómo las explicaba, pero el libro lo explica de una manera... claro, luego me enteré que cuando explicas esto, que de un lado se quita y hay que ponerlo en el otro para compensar, pero claro, si no lo ves, yo decía "¿pero cómo sale esto?" y me fui la biblioteca y de todo para ver... y todos decían lo mismo ¿no? la manera... pero no... lo explicas si estás muy metido en tema, entonces para personas así como yo, que hace años que no tocamos, como... necesitaríamos una explicación extra ¿no? Y entonces, a raíz de que viese esto, pues la nena ha mejorado bastante la nota, porque lo veo yo, lo ve ella, y entonces entre las dos pues..."

b) Transformative elements

The analysis of the data collected from parents present “participation” as a positive component. Families involved in the fieldwork claim that teaching is a responsibility shared with teachers. Martina has no doubts about it:

> “Martina: For example, regarding responsibility, discipline... if they [teachers] say it just to us, then they [children] are not present, and then [teachers] will say it to them [children], but we need to work the three parts together. It would be great to have a talk with our children and us all together, where either a parent or a child would be able to raise their hand (...)”.

There is a positive request for a space for dialogue (Díez-Palomar & Molina, 2009) to work all together in doing mathematics. When we asked Pilar, another mother involved in the workshops, what she would like to ask to the middle and high school teachers if there were the possibility to do it, she said:

> “Pilar: ¿What I would like to propose? Well, like I was telling to you... More contact between teachers and parents in order to see how the child is doing... or if for instead you see... if I see, look, for instance, my daughter now she is really concerned about the tests... or something else, I do not know. Like the teacher: “I’m feeling like she is distracted”... And maybe he can feel her as distracted, but maybe what I’m seeing is like she is overwhelmed, so here it is not communication, and of course, I would like to tell my daughter “let’s go to talk to the teacher”.[19]

All parents interviewed are aware that it is necessary to establish links between the center (teachers) and the families.[20] Thus there is a motivation to encourage their participation within the center. This evidence is consistent with previous contributions drawing on the study of the school practices as a classroom practices, focused not on the triangle made by teacher-student-content, but on a more complex network of components, such as family, peers’ group, TV, Internet, etc. (Aubert, et al., 2008). That means that we need to analyze the school practices according to this complexity of components, not just regarding the school practices as a object of research framed just within the triangle teacher-student-content.

Another important consideration is teachers’ attitude towards families. As there are teachers that raise barriers because that they may feel their work is being questioned, there are others that encourage the relation with the families and they make it more easy. This positive attitude from a certain number of teachers encourages family involvement. Pilar’s quote is an example:

[19] “Pilar: ¿Qué sugeriría? Pues lo que te digo, más contacto así, dijéramos, de los profesores y los padres, para ver tu hijo como va o que… o si por ejemplo tú ves que… si yo veo, a ver, por ejemplo, mi hija ahora por ejemplo, está más angustiada por los exámenes, o algo, lo que sea, pues yo qué sé, cómo a lo mejor el profesor: “es que la veo despistada”, y a lo mejor él la ve despistada, pero a lo mejor yo lo que veo es que va agobiada, y entonces ahí no hay una comunicación, y claro, yo ponerle a mi hija una nota “vamos a hablar con la profesora”.

[20] También es cierto que no todos los docentes que han participado en ambos estudios tienen una opinión tan positiva de la participación de las familias, e incluso hay quien afirma que la responsabilidad de los familiares no pasa de asegurar que llevan al hijo/a a la escuela o que va al instituto, y que no tienen que involucrarse en los aspectos académicos. Eso es responsabilidad del docente. Esta actitud, en cambio, nunca se produce entre las familias.

"Pilar: I used to go to the advisor, because I guess that he is the one who is in charge, and then, any kind of difficulty that she has had, I let her know, and I say to her: "any problem that you may have, you need to talk with him". As mine [my daugther] is not so complicated, then she has no problems, so it is not necessary that... I have been coming to the meetings, and he always opens the door... this is true, always... either this teacher that has been with her for two years, or the one that she had first. I mean, they invite you to a meeting, and then they left the door open to any question that you may have... They do not say "this meeting and that's it", and maybe they can say to you "if I am not calling you it is because I do not see any problem", but they leave the door open just in case you want to ask them something, you can meet with them. So, I have never had any problem with this".[21]

In addition to these elements (that we call *individual transformative elements* or *agency*) there are also parents' own attitudes. For instance, Monica explains how she has a pro-active attitude to overcome the difficulties and to find the ways to help her daughter with mathematics. The next quote illustrates how Monica appeals to all resources available, to reach her objective:

"Monica: For instance, I, at home, well, when they were... (...) I played games... Now, it is more difficult for me, so I try, if I know, to explain it. If not, sometimes I look for information on the Internet, a good web site, for example, is the one from Descartes[22] that I was looking at the other day that has graphs on it. So then I got it more or less and then I explained it to her, so we worked on it both together and... For example, yesterday we had a question (...) about the equations and the textbook explains it very bad, right? Then, you need to explain "x", then "y", then "zeros", but there is not a practical example. So I was not understanding and she was not understanding either, then we were saying that this was new for us, then she took out the notebook and as I was not understanding, I said: "OK, let's go to Descartes". I was looking and it was OK, there was a graphic example and when you move yourself, the graph also moves and then we got it, and there is an example and we were... and then what I thought that was so good was that there were other web sites to explain it, but they do not show you in a graphical way, and in this other web page I was able to understand it quickly and very well".[23]

[21] "Pilar: Normalmente siempre voy al tutor, porque supongo que es la persona que está de referencia, y entonces, cualquier problema que haya tenido o eso, yo a ella misma ya le digo "tú cualquier problema que tengas o tal, lo comentas con él". En principio, normalmente, es que tampoco la mía es muy complicada, entonces, como en principio no tiene problemas y esto pues no ha hecho tampoco falta que… he venido a las reuniones que me ha indicado, y él siempre me deja la puerta… eso es verdad, siempre… tanto este profe que ya lleva dos años con ella, como el primero que tuvo, o sea, te hacen esta reunión y te dejan la puerta abierta a cualquier cosa que tú… no te dicen "esta reunión y ya está" y a lo mejor te dicen "si yo no te llamo es porque yo en principio no veo ningún problema", pero te dejan la puerta abierta a que si tú quieres comentarles algo o lo que sea puedes quedar con ellos, o sea, nunca he tenido ninguna pega de nada".

[22] Descartes is a web page from the Spanish Educational Board with all the national curriculum on-line. See: http://descartes.cnice.mec.es/

[23] "Mónica: Yo, por ejemplo, en casa, bueno cuando eran… (…) hacía juegos o… ahora que es más complicado hasta para mí, procuro, si sé, le explico, si no, a veces busco información en Internet, una página por ejemplo que está muy bien de Descartes, que estuve mirando el otro día, que explica con gráficos, y entonces yo más o menos veo que sí que lo capto y se lo explico a ella o lo vemos entre las dos

The context for this quote is the workshop of mathematics that has been working during the last three years. One of the key elements of this workshop is to distribute resources to families so, afterwords they could be able to help their children and to search for answers when needed. In Spain the Educational Board has a web site where you can find available on-line a plethora of resources to support your work at home. This web site (called *Proyecto Descartes*) was provided during the workshop. When Monica talks about Descartes, she is referring to this resource, and she explains how having it gave her the possibility to help her child. This is what we call here *educative elements of transformation*.

Lastly, another transformative element that encourages family involvement is to give parents the opportunity to participate in the process of designing the content and the learning planning (in our case, the workshops of mathematics). All topics, resources, educative strategies, etc. already used in the workshop have been agreed to by both families and teachers. This is a key element, because people participanting in the workshop attend that workshop because they have concrete requests and they are searching for a space to ask them. For example, after a few sessions Monica (during the first year of the workshop), was starting to come with her daughter's homework, to ask for questions and look for answers during the workshop. Doing this she was feeling that she was learning exactly what her daughter need, so she was able to help her at home to learn mathematics. Her daughter, Carolina, was a girl starting middle school. She was having some troubles with understanding mathematics. After Monica's participation in the workshop, Carolina' scores improve dramatically. Her mother, Monica, explains that because of their involvement in the workshop, she was able to change her habits with her daughter: from not being able to help her with her mathematics, to working with her collaboratively. After several months Carolina started to join her mother in the workshop. Two years later she (Carolina) was one of the speakers in a panel during the conference celebrated in a Catalan university, to present the main results of the study. The transformation involved in this process contributed to help Carolina improve her grades in mathematics.

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y… por ejemplo, ayer mismo teníamos la duda de (…) las ecuaciones afines y el libro lo explica muy mal, ¿no? entonces, explica las x, las y, son ceros, pero no te pone ningún ejemplo práctico, entonces claro, yo no lo entendía y ella no lo entendía, entonces estuvimos hablando que nosotros no lo habíamos dado, entonces sacó los apuntes y como que no me aclaraba, digo "pues nada, nos vamos al Descartes", estuve allí mirando y muy bien, había un ejemplo gráfico y entonces justo cuando mueves también se mueve la gráfica y lo vimos, y pone un ejemplo, y lo fuimos… y entonces lo que vi muy bueno es que había otros sitios, páginas web que explican pero no lo ves gráficamente y ahí lo captó en seguida y muy bien".

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Situating Mexican Mothers' Dialogue in the Proximities of Contexts of Mathematical Practice

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An exploration of seven Mexican mothers' dialogue with themselves, with students, and with a teacher, around problem solving, revealed that dialogic interspaces can create proximities of mathematical practices where participants can transform and appropriate each others' dialogue.

This paper explores Mexican mothers' dialogue with themselves, with students, and with a teacher, around problem solving. The problem motivated this report—student low performance in classroom-based problem solving that contrasts with successful performance in an after-school program—resonates with the different problem-solving performance in different contexts reported by situated learning theorists (Lave & Wegner, 1991; Brown, Collins, & Duguid, 1989; Young, 1993). I argue that at the core of this differential performance is the disconnect that inspired this paper: *la aproximación de diálogos*, or the bringing together of different dialogues around problem solving.

I followed Bakhtin's (1981) notion of dialogue as more revealing and intense not when it is located within well-defined contexts—where it become unchallenged and authoritative—but rather when it is situated in what I call the *proximities* of contexts that include mathematical practices (school-based mathematics and everyday mathematics; children's mathematics and mothers' mathematics; teachers' mathematics and mothers' mathematics). These proximities are interspaces that create and are created by dialogue. I also used Belenky et al.'s (1986) metaphor of voice to address the question: When mothers' dialogue about problem solving is positioned in proximity with (a) themselves, (b) students, and (c) teachers, to what extent is their dialogue appropriated by other mothers, by students, and by teachers? The question is relevant because mothers' dialogue is hardly heard in well-defined contexts such as schools; mothers have inhabited these contexts (home and school); and they have an interest in children's mathematics education that is proximal to that of teachers.

Results

Mothers' dialogue with themselves created multiple opportunities to transform this dialogue. Far from automatic, this transformation occurred as participants took common ownership of problems. Examples include: not knowing the standard division algorithm or relearning the concept of average; fearing that their children may develop voicelessness in mathematics; and recontextualizing problems for each other. In these examples, mothers reframed dialogue by restructuring the dialogic interspace to be inclusive of everyone's dialogue. In contrast, mothers' dialogue remained unchanged when they were told by others what to do, as when some mothers told others to replace lengthy additions or multiplications with the unfamiliar division. This indicates that transformative dialogue requires proximity; that is, dialogue must be structured in such a way as to allow participants to see something in the others' dialogue that resonates with what they are doing and how they are doing it.

In the mothers-students interspace, mothers used dialogue in contrasting ways. By encouraging male students to develop their own solution methods and supporting their reasoning with multiple recontextualizations, mothers' dialogue withheld any expectations regarding how students should solve problems. In contrast, by directing the female student to perform operations beyond the student's understanding, mothers communicated expectations that positioned the student as a silent participant. Interestingly, silence is the prevailing voice when mothers described interactions with husbands (e.g., "pero pues yo nomás miro" [*but, oh well, I just look*]) or when recalling past experiences with mathematics (e.g., "yo nunca fui buena para eso" [*I was never good at that*]). More mother-student interactions would be required to qualify these patterns as more than tentative. As tentative, though, it seems that mothers' may be communicating different expectations to male and female students. These expectations, as enacted in mother-student interspaces, may be mediating the process of dialogic appropriation.

Finally, teacher-mothers interspace showed the greatest resistance for appropriating or transforming dialogue. This resistance was achieved by a separation of dialogues that the teacher managed to established by picking up divergent lines in the conversation; by listening and respecting the mothers' ideas while maintaining his ideas separated; and by reframing dialogue in a manner that reifies the methods, practices, and ways of talking that prevail in schools. The teacher's separation of dialogues was not contested by the mother's dialogue. This is not surprising, given that this interspace is the least explored of the three, with almost no history for either mothers or teachers.

These results suggest that transformative dialogue cannot occur by imposition but by the act of *aproximación*—between how *we* problem solve and how *they* problem solve. The main argument I raise is that people's ethnomethodologies—their *this is how we do things here*—(Garfinkel, 2006) can undergo significant transformations when the dialogue of others is appropriated in these proximities of practice.

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K-12 Family Engagement-Reactor: Enrique Murillo

WHAT: CEMELA-CPTM-TODOS Conference
WHO: Enrique Murillo (California State University-San Bernardino)
PLACE: Tucson, AZ
DATE: March 5, 2010

TRANSCRIPTION

Enrique

What I saw from the papers and really hear from many of the parents... Lo que escuché fue una descripción de los espacios [*What I heard was a description of the spaces*]; the spaces. These in-between spaces, interspaces... I don't know what you want to call it, but what I see is that everyone said the same thing: that these spaces hold the most potential for transformation. Right, if I'm correct? You guys let me know if I'm hearing the same thing. Lo que estoy escuchando es que esos espacios tienen la potencia para transformar las relaciones [*What I'm hearing is that these spaces have the potential to transform the relationships*]. What I like is that it gives us a model on what to do with parents. We often, in schools, from a school perspective, we really don't know what to do with parents. We give a lot of lip service... "Oh, yes! Parental participation, but when it comes down to it, fundamentally, there's no space for parents in schools. El problema que tenemos es que decimos una cosa "¡Oh, sí! Hay que invitar a los padres y crear enlaces", pero la verdad es que por el punto de vista de la escuela, no sabemos qué, cómo relacionarnos con los padres [*The problem we have is that we say one thing "Oh, yes! Let's invite parents and create links," but the truth is that from the point of view of the school we don't know how to make relationships with parents*]. So I see and hear models. That's the exciting part; how to deal with parents. Quite typically schools just give like these middle class tips to parents: "Have magazines on your coffee table" and "Help your kid with homework." That's the extent of it. What I see here is something that can really bring about a shared responsibility or governance. Muchas veces nomás damos tareas a los padres como voluntarios. [*A lot of times we just give parents volunteer type of tasks.*] We just give like... and it's often menial tasks, volunteer tasks. It's not a meaningful relationship and that's bothersome that we don't have horizontal and reciprocal relations with parents. We have one that's very vertical, very top-down. There's a lot of research over the last hundred years that shows that schools don't know what to do with parents, but they actually go out of their way not to have a relationship with parents. Hay muchas investigaciones que dicen más o menos, una cosa es que las escuelas no saben cómo relacionarse con los padres, pero ellos con intenciones tratan de evitar de no tener relaciones con los padres [*There is a lot of research that say more or less, one thing is that schools don't know how to relate to parents, but they intentionally try to avoid having relationships with parents*], and that's very troublesome. There's really good documented work there. Again, other than just help volunteering. Think of the stereotypical PTA, PTO parent, at least in the United States, being a good parent means to acquiesce to the school administration, right? Ser a "yesero." Como decimos "yesero"

[*Being a "yesero". Like we say "yesero"*], 'cause you say, "Yes," right? Or "yesera." "Yes, yes..." and you just go along; you acquiesce. Here, the parental involvement is mediated by the institution of schooling. Schools oftentimes undermine the relationship with parents. It's unfortunate, and I'm just repeating what the research says. I'm not making it up. Although I was a schoolteacher, so I know about the awkward relationship we have with parents. I remember not knowing what to do with parents when they came into my classroom because here you are trying to orchestrate and trying to do stuff and then... because there's no shared governance, you just "OK, yeah. Cut these out and do this and help me do that," so there's no relationship there. That is really the hard part.

Another issue is that parents, in particular Latino parents, we do participate in our child's education, but it doesn't show up on the school's radar. Sí apoyamos la educación de nuestros hijos, participamos, pero las escuelas no conocen las maneras en que realmente ayudamos a nuestros hijos. No estamos en su radar. [*We do support our children's education, we participate, but the schools don't know the way in which we really help our children. We are not on their radar.*] We don't want to show up on the radar. What I see is these spaces are really important for the potential for transformation at the highest in these spaces, almost like a liminal pace to transform the asymmetrical power relations. I see the potential to shift some power back to families, back to parents. If you think about it, we kidnap kids. That's what we do. Basically we kidnap kids, and nowhere else but in schools are the largest group of non-criminals forced to spend so many hours in an institution. `(Laughter)` I'll try to translate it into Spanish... Pero secuestramos a los niños. Es lo que es la escuela de una manera. Jugando, pero es la realidad. Sacamos a los niños y decimos "Ya, ustedes ya..." [*But we kidnap kids. In a way that's what school is. Joking, but that's the reality. We take out the children and say "That's all with you..."*] `(Makes a gesture as if saying "Stay away")` "Leave it up to us. We'll take care of it." Y ese es parte del problema. Entonces veo que hay mucha potencia para poder canalizar, cambiar las dinámicas de poder. [*And that's part of the problem. So I see that there is a lot of potential to channel, to change the power relations.*]

The dynamics here can change where teaching-learning is really a shared responsibility, and this came up a few times; a lot of different people kind of said it. It reminds me of the kind of the Freirian perspective. Oftentimes we don't bring up Freire explicitly anymore, but really kind of shift from the passive to the active, from the object to the subject, right? Creating ways in which parents can participate and shape curricular decisions. I think that's exciting. It gives way to global concepts but with a localized curriculum. One thing I think is very exciting... Una cosa que me gusta mucho es cómo vamos a mejor explicar la relación entre el lenguaje y matemáticas. [*One thing that I really like is how we are going to better explain the relationship between language and mathematics.*] I think that's an under-theorized concept. What's the relationship between language and mathematics? We have ideas. There's all the good research from all the folks here in the room, but it's still very under-theorized. So I think I would encourage anybody who is working on those topics, I think that's the way to go. Be more explicit in those linkages.

Let's look at mathematics as a cultural system. Let's be very explicit that there's no such thing as kind of the true objectivity, Truth with a capital "T", but really it's a cultural system. It's a way of being in the world. One thing that I saw that was really key is the multiple roles adults can take on as learners. Cómo es que los padres, los adultos, ustedes, vuelven a ser alumnos, a aprender. Eso me dio mucho gusto... [*How it is that parents, adults, you all, become students again, to learn. I was glad to see that…*] because we never stop learning. We think of education in the broad cultural sense. School is what we do to kids, but education is what we do over our lifetime. Education is the stories women tell when there are no men in the room, right? Or the stories our grandparents passed down to us. That's education, and schooling, they kind of make that difference. So everyone 's in agreement that parents are a pedagogical resource, not a problem; that we're not deficit. Los padres no somos problemas. No somos los obstáculos, si no un recurso. [*We as parents are not a problem. We are not obstacles, but a resource.*] Couple of things here: Higinio's clips were really powerful. I want to see a full fledge documentary. That's what I want to see. (Higinio: "It's in the making") That's the first thing I said: "This is powerful stuff here," because one thing is to just give quotes. Think of like the typical ethnography. We can pull out quotes from an emic perspective, la, la, la; but it's not the same as seeing it. That's kind of what I did. I've worked on three documentaries and one thing is to just OK, we talk about it and we talk to ourselves, but you make a documentary and all of a sudden, you're reaching out to thousands and maybe millions but these thousands of people; the potential. So that was real good.

You know, what was missing, although it was touched upon… I want to see more from the teachers. How are the teachers transformed? And I'll build on Luis Moll's work, teachers are strangers to the very community in which they're teaching, and so like how are they transformed by these experiences? That was touched upon a little bit, but I want to see a little bit more of that. Lo que estoy preguntando es los maestros, dónde entran ellos en la fórmula, porque me imagino que ellos también deben de ser transformados, deben de cambiar de una manera u otra con la relación que tengan con los padres. [*I'm wondering about the teachers, where do they fit into the formula, because I would imagine that they should also be transformed, they should change in a way or another on the relationship they have with parents.*] So I would like to see a little bit more. I'm sure it's there, people have it, but I want to see more of that. I like the way this model is set up because there's practitioners and researchers so in a way, kind of, "How do we help our teachers help our kids?" I see that. Para las madres de aquí de Tucson y de Chicago: ¿Dónde están los hombres? ¿Dónde están los papás? [*For the mothers here in Tucson and in Chicago: Where are the men? Where are the fathers?*] Where are the men? It's my question. Where are the men? Hey, I'm a father and I see myself missing in the equation, and this is part of kind of the research that I've done in diasporic communities and the kind of small communities, and oftentimes there is what we call the "badger society", just a bunch of young men, but then when the women show up are the ones who create, facilitate community... and we start thinking about imagined spaces. OK. What does it mean for there to be a community now? So the women are the mediators. The women are the ones, and not really the men. Every so often men show up in the research, but it's the women that go and negotiate with the social workers. They are the ones who talk to the teachers. They are the ones who have to defend; in a sense be advocates, and oftentimes the only advocates that their children have.

Entonces, ¿dónde están los hombres, los papás? Porque no es justo que solamente las mujeres son las que están ahí batallando, peleando por los derechos de sus hijos. [*So, where are the men, the fathers? Because it is not fair that the women are the only ones struggling, fighting for the children's rights.*] The good lesson is that we have to go beyond "¿Cómo se porta mi hijo?" [*"How does my child behave?"*] We're stuck at that. "¿Cómo se porta mi hijo?" [*"How does my child behave?"*] And we never move beyond that, so I like this model here. One last question: and this is a little larger, kind of what the macro-question is. Right now, with all the problems with the economy. How does this fit in the economy? 'Cause we know that our economy, and maybe it has to do with the larger STEM, the mathematics, but our economy is based on human capital and technical skills. So how does this translate to help all of us in the larger macro-sense? It's great what's going on in these small spaces, but how do we create these small spaces and make it into a formal dimension of school culture and how does it contribute to high school and college completion rates? Because after all the future of our economy depends heavily on the educational outcomes of Latino students.

Family Engagement Discussion

The following section presents comments from the discussion that took place after the Family Engagement research presentations, the parent panel and the reactor. The participants met in small groups that included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. This strand also included some of the mothers who had been part of the parent panel. We have captured multiple simultaneous discussions and have attempted to be as faithful as possible to the participants' comments.

The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

Whereas other working groups reflected on the connections that exist between this strand and the other strands as a separate reflection question, this group chose to respond to the connections question within the discussions of the other questions. The following summary represents common themes identified within and across the working groups.

What do we know and what are the implications for practice and research?

Based on the research studies presented as part of the Family Engagement Strand, along with the poster sessions, and our own professional experiences we are able to state what we believe to be true about the relationship of family engagement and the learning of mathematics for English Language Learners (ELLs) and particularly with Latinos/as. There is abundant research that documents that when families are involved with their children's education the children have greater achievement (Aspiazu, Bauer, and Spillett, 1998; Epstein, 2001; Henderson et al., 2002). However, current interactions between Latino families and school reveal there is much work to be done to have meaningful involvement of Latino families.

To change the present relationships between Latino families and schools we will need to examine and be aware of commonly held myths about Latino families. The belief that Latino families do not care about or want to be involved in their children's education has been well documented in research as untrue (Rodriguez-Brown, 2010; Valencia and Black, 2002; Zarate, 2007). However, Latino families and schools often hold different definitions of what involvement in education is (Civil and Menéndez, 2010; Zarate, 2007). Latino families are frequently viewed from a deficit perspective and the resources they have to offer schools go unrecognized or are not valued.

Schools generally lack organizational goals or objectives for how to involve families in the educational process (Zarate, 2007) and there is not any institutionalized form of parent participation. Often schools have defined parent involvement to be the performing of menial tasks or participating in parent organizations to support the requested administrative agenda. They do not have horizontal and reciprocal relations with parents but instead, they have vertical, top-down relationships. Schools not recognizing the resources that Latino families have to offer have also stratified parent involvement. Some parents are assigned to bring food while others are asked to speak to the class about their work. Latino families become aware of this deficit perspective held by the school.

It is important for teachers to learn about their Latino families and culture, and not over generalize. Teachers must also broaden their definition of resources that can support education, and may need guidance from experts including community members to recognize these resources. The work that Latinos do, may or may not reflect the amount of education they have or their capabilities. In order to really know about the resources families have to offer, teachers need to know their community. By doing home visits and taking on the role of an ethnographer, the teacher can discover the assets or Funds of Knowledge present in the home and utilize them in the classroom (González, Moll, and Amanti, 2005). It should be noted that home visits might be unfamiliar and uncomfortable for both teachers and families. Any fieldwork must be respectful and must be carefully orchestrated to relieve any tentativeness families may have. Visitors need to thoroughly understand the purpose so as not to reinforce dominant deficit ideologies and practices.

Other fieldwork might include shadowing a parent for a day, having apprenticeships for teachers in the community in unfamiliar contexts (not tutoring, etc.) or positioning parents as the teachers and the teachers as the learners. Teachers could also learn about their students' families through interviews, or by opening their classrooms for parents to share experiences, including how they learned mathematics and how they use mathematics in their everyday life. Teachers will, of course, need to be compensated or supported for required time to be involved in such activities. The work of teachers needs to be redefined to include extended work with families. Structures must be in place that allow for it to be feasible for teachers—not as an add-on without support.

Fieldwork also offers valuable experiences for white middle-class preservice teachers to give them an opportunity to learn about communities and cultures that are different from their own. Preservice teachers might also benefit from experiences that place them in the position of being the "other" in a culture or society. Exchange programs or living with a family could offer them this perspective, along with self-ethnographies reflecting on when race, language or culture became an issue for them. It's important to disrupt the dominant narratives about Latino families. Some critical readings for teachers and preservice teachers should include *Beyond Heroes and Holidays: A Practical Guide to K-12 Anti-Racist, Multicultural Education and Staff Development* (edited by E. Lee, D. Menkart, and M. Okazawa-Rey), and *Subtractive Schooling: U.S. Mexican Youth and the Politics of Caring* (by A. Valenzuela).

Latino families have often had negative personal experiences in school and feel unwelcome in the schools. The added security measures enforced in schools have factored accentuated these feelings. Families feel unable to visit the school without an appointment. Schools with parent liaisons have been more successful in linking families and schools together and creating spaces where parents can feel welcome and comfortable in the school. Liaisons are helpful in developing home/school relationships that support parents across grade levels and also across school levels. Every school needs to have a designated space where families are able to come without appointments, etc. and have access to some school staff.

Superficial and formal communications have also contributed to the problem. Personal communication with parents usually occurs only when there is a problem at school that needs to be addressed. Otherwise, communications are generally impersonal, only of an informative nature and do not solicit input from the families. Accessibility to teachers is often difficult. Immigrant parents from Mexico have experienced a different culture in schools in Mexico, where teachers were available to speak with parents at lunch and after parents' work hours.

Parents often understand their child's learning style better than the teacher does, and it is important that they be able to work together with the teacher to meet the student's needs. If a student is having difficulty understanding one model or representation, other approaches need to be used. Parents are more receptive when teachers focus on a student's potential and share relevant and meaningful information when addressing concerns.

Language barriers factor into parents feeling displaced in their children's educational process. When students are placed in English only classrooms, Spanish-dominant parents can no longer participate in the classroom community or support the mathematical learning. When teachers only speak English they cannot effectively communicate with parents. English has been given an elevated status being the language of instruction and teachers often view parents who do not speak English from a deficit perspective. Changing this perception is imperative. Teachers need to be aware of the resources and support families can provide. Parents must understand how their children are placed in classes, sometimes for the sole purpose of learning English. They must demand that their children not be hindered academically by these placements. Parents must have all available information about bilingual education. Then, if they so choose, they can become advocates for their children to have access to bilingual programs.

Other home mathematical resources often go unrecognized or create tension between families and school (Díez-Palomar, 2010). Parents may have learned mathematics differently or use algorithms not taught in schools. When working with their children, parents are often told "that it is not way they are supposed to do it." Teachers also show resistance to this knowledge and may distance school knowledge from daily life knowledge (Dominguez, 2010). Parents and teachers need to work together to explore the mathematics being taught and recognize and validate the different knowledge that exists. It is important that parents and teachers participate together and learn from each other. How this is facilitated can determine its success. Teachers cannot be "instructing" the parents but must be learners as well. Spaces must be created for families and teachers to come together, and we must not rely on the school institution to do it. Mathematics and Parents Partnerships in the Southwest (MAPPS) and CEMELA's Math for Parents and Tertulias provide examples of how this can be successfully implemented (Allexsaht-Snider, 2010; Civil & Menéndez, 2010).

Schools need to be sensitive to other barriers that keep parents from participating. Working parents need flexible schedule options; childcare and transportation can be issues. Specific to supporting their students' mathematical learning, schools must also recognize that some parents may have "math anxiety" and will be tentative about participating. Safe, non-threatening environments where parents can learn together and feel comfortable are necessary. Parents can encourage other parents to participate by sharing their experiences. Surveys designed to understand the needs of the families and discover what programs or content workshops they would like would be helpful.

Immigrant parents often comment on the difference in instruction between Mexico and the US. They perceive the curriculum in Mexico to be more accelerated, but also comment that there was more memorization and less focus on reasoning. Attention needs to be given to binational initiatives between the US and Mexico to examine and support the education for transnational migrants. International teacher collaborations focused on teaching strategies, technology, and culture need to be supported.

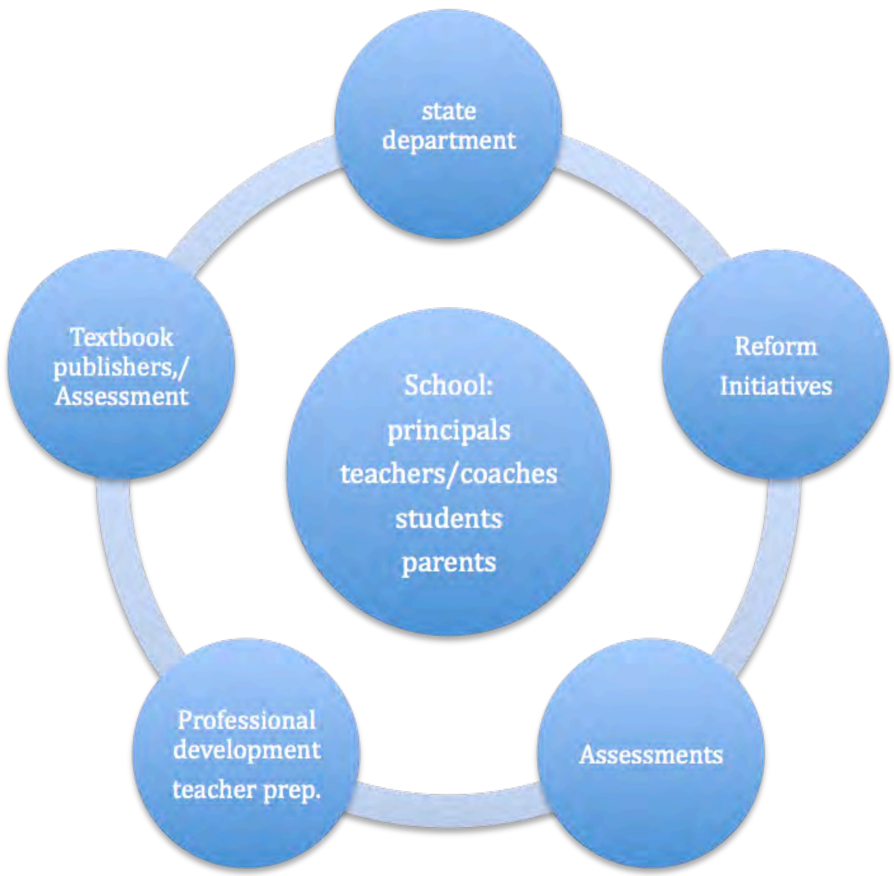
Because the US school culture may be unfamiliar to many immigrant Latino parents, help negotiating the system and developing awareness of available resources and how to advocate for their children should be supported. Parents may feel intimidated by the school system and personnel, and are unsure how to advocate for their children. More information needs to be shared about the process for evaluation and placement of students, and the resources to which children are entitled.

Considering that the Latino population is the fastest growing population in the U.S., and that "Latinos are the least educated of all major ethnic groups" Gándara, 2010, p. 24), it is critical that Latino parental involvement in mathematics is addressed for the good of our country's economic growth. We need national policies relating to the education of Latinos. Systemic initiatives to change the culture of our schools to better accommodate Latino parents need to be implemented. Parent organizations outside of school need to be developed to support Latino empowerment due to the power relationship between schools and parents. Legislative educational policies that undermine the success and support of our Latino students, especially English-only requirements, and college enrollment and tuition policies need to be addressed. When immigrant students do not have access to funding for higher education, or must pay non-resident tuitions their future educational goals are compromised. There needs to be S.T.E.M. initiatives that specifically focus on supporting our Latino students. The satellite model represents the idea that the school context must always consider the influence and interaction of the satellite entities that influence parental engagement and how students learn (see next page).

National Policy/Accountability: NCLB

National Policy: NCLB

Research



What questions do we need to research further?

Participants identified the following questions as needing further research and investigation:

- How does the mathematics curriculum in Mexico compare to that of the U.S.? Do students in Mexico perform better than those in the U.S., and is the mathematics a higher level? (We need more parents involved as research partners.)

- How does the role of Latino models affect students? (Mexican students in Mexico see Mexican professionals, while Mexican-American students in the U.S. often only see Mexicans in a negative light.)
- What cultural knowledge is necessary to know how to effectively negotiate the issues of involvement and change in schools?
- How do we institutionalize systems to create teacher/school/community relationships that support Latino families?
- How do we support and empower families to become advocates for their children?
- How do we move communities (administrators/teachers/families) with little engagement to full engagement?
- How do we encourage teachers' change and development? What creative ideas do we have to address issues of teachers' time?
- How do we get *all* teachers committed to working with parents and working with parents as partners? How do we help teachers understand how to build reciprocal relationships with parents and create informal spaces for dialogue?
- How can researchers connect with parents to advocate for best practices in instruction that use home language as a resource during the creation of school improvement plans?
- How can policy support parents in supporting their children?
- How can we motivate/support parents to take part in the day-to-day spaces already available in many schools? Who are the other community stakeholders who can support parents in advocating for their children's learning?
- What principles or ideas from CEMELA need to be shared with state departments of education to support systemic practices?
- How do we connect parent/teacher/student research to the larger school context and the policy context?
- How do we redefine parent involvement and document other forms of involvement? How do we change how it is measured?

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CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Teacher Education and Professional Development

Section 6 of 9

Chairs: Virginia Horak, University of Arizona
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Tucson, Arizona March 4 -6, 2010

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Embracing the Inherent Tensions in Teaching Mathematics from an Equity Stance

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The paper presented by Rochelle is published in Democracy and Education

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TEACHER EDUCATION AND PROFESSIONAL DEVELOPMENT WITHIN CEMELA: EXPLORING THE REFLECTIONS OF TEACHERS OF LATINO/A STUDENTS

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Willamette University

Barbara Trujillo
The University of New Mexico

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A jointly-designed research study was undertaken with teachers from two multi-grade study groups in elementary schools with high percentage populations of Latino/a students. Teachers reflected on the implementation of a rich mathematical task, as well as their practice in general, through video interviews and study group sessions. Findings show that teachers reflected on multiple areas of their instructional practice, including those practices that teachers consider effective and specific factors that impact practice. These findings suggest that the teachers had high expectations of their Latino/a students as well as the importance of engaging in professional development as a means to reflect and examine student work.

INTRODUCTION

One of the missions of the Center for the Mathematics Education of Latino/as (CEMELA[1]) is to investigate the interplay among mathematics, language, and culture. In regard to teacher education the Center aims to "strengthen teachers' ability to promote Latinos' achievement in mathematics in K-8 classrooms through their expanded knowledge of mathematics and the use of linguistically and culturally responsive learning environments" (CEMELA website). Under the auspices of this mission, researchers involved with two teacher study groups from two CEMELA sites undertook a study to better understand how elementary teachers of Latino/a students reflected upon their own mathematics instructional practice. Of interest was to note what these teachers attended to in their reflections and how the findings compared across locations.

The primary goal of this work was to contribute to the emerging literature on mathematical professional development for teachers of Latino/a students. This work is of particular importance. Although improving mathematics teaching and learning for all students receives particular emphasis in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, NCTM, 2000), the document is not explicit in regard to the cultural and linguistic needs of students. On the other hand the literature addressing multicultural education and equity pedagogy (e.g. Banks & Banks, 1995) does not take into account specific academic content domains.

THEORETICAL FRAMEWORK AND RELATED LITERATURE

Research has shown that professional development experiences should help teachers develop deep understanding of content knowledge and identify students' conceptions and misconceptions (Franke & Kazemi, 2001; Garet, Porter, Desimone, Birman, & Yoon, 2001). However, it is also important for professional development experiences to help teachers enhance their knowledge of student diversity and of how culture and language interact with their instructional practices and student learning. One way to promote this professional growth is through reflective practice in which teachers are encouraged to intentionally critique their work through a lens of improvement (Hatten & Smith, 1995; Schön, 1983). Schön (1983) and Manouchehri (2002) argued that this type of reflection should be a central component of teacher professional development as it opens up possibilities for teachers to revisit the different dimensions of their practice. It is this framing of reflection as central to teachers' learning process that is the focus of our work.

We drew upon research that highlighted the potential for professional development experiences such as teacher study groups to form learning communities (Ball & Cohen, 1999; Franke & Kazemi, 2001; Kazemi & Franke, 2004). These types of learning communities, argued Crespo (2006), can create a context for teachers to

[1] CEMELA is a Center for Learning and Teaching supported by the National Science Foundation, grant number ESI-0424983. Any opinions, findings, and conclusions or recommendations expressed in this document are those of the authors and do not necessarily reflect the views of the National Science Foundation.

collaborate on constructing mathematical understanding as well as reflect on their teaching practices and students' mathematical learning.

Our study was also informed by research that proposes teaching practices that support Latino/a students and second language learners in the mathematics classroom. Participation in mathematics reform classrooms involves students making conjectures, evaluating the mathematical strategies presented by others, and using mathematical language to express their ideas (NCTM, 2000). Communication plays an important role in these classrooms as students must organize and consolidate their understandings of mathematics through discussions of their findings with peers and others. In this context, all students, especially English Language Learners (ELLs) and bilingual children, must be presented opportunities to participate in mathematical discourse practices that draw upon the multiple resources these students use to communicate mathematically (Moschkovich, 2002). Khisty's (1997) research furthered our understanding of what teaching practices would support success in this type of environment by suggesting that teachers of students from diverse linguistic backgrounds may need to understand multiple factors that influence group interactions and thus mathematical understandings.

METHODOLOGY

Researchers from two different CEMELA sites jointly designed a professional development experience for teachers from two different study groups. Teacher study group 1 (TSG1) had the participation of eight teachers from grades K-5 and three different schools. The classroom experience of the two male and six female teachers, five of whom were Latino/a and three of whom were Caucasian, ranged from 5 to 20 years. Teacher study group 2 (TSG2) had 10 participating teachers; nine from grades 3-6 and one from the Gifted and Talented Education program. These teachers, seven Latina and three Caucasian, were all female, from three different schools, and ranged from 3 to 27 years of classroom experience. The schools had on a collective average a 90% Latino population and a 31% classification of English Language Learners (ELLs).

Both study groups had as a central focus the analysis of student work. TSG1 framed their professional development using the Cognitively Guided Instruction approach (CGI; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) as a way to help teachers rethink how to integrate problem solving into their mathematics instruction. Under this framing, study group sessions investigated mathematical problems and analyzed student work produced as a result of particular CGI-type problem-solving activities. The development of TSG2 was guided by previous research with teacher study groups (Arbaugh, 2003; Kazemi & Franke, 2004). This research showed that analyzing student work has the potential to influence teachers to not only investigate mathematical content on a deeper level but also to reflect on their classroom practice.

In order to develop a cross-site project, the researchers employed a strategy similar to the SATTR model (Crespo & Featherstone, 2006). Teachers within both study groups collectively engaged with and reflected on the implementation of a "rich mathematical task" (Crespo & Featherstone, 2006, p. 99). The task selected was a fourth grade National Assessment of Educational Progress (NAEP, 1996) geometry measurement problem specifically asking students to compare the areas of two figures. The particular task selection was intentional for several reasons. First, while research has shown that Latino students are out-performed by white students on the measurement strand of the NAEP (Lubienski, 2003), others studies have shown that Latino students can participate meaningfully through their mathematical discourse (Anhalt, Fernandes, & Civil, 2007). Second, the "comparison of areas task" served to benefit both students and teachers. In the latter case, teachers would be provided multiple opportunities to reflect about the mathematical concepts students need to understand and solve the task as well as what issues of language are involved in comprehending the task. Moreover, as the nature of the task allowed for students to use a variety of strategies, teachers could reflect on such pedagogical moves as what materials to make available for students and how they discovered student misconceptions.

Two back to back sessions of the study groups were used to explore the mathematical task and reflect on its implementation in the classroom. Specifically, session one focused on having the teachers engage with the task as mathematical learners (i.e. they solved the task themselves and discussed their different solution processes) and reflect on what specific adaptations they would make for each of their own classrooms. After each teacher implemented the task, teachers met for the second study group session to discuss the specific modifications they made for their class, including how they introduced the task and what materials they made available, as well as to analyze their classes' work on the task.

Multiple data were collected for this study. At each site, both study group sessions were videotaped as well as selected classroom sessions (three at each site). Follow-up interviews with the teachers whose classroom sessions were videotaped were also conducted. These videotaped interviews were semi-structured, followed similar protocols across all participants, and used selected video-clips of the teacher's classroom implementation. These selected clips were essential to the interview as they provided the teachers the opportunity to reflect on different aspects of the task implementation, which in some cases took place months earlier. Relevant scenes were chosen to highlight (a) the task introduction, (b) the materials provided to students to work on the task, and (c) the different strategies students used to work on the task and some of the challenges they confronted.

Both sites analyzed the data using elements of Grounded Theory (Strauss & Corbin, 1998). First, with the help of TAMS Analyzer, each individual site team openly coded the transcripts of the follow-up interviews and the second study group session and looked for themes in the teachers' reflections. Next, research teams refined their emerging codes, both individually and collectively, with the research teams coding one transcript from the other site to ensure data reliability. Finally, the two research teams met in person and virtually to look for cross-cutting themes.

The following seven categories were developed after the data analysis was complete: (a) Teacher Expectations, (b) Factors that impact practice, (c) Practices considered effective, (d) Issues, (e) Knowledge of students, (f) Recognition teachers have about their practice, and (g) Teacher notions about a variety of topics.

FINDINGS

In analyzing teachers' reflections and subsequently organizing them into categories, data reveals that teachers reflected mainly on the multiple practices they consider effective in teaching and supporting their students to achieve mathematical understanding (category c), as well as the factors impacting their work in the classroom (category b).

Reflections on Practices that Foster Mathematical Understanding

Teachers from both TSG1 and TSG2 reflected on what practices were more typical and important in their instruction and that seemed to be more effective to promote Latino/a students' mathematical understanding. Among these practices, teachers identified: using appropriate mathematical vocabulary, creating learning situations that foster peer interaction, supporting students to become active thinkers and independent decision makers, providing varied materials and resources to solve problems, and supporting student learning through the review of concepts, the validation of their responses and strategies, and when needed, using students' native language. It is important to consider that language policies are different at each site. Whereas the state of TSG1 promotes bilingual education, the state of TSG2 does not. Teachers' words clearly illustrate the significance of the most relevant practices identified at both TSGs:

1. About the importance of developing students' academic language and using appropriate mathematical vocabulary, Ms. Alvarez from TSG2 explained in the interview:

> I think about the language. As I'm saying things I'm trying to think of the correct language. And to make sure that I'm using the mathematical terms […] so I go slow because I'm always trying to think of how to say it correctly and using the vocabulary, so that they in turn will use it also. (Interview, May 2007*)*

2. In relation to developing students' mathematical vocabulary and the need of using their native language as a way to support students' learning, Ms. Salas from TSG1 commented:

> A lot of the kids in here would need [Spanish] and it would help them. And because you just can't give it to them all in English because then it would hinder their learning, this way you explain both English and Spanish and "I can do that, I understand what she is talking about now…" (Interview, May 2007)

3. Teachers underscored the importance of creating learning situations that would encourage students to interact and learn from each other. They drew on the belief that students benefit from collaborative work within problem solving contexts. Mr. Sloan, from TSG1 explained:

> If the environment is set up correctly, the children will learn just as much or more from each other than they will from your instruction. […] In something like this, I remember there were a couple of places where kids were doing as well or better a job than I would be doing explaining to another child how they got the answer. (Interview, May 2007)

4. In connection to teachers valuing students to learn from each other is their emphasis in promoting learning situations that would support students to be active thinkers, to

develop and apply higher thinking skills that they could transfer to other situations in which they are required to problem solve. Ms. Alvarez from TSG2 explains her expectations:

> I want them to be able to think on their own. …I've always told them, when you are solving a problem try and, I try and teach them different ways, different strategies, to solve something. And I want them to be able to reason and think, okay here I have a problem, what can I do to figure this out? (Interview, May 2007)

Reflections on Factors that Influence Practice

Data indicated that among the most influential factors on what teachers teach, what they do in the classroom, and what they consider relevant for students learning are (a) the adopted reform curriculum, (b) their knowledge of students in terms of dispositions for learning, language, prior knowledge, and mathematical understanding, (c) their previous personal and professional experiences as learners and teachers of mathematics, (d) their expectations or what they value for students to learn and understand, and (e) the TSG professional development experience.

It is important to notice that both groups of teachers identified the adopted reform curriculum as their referent to decide what to teach. In addition, these teachers reflected on the importance to have high expectations and to build knowledge about their students, especially to be able to understand what students are able to accomplish, what their struggles are, and what they know that might help them to be successful with problem-solving. Mr. Sloan (TSG1) thinks that teachers

> should never go into something with a… predetermined notion of whether or not the kids will be able to solve the problem. My attitude is… anything I give them they should be able to do it, and if they can't do it immediately then maybe you just need to be give them a few more tools, a little more background knowledge. But I think way too often we kind of have preconceived notions of what they're capable of and so they'll live up to those expectations. So if they are low expectations, they live up to them. (Interview, May 2007)

Teachers' personal, educational, cultural and linguistic experiences, as well as their experiences in the classroom, seemed to affect the way they perceived effective teaching and the conditions that foster students' learning. Ms. Alvarez's (TSG2) memories illustrate this point:

> You know we didn't grow up that way. Where we had . . . to memorize everything, and some things I'm learning along with them. I told them, you know I memorized, like when we were working with fractions, especially with division I told them, you know all I memorized, was you turn them and you multiply. And I go "I didn't understand why. And now you guys are lucky because you are understanding 'why,'" and I learned along with them. (Interview, May 2007)

Professional development was a relevant factor affecting the way teachers interpreted students' work and the instructional decisions they made to provide for students' learning needs. Teachers' raised questions, expressed doubts, and understandings they have gained through the TSG process. For instance, Ms. Segovia (TSG1) revisits her practice based on her insights of students' understandings:

> You see what they did and what they didn't get and it shows you how you taught it or what you didn't teach, … for some reason they just didn't understand it the way you taught it. Or maybe something in your thinking, [you need to ask yourself], what am I not doing? Do I need to go back and review myself to see what's missing? (Interview, May 2007)

DISCUSSION AND IMPLICATIONS

The findings from our study highlight several important findings and implications for future research. First, as the results from the study show, teachers reflected on various components of their instructional practices with Latino/a students. Of particular relevance is that teachers at both sites reflected on their deliberate focus to model academic language in their instruction, specifically referring to their use of and their students' development of appropriate mathematical vocabulary. Additionally, teachers reflected on their practice of using Spanish to support students' mathematical understanding. Through their explanations we can see that the teachers use and incorporate Spanish into their mathematics teaching as a means to support students' comprehension of mathematical concepts, a method congruent with suggestions from leading researchers in the field (e.g. Khisty, 1997). These explicit reflections showcase that teachers of Latino/a students can and do have high expectations of their students and support them in the manner set forth by the NCTM (2000).

One underlying purpose of the task implementation was to understand how teachers discuss issues of language and culture. Given this focus, it was interesting to note that the teachers rarely discussed these elements in regard to students' mathematical understanding. We hypothesize that perhaps these teachers internalized the identity of their students as Latino/a students, thus making it difficult for them to explicitly reflect on certain components of this identity in relation to the teaching and learning process. We suggest that further research be done in this area.

Although the purpose of this study was not to explore the impact of doing cross-site research, the opportunity to reflect about the limitations, strengths, and challenges of doing this type of research served as an unexpected outcome that benefited all researchers involved. Through this research, we were able to explore the methodological implications of designing and developing research in two ongoing teacher study groups that were pedagogically and contextually different (meaning they had slightly different purposes, content and were situated in different social and educational contexts). In relation to this, the work done throughout this study informs us as researchers and collaborators about:

- how cross-site, cross-specialty collaboration can occur, including the time required to construct a common project, the negotiation of theoretical perspective, and the methodological approach taken;

- the importance of constructing a common approach to data collection such as the interview protocol;
- the benefits of using technology (such as TAMS Analyzer, Skype, and Google Documents) as a means to virtually discuss, negotiate, and document the different conceptual and methodological aspects of the study;
- the challenges of writing and authorship; and
- the strength of collecting data from different sites and the potential to generalize some of the conclusions while taking into consideration the limitations embedded in comparing and contrasting data collected in different contexts.

Ultimately our study, highlighted by the teachers' reflections, explicates the value teachers of Latino/a students placed on participating in a professional development community. The teachers in this study were able to gain both an individual and a collective understanding of how their students made sense of the concept of area and what might be ways to promote student understanding. Furthermore, through the context of a professional learning community, the teachers were able to draw upon each others' insights regarding both their successes and challenges during the task implementation which was evident when teachers commented on specific strategies they wished they had incorporated. Thus, the findings of this study underscore the importance of engaging in professional development as a means to reflect and examine student work in an intellectually engaging manner.

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USING CULTURE AS A RESOURCE IN MATHEMATICS: THE CASE OF MEXICAN AMERICAN PRE-SERVICE TEACHERS IN A BILINGUAL AFTER-SCHOOL PROGRAM[1]

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This paper explores Mexican American pre-service teachers' use of culture – defined as social practices and shared experiences - as an instructional resource in mathematics. The setting is an after-school mathematics program for children of Mexican heritage. Qualitative analysis of the pre-service teachers and children's interactions reveals that the nature of the mathematical activities affected how culture was used. When working on the "binder activities," pre-service teachers used culture only in non-mathematical context and mathematical discussions were English dominant. When working on the "recipes project", however, culture was used as a resource in mathematical context and the amount of Spanish used in mathematical discussions increased significantly. Implications for mathematics teacher preparation of Latinas/os are discussed.

It is widely accepted that in order to improve the mathematics education of culturally diverse students, teachers must value and draw on their students' interests and every day practices with their families and in their communities (e.g. Averill, Anderson, Easton, Te Maro, Smith, & Hynds, 2009; Gonzalez, Moll, & Amanti, 2005; Lipka, Webster, & Yanez, 2005). However, there still remains a question of how to prepare teachers to use students' cultural experiences as instructional resources in mathematics (Gay, 2009). The work that has been done in this area has primarily focused on helping White (usually English monolingual) pre-service teachers to teach students who are racially and ethnically different from themselves (e.g. Ensign, 2005). As a result, minority pre-service teachers are usually left to figure out on their own how to best use their cultural knowledge as an instructional resource, as if it is assumed that simply by virtue of their backgrounds they will have the skills and knowledge to do so (Villegas & Davis, 2008). Pre-service teachers who share similar backgrounds with language minority students (LMS), such as Latinas/os, however, may need different kinds of support than their White English monolingual counterparts, to incorporate their home language(s) and cultural knowledge into pedagogical practices in mathematics. This is particularly relevant to Latinas/os since they are the fastest growing minority teacher group (Strizok et al., 2006), they tend to teach Latina/o students (Villegas & Davis, 2008), and most of them speak Spanish. Nevertheless, the emphasis on White and English monolingual teachers in the literature on teacher preparation for culturally diverse students pays scant attention to the preparation, support, and empowerment of Latina/o teachers. The issue, therefore, becomes how to help Latina/o pre-service teachers build on the unique strengths they bring into teaching, particularly their knowledge of Spanish and possible familiarity of their students' living experiences. Moreover, what kinds of experiences do Latina/o pre-service teachers need in order to develop and incorporate their knowledge of their students' home language and lived experiences into pedagogical practices in mathematics?

This paper seeks to address these issues. Specifically, this paper provides some insights that were gained on how four Mexican American pre-service teachers used their cultural knowledge as a resource while facilitating mathematical activities with Latina/o bilingual children in an after-school mathematics program.

As noted previously, the emphasis on White and English monolingual teachers in the literature on teacher preparation for culturally diverse students pays scant attention to the preparation, support, and empowerment of Latina/o teachers. However, studies on Latina/o teachers point out that it should not be assumed that simply because students and teachers share home language and ethnicity that the teachers will know how to connect student life to school curricula (e.g. Téllez, 1999, 2005). Nor should it be assumed that Latina/o teachers, merely by virtue of their ethnicity possess a natural aptitude for teaching Latina/o students. Although it may be beneficial if the cultural and linguistic background of the teacher is similar to that of the students' (Quiocho & Rios, 2000, Valencia, 2004) such a similarity is not sufficient to ensure that the teacher is able to connect to his or her students (Cahmann & Remillard, 2002) and use students' home language and culture as instructional resources in mathematics. For example, Gordon (2000) found that cultural connections were sometimes difficult because of class differences between middle class Latino/a teachers and their working class/poor Latino students. In fact, many second generation Latina/o teachers may have little in common with life in Mexico and as a result, little understanding of recent immigrant children and their families (Riegelhaupt & Carrasco, 2000; Walker de Felix & PeÒa, 1992). Third generation Latina/o teachers whose family traces its heritage to Mexico City may have a difficult time understanding, much less legitimizing, the culture of a family that most recently emigrated from rural Mexico (Téllez, 2005).

Challenges in using culture as an instructional resource in mathematics have also been reported even when teachers and students are able to connect at a personal level. Aguirre (2007), for example, found that first year Latina/o mathematics teachers who felt strong connections with their students, faced multiple challenges when attempting to incorporate student's cultural experiences in their lessons. In another study, Téllez (1999) found that Mexican American student teachers who had similar backgrounds with their students used very little ethnic expression during instruction and used their cultural knowledge only in non-academic aspects of the lessons they taught.

Thus far I have highlighted the complexity of using culture as an instructional resource even when teachers and students share -or are perceived to share - similar cultural backgrounds. If we want to prepare bilingual Latina/o teachers to use culturally responsive practices in the teaching of mathematics, there is much we need to understand further. The purpose of this paper is to describe how a group of Mexican American pre-service teachers used culture - defined as shared experiences and social practices - as an instructional resource in mathematics through their participation in a non-traditional field experience. The setting of the study was an after-school mathematics program, namely *Los Rayos de CEMELA,* housed in an elementary school that serves working class students of Mexican descent. First I describe the study, its setting, participants, and the methods employed. Next I discuss how the nature of the mathematical activities affected the pre-service teacher's use of culture. I close my discussion with some concluding thoughts related to the implications for the mathematics teacher preparation of Latinas/os.

LOS RAYOS DE CEMELA

The work presented here draws on a wider dissertation study that explored how Mexican American pre-service teachers use language and culture as instructional resources in mathematics. It reflects current work carried out by the Center for the Mathematics Education of Latinas/os (CEMELA)[2], which focuses on the research and practice of the teaching and learning of mathematics for Latino/as in the United States through the integration of socio-cultural theory, language, and culture. CEMELA has created after school projects at two of its sites, one of which, *Los Rayos de CEMELA*, is the source of the present study. *Los Rayos* is a general adaptation of the work of the *Fifth Dimension* (e.g., Cole and the Distributed Literacy Consortium, 2006) and is guided by other similar projects (e.g., Gutierrez, Baquedano-Lopez, & Alvarez, 2001; Vasquez, 2003). These works have utilized the after-school as a way of understanding literacy; CEMELA has extended the work to consider mathematics. As in the work of the *Fifth Dimension* and related projects, *Los Rayos* involves pre-service teachers who participate as facilitators.

While *Los Rayos* was designed to investigate the linguistic and cultural resources bilingual Latina/o students use in mathematics and that support their mathematics learning, it simultaneously serves as a non-traditional field experience for pre-service teachers as they participate in a unique context where they must form interpersonal relationships with students, negotiate mathematical ideas, and engage in dialogue across two cultural languages (Spanish and English). *Los Rayos* is a setting where pre-service teachers and students naturally talk about their experiences and their interests. A group of Spanish speaking mothers often join *Los Rayos* and participate along with the children and the pre-service teachers in many different ways. This gives the pre-service teachers the opportunity to interact with children's family members, thus giving them multiple opportunities to recognize and draw on the children' interests and every day practices with their families and in their communities. In addition, the mathematical activities are intended to be open-ended and to require students to experiment, develop multiple strategies, and communicate their reasoning. Children can choose what activities to work on, with whom, and could change or redirect activities as they wished but within the limits of being part of a group. Furthermore, they are encouraged to work collaboratively and pre-service teachers are encouraged to capitalize on childrens' ideas, comments, and any other resources they presented. This is relevant to this study because, again, the environment offered the pre-service teachers many opportunities to choose to use childrens' experiences and/or any shared experiences they might have since they came from the same cultural background as the students. This design feature was intended to open the way for facilitators to, in turn, draw on these experiences to support conceptual development (Khisty & Morales, 2007). This work assumes that learning at any age occurs in a social context (Vygotsky, 1978) that emphasizes active dialogue among participants (Wells, 2001). Furthermore, it assumes that what is known by an individual is the outcome of continuing co-construction processes that depend on multiple opportunities to encounter and make sense of challenging new experiences (Wells, 2001).

In *Los Rayos* the pre-service teachers are not expected to teach or tutor in the traditional sense. They are afforded the opportunity to experiment and to use a multiplicity of resources with small groups of students while doing mathematical activities, unlike in a student teaching experience where there are constraints due to the mentor teacher, the curriculum, or the size of the class. Furthermore, because of the nature of the after-school and because it was not a typical field experience context, the pre-service teachers had a good deal of freedom to engage with children as they wished or to make decisions about how to use culture as a resource for doing mathematics. This "open-ended" environment provided me with opportunities to observe, in essence, the decisions they made that relate to the purpose of this study.

Thus far, I have provided a general overview of *Los Rayos*. However, every semester was unique in the sense that, the mathematical activities were different, the group of facilitators was different, the level of the mothers' participation was different, etc. Thus, I will now turn to describe the kinds of activities that took place in *Los Rayos* during the time of the study, since they are most relevant to the findings presented in this paper.

The mathematical tasks in *Los Rayos* revolve around several dimensions of play, they are open-ended, and require students to experiment, find patterns, develop multiple strategies, and to communicate their solutions. During the time of the study presented here, the mathematical activities in *Los Rayos* consisted of a collection of mathematical tasks adapted from existing curricula that emphasize problem solving, and included non-routine challenging problems that were non-remedial and focused on various topics such as fractions, logic, geometry, patterns, etc. These tasks were provided in both Spanish and English, were grouped according to the topic they addressed, they were placed in binders, and everyone referred to them as "the binder activities" and this is how I will refer to them as well. Each group had the freedom to choose which task to work on in any given session.

On the second half of the duration of the study, however, the context of the mathematical activities changed. During this time, all students had to work on the same project whose mathematical goal was to support children's proportional reasoning. This project, which everyone came to refer to as "the recipes project," and this is how I will refer to from now on, consisted of a sequence of activities designed to ultimately lead each group to create a recipe and prepare a dish for the end of year party in *Los Rayos*. Each group was responsible for creating a unique dish, which was chosen at random from a list of five dishes given to the students: guacamole, salsa, cupcakes, jello, and lemonade. Project activities involved finding out which orange juice recipe from a list of different orange juice recipes is more orangey, creating a perfect recipe for the groups' dish, going to the local grocery store to buy ingredients for that recipe after deciding which items were the best bargain, making mole – a special and favorite dish among Mexican families - with a group of mothers who posed mathematical problems in the process, figuring out how to magnify the recipe to serve all participants in the party, and finally making the dish for the party.

I now turn to describe the pre-service teachers who were observed at the time of the study.

THE PRE-SERVICE TEACHERS

Jose, Juanita, Maria, and Lupe[3], the four pre-service teachers were undergraduate students at a large University in the Midwest. At the time of the study, Jose was a junior in the elementary education program, Maria was a sophomore in the secondary mathematics education program, and Lupe was a freshman in the elementary education program. Juanita was an undeclared major in her sophomore year who, at the time of the study, was strongly considering entering the elementary education program. For the purposes of this study I refer to her as a pre-service teacher even though she was not officially an education major during the period she was observed.

Jose and Juanita self identified as Mexican American, while Maria and Lupe self identified as Mexican. All four participants' home language is Spanish and their parents are immigrants from Mexico. Moreover, all four participants were brought up in predominantly Latino communities. In fact, Maria grew up in the same community with the children in *Los Rayos*.

METHODS

The four pre-service teachers were observed as they worked with small groups of fourth and fifth-grade Mexican American students in *Los Rayos* twice a week for one and a half hours each time for approximately nine weeks. All sessions were videotaped and transcribed. Additionally all pre-service teachers were required to take detailed field-notes of their interactions with the students. Finally, during this nine-week period, all participants met in weekly a two-hour debriefing seminar where they discussed the happenings in *Los Rayos* and worked on the mathematical tasks that were part of the *Los Rayos* curriculum. All seminars were also video-taped and transcribed.

Data were analyzed using a constant comparative method (Glasier & Straus, 1976) in which patterns of how participants used culture as they facilitated mathematical tasks became apparent as the data were continuously examined. In conceptualizing culture, I adopted a "process approach" (Moore, 1987,p. 729) by which culture is understood and examined as lived experience. Thus, the emphasis is on social practices. In this view, the processes of everyday life, in the form of daily activities, emerge as important. These daily activities are "a manifestation of particular historically accumulated funds of knowledge that households and communities possess and actually transform through their daily activity" (González, 2008, p. 96). In other words, culture is not understood as being static or of being composed by a collection of traits that are used to characterize and categorize groups of people. Instead, it is viewed as being dynamic, multidimensional, and as constantly changing as people constantly draw on multiple cultural systems in their daily activities.

When examining pre-service teachers' use of culture in their interactions with the children, I focused on pre-service teachers' and students' shared lived experiences. These shared experiences were revealed through discussions that participants - pre-service teachers and children - had about their daily practices and interests. For example, new shared experiences also emerged as participants worked together in *Los Rayos*. Thus, my unit of analysis was how pre-service teachers used these shared experiences as tools in their mathematical interactions with the students.

My data analysis followed the four cycles outlined in Creswell's (1998) *data analysis spiral*. During the first stage, I went through the video summaries and transcriptions of the after-school sessions and identified the sections in which students talked about their experiences and interests. Here I did not isolate sections that took place when the groups were working on the mathematical activities. Instead, I looked at the entire sessions since children discussed their experiences and interests with the pre-service teachers before, during, and after the mathematical activities. Next I identified sections in the pre-service teachers' field-notes and the transcriptions from the debriefing meetings where pre- service teachers noted students' interests and experiences. I also identified sections where pre-service teachers described their own experiences as relating to the childrens' experiences.

During the second stage I read through the selected pieces of data as described above and made a list of the experiences and interests that each child and pre-service teacher had mentioned. I used this list for my codes in the next stage. During the third stage, I coded the transcripts from the after-school sessions that revolved around the mathematical activities using the list generated in the previous stage. Here I focused on coding for the apparent purpose each experience or interest was used. For example, I looked at whether it was used to motivate the students, or to relate the activity to students' lives, or to better explain a concept, etc. During the second round of coding I divided the codes into two categories: those which reflected childrens' experiences and interests, and those which reflected shared experiences with the pre- service teachers.

Finally, during the fourth stage, I selected the relevant data along with my notes to represent the themes that emerged and began drafting my interpretations of how pre- service teachers used culture as a resource in their mathematical interactions with the children. I now turn to describe the findings.

PRE-SERVICE TEACHRS' USE OF CULTURE

As noted earlier, the after-school is a setting where pre-service teachers and students form interpersonal relationships and naturally talk about their experiences and their interests. This design feature was intended to open the way for facilitators to naturally, in turn, draw on these experiences to support conceptual development (Khisty & Morales, 2007). The data suggests that the nature of the mathematical activities affected how pre-service teachers used culture as a resource. Specifically, when doing activities from the binders, pre-service teachers used students' interests only in non-mathematical contexts as a way to connect with the students and they used games that involved mathematics as culturally alternatives to the activities from the binder. During the recipes project, however, students' experiences were naturally used in mathematical contexts and used their shared experiences with the students' as a resource in mathematical context.

Their Use of Culture While Working on "the Binder Activities"

During the after-school sessions, students often talked about their interests and experiences. This occurred naturally as part of their interactions with the children in their group and was something the pre-service teachers encouraged. Whether it was before, during, or after the groups worked on a mathematical task, students talked about several things that were part of their regular social practices such as playing video games, soccer, basketball, music; watching cartoons such as *Pokemosn, the Simpsons, Sponge Bob Square Pants, Family guy;* and using the internet to connect with other students through *My Space*. They would also talk about other experiences such as getting their nails or hair done, and other experiences that involved their friends and family members. Usually, these discussions around students' interests and experiences were demarked from the tasks, that is, they were not related in any way to the mathematics involved. Even when these discussions took place as the groups worked on a task, they were irrelevant to the specific mathematics and were simply side conversations. When this happened, pre-service teachers would try to interrupt these side conversations, as they were seen as irrelevant to the task, and try to get the students back to focusing on the mathematical task they were working on.

There were very few instances where pre-service teachers attempted to relate the mathematical tasks with the students' interests. When they did, the relationship with the task was superficial in the sense that it did not directly relate to the mathematics involved in the task. For example, there was one instance where Jose, a pre-service teacher, brought up an episode from *the Simpsons*. Andre, one of the students, was working on a task which involved creating a circle through drawing a series of lines on a grid. Andre thought that the design looked three-dimensional so Jose brought up an episode where Homer (from *the Simpsons*) appears to be three-dimensional.

> Andre: It's all like 3D and everything!
>
> José: Have you ever seen that Simpson episode cuando Homer se hace 3D que [*where Homer turns into 3D that*] he throws (inaudible) and falls into the media hole?
>
> Andre: Oh! That's the Halloween episode when he went through the wall!
>
> José: Yeah! Exactly that one, that one, that's the one I'm talking about. Que se tira Bart [*where Bart throws himself*] and (inaudible) but he has a rope around him. Remember?
>
> Andre: Yeah, they all look 3D and when Homer goes to Earth!
>
> José: Yeah and they found him in the garbage can.
>
> Andre: (laughs) Yeah!
>
> José: Yeah, that's the one. All right.

In the above excerpt, Jose, related a shared experience he had with Andre, namely an episode from *the Simpsons,* to the aesthetic aspect of the task that Andre pointed to, that is the appearance of the three dimensional design. However, none of the discussion related to the mathematical content involved in the task. In other words, Jose used this shared experience to connect to Andre and both of them connected with the task in a personal level; but this shared experience did not enhance the mathematics involved. Similarly in another session, where Juanita, another pre-service teacher, and her group were assembling a collection of shapes to make a perfect five point star she commented that the star that one of the students, Katia, had made looked like *Sponge Bob's* friend.

Juanita: Aww that's cute. It reminds me of the star from Sponge Bob
Katia: Patricio!
Juanita: I think that's the starfish, right?
Miriam: Patrick!

In the above excerpt, Juanita also connected a shared experience, that is a character from *Sponge Bob Square Pants,* to an aesthetic aspect of the task rather than to the mathematics involved. Again, this shared experience was not used as a resource in mathematics but rather to connect the task with the students at a personal level. In both of the examples mentioned above none of these cultural connections related or connected to the mathematics involved in the tasks.

Thus far, I have described two patterns regarding the pre-service teachers' use of culture during the first few weeks of the after-school sessions when the primary curriculum was the activities from the binders. During this time, pre-service teachers would rarely make connections between the activities and the students' experiences and interests and when they tried to make connections, these had little relevance to the mathematics involved in the given activity. Now I turn to two more patterns of pre-service teachers' use of culture that emerged during the recipes project.

Their Use of Culture During "the Recipes Project"

As described in a previous chapter, the recipes project was intended to have students deal with proportional reasoning. During the recipes project, all the activities were concerned with some aspect of the students' experiences and were initiated with these experiences. Students started with a common activity in which they experimented with mixing juice concentrate with water to create their own orange juice. This exploratory work served to raise awareness about important mathematical concepts related to the preparation of recipes, such as measurement, estimation, and proportions. As they started working on their selected recipe, students were challenged with fractions and proportions problems embedded in realistic situations, for example preparing a list of ingredients with a limited budget and then enlarging the recipe for an end-of-year party. During the recipes project students would regularly bring their experiences from grocery shopping, or cooking to do the recipes and would naturally draw from them to work on the mathematics in the activities. In essence, pre-service teachers did not have to look for additional ways to connect the activities with students' experiences because that was already part of the activities.

For example, during the recipes project, students were to ask their mothers how much certain ingredients would cost. When Juanita and Maria's group was estimating the cost of milk prior to buying it from the grocery store, both Griselda and Lisbeth (students) used their prior knowledge to estimate how much it will cost and this led to a mathematical discussion.

Lisbeth: Oh mi mamá hoy en la mañana compró leche y costó uno noventa y nueve. [*Oh my mom bought milk in the morning and it was only one ninety nine.*]
Griselda: Mine three twenty nine. How expensive is that!?
Lisbeth: No por que son dos galones por cinco dolares. [*No because*

two gallons are five dollars]
Griselda: Oh, dos galones!
Lisbeth: Pero nomás compro uno…dos dólares.. [*but she only bought one for two dollars.*]
Griselda: I don't get it.
Juanita: Dos dólares por un galón? [*Two dollars for one gallon?*]
Griselda: I think it was half a gallon.
Lisbeth: Dos por cinco dólares y uno por uno noventa y nueve. *Two for five dollars and one for one ninety nine?*
Juanita: Pero es un mejor acuerdo para comprar– pero no sería mejor– [*But, it is a better deal to buy- wouldn't it be better*] are you sure? Porque no saldría… Tiene que ser más de uno noventa y nueve porque si es el especial…[*Wouldn't it be… It has to be more than one ninety nine because if it was the special…*]
María: Yeah, porque si los compras separados sale cuatro dólares [*Because if you buy them separately, it's four dollars*] so what's the point of buying two for five when you can get two separated for two dollars each.

In the above excerpt, Lisbeth and Griselda shared their experiences of how much milk costs. Lisbeth's remark, however, that one gallon costs $1.99 while two gallons cost $5 led both Juanita and Maria to counter that remark by explaining that if that was the case then buying two gallon would not be a good deal. From Juanita's and Maria's experience, any time there is a special offer at a store, buying more than one item at once would reduce the per item cost. It appears that Griselda was also puzzled by Lisbeth's remark and suggested that maybe the milk that cost $1.99 was half a gallon instead of one which would then make two gallons for $5 a deal.

The above example is typical of the many instances during the recipes project where both pre-service teachers and students naturally drew from their experiences in a mathematical context and used them to argue their point. I should note here that pedagogically speaking this not the best example because both Maria and Juanita jump in to correct Lisbeth. One might see this as a missed opportunity to have a rich mathematical discussion. However, the pre-service teachers' approach to pedagogy is not the unit of analysis here. Instead, the point is that mathematical discussions that were based on students' and pre-service teachers' daily experiences were the norm during the recipes project. Unlike the activities from the binders, which did not connect to the students' experiences, the recipes project was built around the students' experiences and this led to many instances where pre-service teachers and students used these experiences in mathematical contexts. In essence, pre-service teachers did not have to search for additional ways to connect students' experiences to the activities or the mathematics involved in the activities because this was already part of the project.

Another dimension of using shared experiences to facilitate the activities during the recipes project was recalling a shared experience from a previous session. This is something that did not occur when the groups worked on the activities from the binders as those activities did not build on each other. Due to the fact that the activities during the recipes project were designed in a way that built on each other, they naturally led pre- service teachers to have students recall on their experiences in previous sessions to make sense of and solve the activities.

In the example below, Juanita has Griselda recall the measurements they used during the previous session to make el Maga's orange juice. During that session, as part of an activity that called to compare the strength of different recipes, amongst other things they had mixed certain quantities of orange concentrate and water to make "el Maga's" (the magician's) orange juice. The recipe made orange juice for two people. During the following session, the group had to decide which recipe out of a list would taste the same as el Maga's. In order to do so the group decided to re-create el Maga's orange juice but reduce the recipe to one person. In order to assist the students in figuring out which quantities to use Juanita kept reminding them what they did during the previous session.

> Juanita: La otra vez que hicimos el jugo, ¿cuántas medidas le pusimos? [*The other time we did juice, how many teaspoons did we put?*]
>
> Griselda: Five
>
> Juanita: Five, ¿verdad? Five eran para dos…la cantidad que hicimos era para dos personas. [*Five, right? Five were for two…the amount we did serves two people.*]
>
> Griselda: No…oh yeah!
>
> Juanita: Vamos a suponer. Pero él está diciendo que él quiere para una. Si vas hacer nada más para una, que vas a hacer con las cantidades que usaste? [*Let's suppose. But he's telling you that he wants it for one person. If you are going to make only for one, what are you going to do to the quantities you used?*]
>
> Griselda: I'll put six teaspoons of orange juice and six teaspoons of water.
>
> Juanita: Ok. (she lifts a measuring cup) Pero vamos a decir que vamos a agarrar un vaso y le vamos a poner agua, y es para…para dos personas. Pero si nomás la queremos para una persona: ¿Qué vamos hacer con las demás medidas? [*But let's say that we are going to get one cup and we pour water in, and it is for two people. But if we want it only for one person, what are we going to do with the rest of measurements?*]
>
> Griselda: Medio…medio vaso nomás…medio [*half, half a cup only, half*]
>
> Juanita: Yeah. Umm, what was it? One thing we used – we used this measuring cup and we did…and we used teaspoons, right? We used five teaspoons of concentrate and five teaspoons of water, pero cada cup tenía five teaspoons en cada uno. [*But each cup had five teaspoons in each one.*] So, let's say that that was enough for two people, pero*, but* we want to cut it down to one person. What would we do to our measurements?
>
> Griselda: Two and a half

In the excerpt above, Juanita not only keeps referring to what the group did in the previous session, she also physically points to the measuring devices they had used. She first reminds Griselda that during the previous session they had used five teaspoons of orange concentrate for two people and now they need to make the same orange juice recipe for one person. Griselda's response that they need to use six teaspoons of orange concentrate and six teaspoons of water suggests that she is focusing on the fact that the recipe has to stay the same and therefore use equal measurements of each ingredient. Juanita however is trying to get Griselda to say that they will need half of what they used last time so she uses the measuring cup they used last time and reformulates her original question. Eventually another student, Monica figures out that they will need half of five, which is two and a half teaspoons of each ingredient and the rest of the students agree with her. In other words Juanita re-creates the group's shared experience of the previous session by using both verbal expressions to recall this experience and the measuring devices previously used as tools to assist the students in solving the task.

Just like Juanita, the other pre-service teachers also used this strategy of recalling shared experiences from previous activities regularly during the recipes project. In essence, the way each activity in the recipes project built on the previous ones naturally led the pre-service teachers to keep referring to the previous sessions as a tool to help the students make sense of and complete each new activity.

SUMMARY

In this paper I described a study of four Mexican American pre-service teacher's use of culture as a resource in an after-school mathematics program, *Los Rayos de CEMELA*. Data analysis revealed that the nature of the mathematical activities influenced the pre-service teachers' use of culture as an instructional resource in mathematics. During the first half of the study, the curriculum in *Los Rayos* included the activities from the binder. During the second half of the study the curriculum included a series of activities that together made up the recipes project. When facilitating the activities from the binder, the pre-service teachers did not connect these activities to students' interests and experiences. The few times such connections between students' interests and the activities, the connections were focusing on the aesthetic aspect of the activities rather than the mathematical content. During the recipes project, however, the activities were built around students' interests and experiences and as a result pre-service teachers did not have to make an additional effort to make cultural connections with the activities and the students because these connections already existed. Also, because of the fact that these activities build on one another and functioned as a sequence, each session served as a shared experience between the pre-service teachers and the students and pre-service teachers consistently drew on these shared experiences to help the students make sense of the following activities.

So what does this mean for mathematics teacher preparation? I now turn to draw implications for the mathematics teacher preparation of Latinas/os.

CONCLUSION AND IMPLICATIONS FOR MATHEMATICS TEACHER PREPARATION

If we want to prepare Latina/o teachers to use culturally responsive practices in the teaching of mathematics, there is much we need to understand further. An insight that was gained from this study is that if the curriculum does not naturally draw on students' experiences with their families and in their communities, and does not lend itself to creating shared experiences that teachers can use as an instructional resource, we cannot assume that teachers can use their cultural knowledge for pedagogical purposes in mathematic. In other words, Latina/o pre-service teachers need experiences working with Latina/o children on mathematical activities that are conducive to using both theirs and student's cultural knowledge.

Teacher preparation programs should provide opportunities to work on open- ended community based mathematical projects with Latina/o children. Even though there has been a great amount of work on project-based learning (e.g. Thomas, 2000), teacher preparation programs do not typically provide opportunities for pre-service teachers with project-based mathematical activities. Furthermore, Téllez (1999; 2005) has found that Latina/o pre-service teachers did not use their cultural knowledge in the formal curriculum and has argued that they need more opportunities to explore ways of using their cultural knowledge instruction. The findings of this study suggest that working on a community-based mathematical project with the children in *Los Rayos*, such as the recipes project, enabled the pre-service teachers to draw on their own and the students' experiences and therefore use their cultural knowledge while doing mathematics. In addition to affording opportunities to use their cultural knowledge, a community-based project, such as the recipes project, that involves meaningful parental involvement and out-of-school activities that require going in the childrens' community, naturally lends itself to privileging home culture.

In other words, providing Latina/o pre-service teachers with opportunities to engage with Latina/o children in community based mathematical projects, that involve meaningful parental involvement and activities inside their community, afford great possibilities for them to use their cultural knowledge as a resource in mathematics content.

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Teacher Education and Professional Development-Reactor: Dorothy White

WHAT: CEMELA-CPTM-TODOS Conference
WHO: Dorothy White (University of Georgia)
PLACE: Tucson, AZ
DATE: March 6, 2010

TRANSCRIPTION

Dorothy White

I'm going to organize my reaction in a way that may be a little different. What I'm going to do is just highlight what I found were some of the key things that informed my thinking and pose three questions for each presenter and then somehow talk about next directions that I'm thinking we should go into... So let me tell you a little bit about where I am in this place Math Education Research and Professional Development. I wear several hats. One hat is a teacher educator. I work with pre-service teachers preparing them to teach students of diverse backgrounds, both culturally, linguistically, and socioeconomically... and place; I think place is important. I'm also a professional developer. I work with teachers in schools, in districts across the country, so I get the opportunity to learn first hand what it is that we're facing in our schools. I also research how do we support teachers at every level to infuse issues of equity and diversity and culture into the classrooms and teachers. I'm, last but not least, a parent of African-American children in public schools, because I really believe in public education. So I have a host of things that are working either together, or in opposition as I go about my work everyday, so that's the lens by which I am phrasing my comments and suggestions.

I want to say that all of the papers and presentations have challenged me to think more critically about what it means to educate Latino students in particular and all students in general. They've challenged my assumptions and the way we go about this thing called "mathematics education." So I'd like to hopefully share that. Embracing the inherent tensions help us think critically about the achievement gap, and Rochelle (Gutiérrez)and Laura have talked about this and Jeremy (Kilpatrick), and so when Rochelle tells us that the achievement gap is not the be-all, end-all, that it is a slice that we critically need to analyze, I listen. I listen because too often we use the gap as an excuse, and opposed to as a solution or an assessment. It is an excuse to keep certain students marginalized, while uplifting other students. So her work on embracing the tensions talks about this dimensions of equity in playing and changing the game of school mathematics. Playing in the game is making sure that children are passing standardized tests, having high expectations, keeping the mathematics and integrity of tasks and opportunities there; at the same time changing, changing it for the next generation, and for all students, so that the game does not continue. So these tensions for me have helped me think about how I can realistically help pre-service teachers see the reality in schools. Too often, at the university we tell them these things and they go out to schools and they hear very different things and we don't take the time to set up the tension that exists between knowing the students and not knowing them, or teaching mathematics, not teaching mathematics, and also this notion of control. For me, in classrooms, control is a big issue, but as (Name - incomprehensible) told us back in '95, that it's a false sense of control, because kids have control; they can make a break that day just like that. (Snaps her fingers - laughter) So these tensions help us think about what it is to do and learn about teaching mathematics.

So here are some questions that I'm wondering about. How do we help other teacher educators embrace these ideas and incorporate them into teacher education classes and programs? How do I get my colleagues at the University of Georgia to understand that this is important (incomprehensible) that we need to have both in faculty meetings and in our classrooms? How do I help others do that? How do we create a shared language around district personnel and teacher educators/professional developers to move our agenda forward? And I bring this up because I meet mostly with the current district that I'm working with, and we are speaking very different languages. They are speaking the language of student achievement and I'm talking student learning. We are not talking... We don't share the same thing. The achievement gap is the way that they speak and I don't think they're going to change their speak, so how can we change our speak so that they can hear us, because once they are listening, then we can change... and how do we identify and support practitioners to do this work? The nice thing about conferences and hearing the wonderful things is that you see teachers are doing it. You have hope when I go back into my classrooms, and in my classrooms I still see whites are privileged, blacks are reprimanded, and Latinos are ignored... So how do we create a larger pool of teachers who can (incomprehensible) support students, but our pre-service teachers as well. So those are some of the questions (inaudible). Thank you, Rochelle.

Eugenia (Vomvoridi-Ivanović) talked about using culture as a resource, and what her work did was challenge my assumptions about the needs of pre-service teachers of color, and she puts it right out there in my face and say "Hey, wait a minute. We are assuming that if you are a person of color, then you know how to teach people of color because you're colored!" (Laughter) And it does a disservice to everyone involved... because it doesn't help teachers of color, pre-service teachers, learn how to do the job that they're going to be expected to know how to do, and it makes assumptions about their experiences as being shared, and I think about the two teachers that self-identified as Mexican-American and the other two teachers in the study identified as Mexican. Yet, in the classroom we see them all the same. They're just going out there... or worse yet, because you're (incomprehensible) you can teach all Latinos! It doesn't matter if they're from Chile... so it challenges those assumptions. It examines this whole nation of a shared culture and if there are shared cultures, but then when we deal with mathematics, those things can be convoluted at times. The task can either support or hinder pre-service teachers' ability to infuse cultures (inaudible)... And I think we saw that with the binder task versus the (inaudible) task. (Inaudible) one of my questions. How do we incorporate students' in-class experiences within these new spaces? So we have this out-of-school experience, which is still in the school, because it's after-school, but the kids know it's not really school, but how do we make the connections for them, that these experiences can be useful as they learn school mathematics? I think we have to make that explicit. Kids don't make the leap nor do teachers. How does the self-identified cultural identity of the teacher influence the intentions of students' cultural identities and mathematical needs? I was really wondering about that. I have a lot of African-American teachers who believe the same discourse about students: "They're poor; therefore, they can't learn." They don't share up an identity. They don't share a socioeconomic identity and they've adopted the school's cultural identity of who can and cannot do mathematics. I need help, 'cause this is really a problem. Too often, kids are in classrooms... (Incomprehensible) We have a new discipline policy, 'cause that's all we do with discipline. We just… we can't get his up and right. So we don't want to show that we have in-school suspensions, right? We have (inaudible). So now, what we do is we take Sean and we say "Sean. Get your books. You got to go... but don't go to in-school suspension. Just go to the other teacher's classroom."

So, on the books we're down with the numbers of in-school suspensions. Sean goes to the classroom and Sean (incomprehensible). Right? There's nothing done! The teacher, "Welcome, Sean! Come on, grab a seat right here," and Sean is ignored the rest of the day. Can we afford that? Even though the teacher looks the same as the student, Sean, but they don't view themselves as having a shared cultural identity with the students. So, how do we do that? 'Cause it interferes with the mathematical needs and therefore, ability to assess. So, how do we help pre-service teachers learn when to infuse students' culture in mathematics lessons? I think that's still unchallenged. (Incomprehensible) for some time and it's not that simple. And so, is it that we need them, “When you get here, do that”? That's what our district does a lot of. It's just a recipe. Or do we help them become critical interrogators of curriculum and classroom? And I'm going to argue that we need to do that, and that we need to help them find out how to find information to support their needs.

"Exploring teacher reflections," and in this study they examined teacher study groups from two different CEMELA sites over time and I think that's powerful because a lot of times we have this individual projects and we don't take the time to really see where we intercept or, more importantly, design it so that we are looking at these different sites; recognizing that the sites have different elements, but what are the shared ways as both researchers and professional developers? It highlights the teacher's reflections about effective practices. We need to continue to be our teachers' voice, both the good, the bad and the indifferent. In this case, we hear voices of, "What is effective teaching?" This is a shift in our model, because often we tell them what to do and we study how well they did it. So now this is, maybe they should tell us what to do and we should study how well we're listening. (Laughter)

It also provides opportunities for teachers to solve mathematical tasks, reflect on how the students would engage in a task, and discuss what happened in classroom, and to me this is a challenge, because right now we're studying what happens in common planning times, and a lot of time it's not a lot of planning that's productive. "One day I'm going to teach 1.1. What are you going to teach?" "1.2" "OK, and then we're going to do (incomprehensible) test on (incomprehensible)," right? But that's not productive use of time. So engaging them in mathematical tasks can be faced with resistance. "I don't want to do the task. I'm a teacher. I don't have time for this. You're eating up my planning time." So having them understand how important it is. I don't know how well you should know the task before you teach it, but more importantly, to think about how students are going to engage in this task, and then to come back and discuss what happens. I've seen models where we (incomprehensible) student work, but they don't have a collective group of work that we're bringing in. So we deal with the whole notion of "Well, I don't know how this other teacher is going to feel about what I'm presenting," and we're territorial because it is a reflection on us. Maybe we want to say "No, it's not. It's just the students..." No, it's your students, and when you leave, they're going to let you know. So we have that.

So here are my questions that I'm wondering about. "How did the discourse change over time across the two groups?" Did it become more about the mathematics? Did it become more about the students? How did that change over time and what's important about that? I was very concerned that they didn't talk about district constraints, because our teachers are constantly talking about how the district is imposing these things on them, and that's a reality. That's a tension, that if we don't acknowledge, we won't be heard. So what types of districts constraints have they talked about? And given the teachers really discussed issues of language and culture, how should we interpret this? I think a lot of times we're looking for "Oh, they should be saying..." What would we say? What would they say? How do we (incomprehensible)? And I think you've addressed, and I just didn't want to change my slide. (Laughter) And then, our practitioners... That was really hard, because this was on (incomprehensible)... but the thing that I heard was that their experiences changed their perceptions about students and teaching... *their* experiences; that pre-service teachers need to learn about immigration and political views, and one of the things that we're doing now, in our teacher education, is we're bringing in the state district attorney to talk about educational law. We're seeing, our students are seeing things... We are having our pre-service teachers see things in schools that they think are really hurting kids. What's the law? What is that? How do we help them understand? But also, what's the (inaudible) in schools? School cultures can be hostile. They are becoming more hostile because of all the ways that No Child Left Behind, AYP, all of these mandates are imposing, which goes to my next... Assessment practices are stifling teachers and students. (Researcher's name) in the 80's talked about "assessment is the new academic lynching..." that the more you have lower economic status kids, the more testing that we give them. It's stifling both the teacher's ability to teach, but not just teach something but to reflect on what happened and adapt their instruction accordingly because by the time you do that you got to assess again, and it's stifling students' ability to feel good at any time. I don't know about you but I had to play monopoly for a couple of years to really get good at it, and I think I can say most of you were the same way. (Laughter) Same things with mathematics. It's a set of skills, (incomprehensible). So, how do we do that?

All the school's (incomprehensible) professional development must be relevant, and professional learning can be (incomprehensible) to support professional conversations. So here are my questions: How do we provide experiences for teachers to transform their views within existing professional development models? Districts are doing professional development all the time, but as (incomprehensible), they may be not be relevant. How

do we encourage teachers to become leaders in the schools, districts, and math community, to continue and support these efforts?
And then finally, how can we as teacher educators, professional developers, researchers, become what (Name) talks about as these "educational brokers," between teachers and administrators? We can say things that teachers can't say, and so how do we become those brokers? So, as we move forward and think about it and dream about what can be, here are my four next steps. (1) We have to convince all stakeholders that the strategies that we are seeing work well for all students. I'm tired of my students saying that, "If I get some Latino students I'm going to do that!" (Laughter) We have to explicitly help them understand, all stakeholders: teachers, administrators, parents, teacher educators, that these strategies are good for all students. (2) We have to change the discourse in our schools and districts. It's not "these kids;" it's "our kids," and until they belong to us, we will not teach them. The parent is (incomprehensible), because when I'm in a classroom, they are all my kids and they always say "Do you have kids?" "Yes, I do. How can you tell?" We have to change the discourse. (3) We have to engage in cross-cultural research to broaden our knowledge base. We have research on African-American students. We have research on Latino students. How can we start broadening that? Or will we back here in ten years and we will ask the same questions with different people? Instead of "How do prepare White pre-service teachers to deal with Latino and African-American students, how do we prepare Latino teachers to deal with African-American and White students? So we need to really start broadening that, and finally, (4) how do we create models that can be tested in different contexts, cross-cultural, linguistic and socioeconomic? The political landscape is not going to change with the discourse around education. It works (incomprehensible). So we need models that we are testing. What do we agree with those models? Large-scale. It's time to start taking it all. I hope that you have felt, like I have, that we come together to celebrate our successes, (incomprehensible) challenges, and rejuvenate our spirits and missions to move forward. I hope we can continue these conversations and that we will continue the good work. Thank you.

Teacher Education and Professional Development Group Discussion

The participants involved or interested in teacher education, were divided into small groups (6 to 8 people) to discuss the current situation and unsolved issues in the domain of teacher education. The members of the groups were teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. The group discussions were structured by the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The connections question became embedded in the discussions of the other questions. This summary represents common themes identified within and across the working groups.

What do we know and what are the implications for practice and research?

Combining the presented research studies from the Teacher Education Strand, the provided poster sessions, and our own background knowledge and experiences, we are able to state what we believe to be the issues involved in teacher education of mathematics with English Language Learners (ELLs) and particularly with Latino/as.

An overarching theme of identifying and addressing equity issues for marginalized students learning mathematics predominated the presentations and discussions. With an excessive focus on the “achievement gap,” the need to close the gap (“gap gazing”), and the need to have schools and students make adequate yearly progress (AYP), certain dominant perspectives about schooling, achievement and equality are reified “with little concern for issues of identity and power or broadened notions of learning from a critical perspective” (Gutiérrez, 2009, p. 9). How we address this as teacher educators became a central piece of the conversations.

There were three predominant areas debated during the group discussions: teacher preparation program development, pre-service teachers’ preparation, and in-service teachers’ professional development. The main ideas and the common themes discussed within and across the working groups are divided into the three areas mentioned above.

Teacher preparation programs development

A common concern among the participants in the teacher education strand was the lack of agreement among teacher educators regarding the basic knowledge needed for teaching. Teaching is a complex profession that requires skills, abilities and knowledge that are not yet fully understood by many of the members of the communities of educators, administrators, policy makers, and teachers. For this reason, it is a difficult task to develop a common set of guidelines for learning to teach mathematics.

The need to understand what it means to be a teacher in our society created an open space to discuss teachers' lives in actual school systems. To be a mathematics teacher today means not only to facilitate students' learning of mathematical content but also to be able to deal with social issues, parents' and administration's expectations, a multicultural and diverse population of students, overpopulated classrooms, and a perpetual testing system. In this rather unproductive environment, teachers should have a toolkit of diverse and specialized knowledge, which needs to be discovered and debated.

One of the main concerns of the contemporary teacher profession, discussed by the participants at the conference, was the "teachers' political knowledge for teaching." Because of the many social and political issues teachers have to negotiate, teacher educators need to prepare pre-service teachers to navigate the systems while keeping fidelity for mathematical content and staying accountable to the students.

For example, teachers should be prepared to include curriculum projects that address social justice and equity issues relevant to their students' lives using rigorous mathematics as an investigative tool in their teaching. By implementing this critical pedagogy, teachers develop agency among disadvantaged groups of students and push back against privileging the dominant culture (Gutstein, 2006). However, teachers must be prepared to defend such teaching to administrators, parents and students, articulating the connections to the standards and ensuring that the mathematics is rigorous. Also, they must be prepared for student resistance due to a narrowly held definition of what mathematics is as defined by previous experiences in school and the community at large.

Consequently, teachers should be able to create and maintain a continuous connection between the mathematical content, which usually is developed in a closed classroom environment, and the outside social reality in which students and teachers live. Teachers have to be flexible to adapt and use knowledge and strategies from different content areas in their mathematics teaching. These types of adaptations may ease the transition from knowing mathematics to teaching mathematics to teaching mathematics for ELLs. This is a long process that has to begin in the pre-service teacher preparation programs and continue throughout one's teaching career.

At this moment in teacher education, there are no straightforward methods and resources that provide teachers with all the tools needed for their work in today's schools. It may not even be possible to have or create all of these tools. Instead, perhaps teacher preparation programs should focus on creating a disposition in teachers of wanting to continue learning how to teach. Teacher preparation programs need to develop prospective teachers' analytic competence to systematically analyze teaching. Teaching then becomes an "an object of study" and prospective teachers develop the habit of mind of reflecting on their teaching to continually improve it and view themselves as teacher-researchers (Hiebert et. al., 2007).

Another problematic area in teacher preparation program development is the alternative certification route. Often teachers who are certified through such programs are not well prepared and require added support from their peers and administrators. They also often leave the profession after a few years. Teaching requirements vary from state to state and this also can cause difficulty. There may need to be a screening process to identify pre-service teacher candidates that are most likely to be committed to education, open to new ideas, flexible, and life-long learners.

The tensions present in creating more equitable mathematics classrooms for Latina/o students are not understood by many of the teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. These communities need to be brought together in professional development settings and conferences to address coherence in teacher preparation programs, curriculum, assessment and pedagogical practices that are required for addressing equity issues in the teaching of mathematics for Latina/o students and other marginalized groups of students.

Pre-service teachers' preparation

Every small group of participants in the teacher education strand discussions highlighted the essential impact that the pre-service teachers programs have on teachers' professional development. It was unanimously recognized that if we want to make a change we should start from pre-service teacher preparation programs. These essential experiences for the students were identified:

- experience different ways to teach mathematics;
- experience how ELL students feel and learn in a mathematics classroom;
- experience what happens in a real classroom environment.

It is critically important that pre-service teachers experience new approaches to learning mathematics. Without new strategies, teachers often rely on their own past learning experiences for teaching. This often reduces teaching mathematics to basic procedures. Pre-service teachers need to build their mathematical knowledge in order to understand that mathematics does make sense. Only when this confidence is built will mathematics teaching move beyond procedural knowledge. Having deep mathematical understanding will also allow teachers to pursue and uncover students' thinking and possible misconceptions.

Before starting their teaching profession, pre-service teachers should be ready to use students' experiences and culture as resources for teaching mathematics. It cannot be assumed that because a teacher has the same ethnicity or language background as his/her students, the teacher is prepared to teach using culture as an instructional resource in the mathematics curriculum. Therefore, "pre-service teachers need experiences working with Latina/o children on mathematical activities that are conducive to using both theirs and students' cultural knowledge" (Vomvoridi-Ivanovic, 2010, p. 14).

The second area discussed as an important experience for the pre-service teachers was to develop an understanding of how students feel about and learn mathematics. To encourage pre-service teachers to consider and analyze students' thinking, instructors of pre-service teachers have to create realistic experiences of how students learn mathematics. The pre-service teachers need to experience how the ELL students feel and identify what strategies are effective in supporting their learning. The future teachers should experience first-hand how certain strategies such as just speaking slower, using more gestures, or more pictures are supportive to the second language learner. For example, giving live lessons in another language can provide such an experience. These live lessons in another language provide an even richer experience than just giving students' a test in another language.

The third area of experiences that pre-service teachers should have during their training is field and in-school experiences. The participants considered lobbying for earlier field experiences, possibly apprenticeship programs, and carefully selected placements that are reflective of known best practices in the teaching of mathematics. A framework for reflection to be used by the mentor and student teacher together is a powerful tool. The Instructional Context guidelines from the National Board Certification Workbook (Mack-Kirschner, 2005) can help pre-service teachers "see" their class and their students, and then inform their instruction. Videotaping the classroom and then reflecting on the sub-groups in the classroom are helpful in identifying and meeting the needs of all students.

Research shows teachers need to build classroom trust, so students feel the teacher cares about them (Gutiérrez, 2009). Pre-service teachers need to explore the dynamics of developing teacher-student relationships through home visits and meeting families. Pre-service teachers need to be aware of and confront their own biases, cultural stereotypes and deficit thinking that may interfere with having high expectations for all students. Implementing task-based interviews using a cognitively guided instruction framework can help identify strengths that the child has and can be used as an instructional resource.

Through these field experiences, the pre-service teachers may become aware of student resistance and the complex spectrum of students' issues. They need to have strategies that help lessen resistance and students issues. Heather Cavell's (2010) poster presentation documented that the presence of students' resistance and acceptance behaviors were influenced by language use, mathematical identity, connections to culture, and peer relationships.

In-service teacher professional development

The main topics that arose from the discussions about in-service teacher professional development were:

- current teachers' professional development programs;
- new mathematics teaching strategies and approaches;
- new ways to enhance teachers' professional development.

Though these topics were interconnected through a cause-effect relationship, the ideas expressed by the participants become more relevant if they are classified by these three topics.

Many current available professional development programs were criticized for not being productive or relevant to what teachers do and for not focusing on what is needed. For example, some professional development programs for implementing the reform curricula present how to use the websites with textbooks. This is not the inservice needed for teachers to understand the pedagogical underpinnings of these reform mathematics curricula and to change their teaching to enact the curricula in the classroom. Without developing teachers' pedagogical understanding of reform curricula, they do not always recognize new strategies for teaching mathematics as valuable and may reject ideas and continue teaching as they have in the past. Or they may find them too difficult to implement. Then the curriculum often is deemed as a failure, when in actuality the fault lies in the implementation and lack of appropriate professional support.

Standardized testing determines what and how mathematics is taught, stifling both teachers and students (White, 2010). When we closely examine these tests they do not present drilling problems. Because open-ended tests are expensive to grade, most tests are multiple-choice. These tests are high-stakes but it is not understood if these tests assess what the students know or do not know. Because of the pressures of AYP, marginalized students are subjected to even more testing, further reducing the amount of time teachers have to teach, reflect on student learning, and make adaptations.

The metaphor for teaching needs to change from "a race to the top to exploring the mathematics landscape" to ensure marginalized students have access to deep mathematical problems and thinking (Wood et. al., 2010). By changing the metaphor, the current deficit conversations that reify these students' identity may shift to focusing on student thinking and allowing for the possibility that these students will succeed.

Being aware of and addressing the tensions that are a reality for teachers within a school's climate is another important piece in professional development. Teachers' difficulties in implementing reform curricula need to be explored and discussed. Teachers trying implementation of reform curricula, social justice projects, etc. may find resistance and criticism among their peers or even administrators. These teachers will need support on how to navigate these politics. Researchers, teacher educators, and professional developers need to become "educational brokers" between administrators and teachers (White, 2010).

Recognizing the complexity of teachers' lives and the constraints placed on their time is critical. Authentic project-based teaching that is relevant to students' lives and culture will require support to identify resources and connect the mathematics to required syllabi.

Stronger connections between the research community and practitioners are vital. To honor the issue of teacher time, research articles need to be available that are written more "teacher friendly" in terms of length and difficulty. Teachers must also be afforded time in their teaching day to reflect on and discuss the research.

Teaching mathematics with an equity stance is a complex phenomenon, one that cannot be mastered before one begins a teaching career. Learning to teach is a life-long process. The novice teachers' professional development needs will be very different from those of teachers with more experience. When designing professional development, these needs must be addressed. Therefore, professional development needs to be thought of as a continuum and be long-term. Teachers must have support during the implementation of reform curricula. The challenge is how the process of "professional development implementation" becomes a cycle, an ongoing process. Using technology to sustain professional development programs or constructing professional development trajectories may assist in overcoming the challenge.

Participants of this discussion group identified several successful professional development models. These included having teachers involved in:

- focused observations with reflection;
- reflection on videos of their own classes, thinking about what happened with any sub-group of students in their classroom and questioning if they met the needs of all students;
- action research projects with possible collaboration with a researcher;
- interpreting classroom data to inform classroom practices; and
- professional learning communities.

These models can be supported from the outside, using resources provided by universities, and from the inside, using teacher leaders as a resource. Teacher leaders who have the greatest potential for being perceived as leaders by their colleagues need to be identified. They will need a support system to help them become comfortable with this role and to develop their knowledge base for working with adults. A long-term commitment from those that take these positions helps ensure that districts' investment will produce a positive return. Moreover, to maintain the "within system" characteristic of teacher-leader, models should be developed that create leaders without taking them out of the classroom.

Research indicates that an effective professional development model has teachers involved in learning communities (Kondek McLeman, et al. 2010). Therefore, there is a need to design professional development models in which mathematics teachers can be part of a team. Developing communities of teachers who work together to investigate their own strategies may assure that the learned strategies and teacher agency will be more sustainable. For example, *Lesson study* might be a model to look at for professional development. However, there are some challenges with any learning-community model. It is costly and time-consuming, although the benefits can be dramatic and empowering for teachers.

The reality is that when funding ends on a successful and/or meaningful project, the project comes to an end. Sustainability of programs with less funding is a must to further teacher professional development. Technology may be utilized to help reduce the cost of professional development by having it administered concurrently to a variety of sites or through the use of social networking sites.

One more important consideration in professional development is the support that administrators need. They should be encouraged to attend teachers' professional development, but they must also have professional development that is designed specifically for them. Images of what classrooms, teaching, and learning look like when reform curriculum is being implemented should be included. The messages that administrators receive need to be consistent with those that teachers and teacher leaders receive in professional development.

What questions do we need to research further?

Participants identified the following questions as needing further research and investigation:

- What are the qualities of a good task when culture is a consideration?
- How do we prepare teachers from all cultures to teach students from all cultures?
- Based on analysis of novice teachers' practice, what is most important to teach pre-service teachers? What will be most helpful to them?
- How do you determine if a task is authentic and meaningful? What negotiations must take place in order to create and implement an authentic task?
- How do we change the conversation from the deficit model of education that determines our education policy and denies our students the opportunity to learn?

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COUNTERACTING THE (UNDER) EDUCATION OF LATINO IMMIGRANT CHILDREN

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In this article, we draw from cultural historical theory to examine how assimilationist forces embodied by state-level ESOL practices exert pressure on Latino schoolchildren to conform to local notions of literacy grounded in monolithic assumptions of what it means to be "American." Their exceptionality and potential for additive bilingualism are ignored as U.S. Latinos are often framed as limited speakers of English, and routed into remediation tracks that leave most of them academically behind. We conceptualize how a Cultural Identity Plus (C-ID+) policy as a component of a broader lifespan strategy necessary for dismantling "Group Based Inequality" might counteract the damaging caused by current practices.

IN THE SHADOW OF STONE MOUNTAIN: CULTURAL-HISTORICAL IDENTITY(IES), GROUP-BASED INEQUALITY, AND COUNTERACTING THE (UNDER) EDUCATION OF LATINO IMMIGRANT CHILDREN

> "Let freedom ring from the snow-capped Rockies of Colorado. But not only that. Let freedom ring from Stone Mountain of Georgia."—Martin Luther King, Jr.

We live and work just an hour and a half drive north of Atlanta, Georgia—not far from Stone Mountain where, in 1915, the second Ku Klux Klan was born (MacLean, 1995). It was to that same Stone Mountain that Dr. Martin Luther King, Jr., quoted above, alluded in his legendary cry for the dismantlement of Jim Crow. Almost ten years into a new millennium, the stony gaze of Confederate leaders carved into the ridge's granite side remains fixed on a rolling landscape marked by demographic change. Census data indicated that between 1990 and 2000, Georgia's Latinos grew more than 300% to account for approximately 8% or over 860,000 of the state's residents (National Council of La Raza, 2005; The Tomás Rivera Policy Institute, 2004). Burgeoned by industrial poultry production, the construction boom that accompanied and followed the Atlanta Olympic Games, and the carpet industry, a number of counties and communities in metro Atlanta are currently over 25% Latino. By 2010, some are expected to exceed 75%. All the while, the Confederate States of America's Stonewall Jackson, Jefferson Davis, and Robert E. Lee look out from the dome monadnock into the I-85 corridor. Indeed, the past twenty years of the state's unprecedented "browning" has awakened inveterate white supremacism in a so-called "Battle of Georgiafornia" (Moser, 2004). At times, violence has been visceral with local and regional newspapers reporting of hate crimes of which Mexican immigrants and Mexican Americans are too often victims (Bauer, 2009).

In terms of academic achievement, Latinos, both in and outside of Georgia, face what we consider a different, more subtle, but not insignificant type of violence. Failed educational policies and practices framing U.S. Latinos as linguistic minorities have wreaked havoc on the development and future capabilities of U.S. youth. Today, the 2.9 million Latinos enrolled in U.S. high schools are less likely than their non-Latino peers to complete a degree (National Council of La Raza, 2007). Thus, as opposed to being honed for their exceptionality as potential bilingual/biculturals, U.S. Latinos are too often framed as non/limited speakers of English and routed into remedial English instruction (cf., e.g., González, Moll, & Amanti, 2005; Valdés, 2003; Valenzuela, 2002). Critiquing cultural historical forces embodied by such push-in/pull-out ESOL programming, we theorize a counteracting agenda whereby a *Cultural Identity Plus* (CID+) approach is embedded in a lifecycle of sustained support for dismantling "group-based inequality" in Georgia and the nation.

Institutionalizing Achievement Gaps

As in much of the U.S., The Georgia Department of Education's guidelines require that students be administered a "home language survey" upon enrolment to determine if English is their native language, primary home language, or first language. Students whose native, home, or primary first language is other than English are subsequently assessed for English language proficiency using the state-adopted English proficiency screening instrument, the *WIDA-ACCESS Placement Test (W-APT)*. Assigned an English proficiency level on a scale of 1 to 6, students whose score is less than 5.0 are determined to be Limited English Proficient (LEP), are eligible for language assistance services, and may be served through the English for Speakers of Other Languages (ESOL) program.

In Georgia for example, programming for English learners may follow a number of delivery models. These include: 1. Pull-out model outside the academic block – students are taken out of a non - academic class for the purpose of receiving small group language instruction; 2. Push-in model within the academic block – students remain in their general education class where they receive content instruction from their content area teacher and language assistance from the ESOL teacher; 3. A cluster center to which students are transported for instruction – students from two or more schools are grouped in a center designed to provide intensive language assistance.; 4. A resource center / laboratory – students receive language assistance in a group setting supplemented by multi-media materials; 5. A scheduled class period – students at the middle and high school levels receive language assistance and/or content instruction in a class composed of ELs only; and, or, 6. An alternative approved in advance by the Department of Education through a process described in Guidance accompanying this rule. Very little is known about how these practices reduce achievement gaps. Nevertheless, they have, as in the case of Georgia, been embraced by K-12 institutions working with large new Latino immigrant settlement.

Figure 1 provides a breakdown of student achievement on the Georgia High School Graduation Tests (GHSGT).

Figure 1

Georgia High School Graduation Tests (GHSGT). Percentage of 11th-Grade 1st-Time Test Takers at Each Performance Level: Comparison By Race/Ethnicity (Georgia Department of Education, 2009)

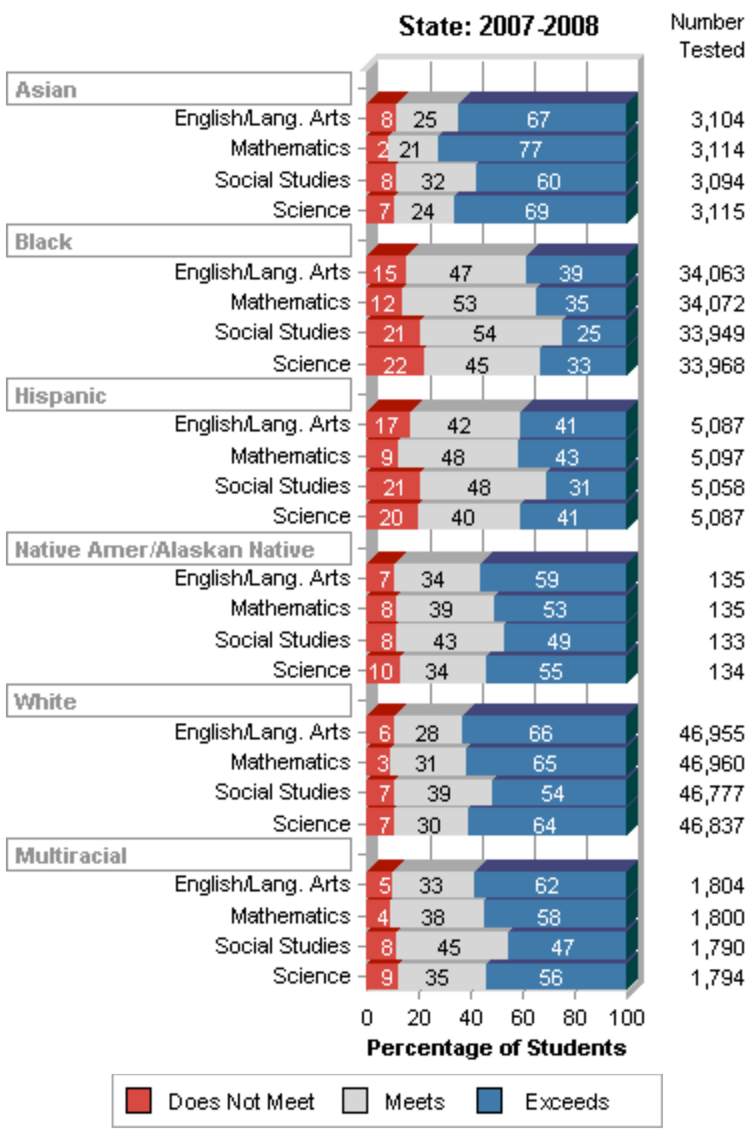
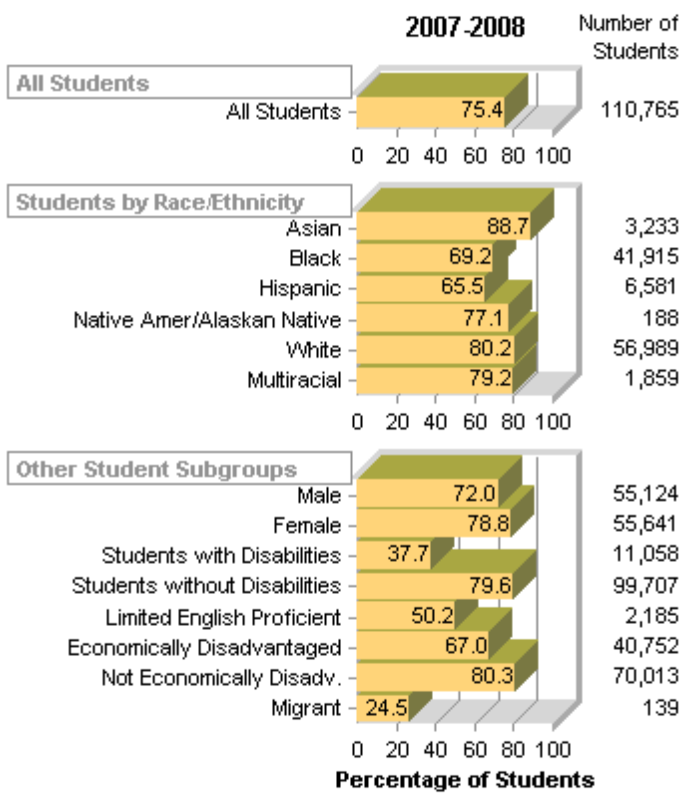


Figure 2 indicates the 2007-2009 graduation rates. Here the graduation rate reflects the percentage of students who entered ninth grade in a given year and were in the graduating class four years later. Statewide, 6,581 students in the racial/ethnic category of Hispanic graduated at a rate of 65.5%--lower than any of the other five categories. The subgroup of “Migrants” fared the worst with only 139 graduating at 24.5%. Limited English proficiency students fared better than “Students with Disabilities” but worse than the “Economically Disadvantaged.”

Figure 2
State of Georgia 2007-2008 Graduation Rates (Georgia Department of Education, 2009)



While the numbers in Figures One and Two represent a modest change from past years, the gap between dominant and non-dominant communities is, nevertheless, significant except for Asians—a case beyond our scope that focuses on the LEP to Hispanic school achievement/teaching gap . The percentages represented in Figures One and Two are not simply located in the minds of Latino students. Rather, group based inequality is (re)produced by the prevailing limitations still being imposed on children’s potential by systemic under-education practices. The gap is also located in the deficit thinking of a dominant majority largely reacting, from a sociogenesis standpoint (Vygotsky, 1978) to a new *invasion* that is often threatening to nativist identities (Brimelow, 1995; O'Reilly, 2006).

Indeed, as Valenzuela (1999) argues, “In a world that does not value bilingualism or biculturalism, youth may fall prey to the subtle yet unrelenting message of the worthlessness of their communities” (p. 264). From our own point of view, resistance to bilingual and multicultural education, the de facto segregation of Latinos in U.S. schools, the proliferation of high stakes testing, and a culture of "accountability" are the legacy of white supremacist ideologies of American schools to which Latinos and other ethnic and racial minorities have been subjected (Spring, 2004). The failure of Latinos to achieve at grade level standards especially in gateway STEM/college areas is an inequality-reproducing system.

As an aside, we note that the category of "Latino" depends largely on who defines or claims it in relation to specific cultural histories that interact within a predominately White, Eurocentric society. A Mexican-American businesswoman with multiple generational roots in San Antonio, Texas has a very different "history-in-person" (Holland & Lave, 2001) than a transnational, first-generation immigrant from rural El Salvador currently living in Charlotte, North Carolina. In the context of Georgia, Latino is often used as an implicit synonym for "Mexican/Mexican-American". In effect, The National Council of La Raza (National Council of La Raza, 2005) reported Georgia's Latinos of Mexican descent as representing the largest "Hispanic subgroup" at 67.8%. Latinos of Central American origins—Guatemala, Belize, Honduras, El Salvador, Nicaragua, Costa Rica, and Panama—numbered the second largest subcategory at 11%.

Instead of viewing the children of immigrants as a potential threat to the stability of this country, schools could better recognize their primary cultural identities *as assets that can be developed for the nation's global economy.* Our definition of cultural identity is not limited to the advantages associated with additive bi/multi linguistic literacy. On the contrary, this additive model of cultural identity development is concerned with the forging of inter-culturally distributed cognitions, attitudes, and the affective-behavioral patterns that result in achieving broader, richer identities for a variety of social contexts As such, Cultural Identity Plus (CID+) lies at the other extreme of deculturalization and represents a path to promote a healthier, smarter future society freer of gaps related to violence and related poverty costs. It does not reduce academic worth and aptitudes. It does represent a different valued outcome that is discouraged or ignored in the high-stakes world. We believe it is instrumental in reducing academic gaps and a part of a coordinated comprehensive restructuring of education that is urgently needed.

In sum, while guidelines for identifying English learners abound, relatively little research exists that might support choosing one of the various models of instruction for English learners over another. The effectiveness of transition programs driven by a sheltered instruction paradigm has been largely unverified for both English learners and non-English learners. We note that not all Latinos are English second language learners, nor are they all proficient in Spanish. However, the rapid pace of Latino settlement in Georgia and along the entire I-85 corridor creates a group that is mostly of Mexican origins with a very different past cultural history than, for example, U.S. Latinos in the Southwest or from other countries. To prevent the massive gaps of this growing and often united ethnic group, an imperative for sustained counter activity at the educational policy level is needed. In this article, we focus on counter-acting the affective-relational domain (Comer, 1980) related byproducts of deficit-oriented practices on Latinos' identity, self-esteem and achievement motivation. These secondary identity variants impact a learner's self concept that in turn mediates academic and other aspects of development. The activity contexts that these children encounter at school, as presently configured by remediation/tracking practices, and enacted principally by majority authority -laden educators, can be changed to mediate stronger, broader identities for all students and their educators. Below we outline the theoretical basis for improving education by activating dormant language and cultural resources in ways that favor positive human development.

Cultural Historical Perspectives of Identity(ies) and Development

Conceptualizations of identity(ies) and how they come to be vary by disciplinary and/or theoretical orientations. Specifically, our understandings of identity are enriched by psycho-social models in human development (Erikson, 1968; Vygotsky, 1978). A cultural historical theory represents a rich meta-paradigm that unlike the other major forces in social science resists reductionism in its plasticity. Multiple lenses for studying human development have drawn from the translated works of Vygotsky (1978; 1986) and his contemporaries such as Luria (Luria, 1976, 1979); Bakhtin (1981; 1986) and the subsequent writings of their American translators and scholars (Cole, 1996; Cole, Engeström, Vasquez, & University of California San Diego. Laboratory of Comparative Human Cognition., 1997; Lave & Wenger, 1991; Scribner & Cole, 1981; Wertsch, 1985, 1998, 2002). However, identity development at the psychological and social levels still baffles academic communities struggling with the semiotics shaping individuals with cultural "kits" interacting over time and spaces (Wertsch, 1985, 1998, 2002).

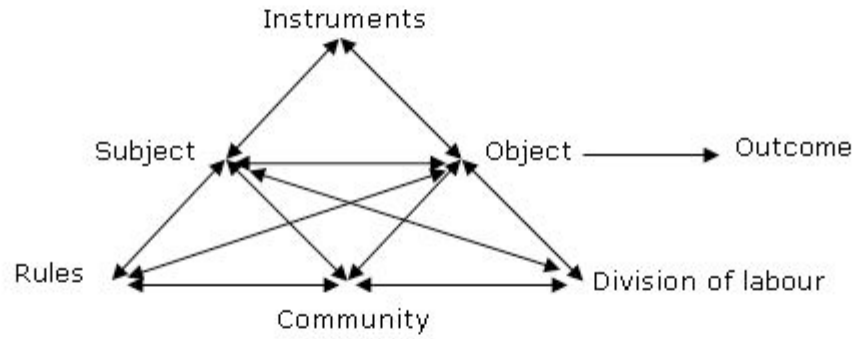
Central to a socio-cultural perspective is the notion of social origin of human consciousness including identity or self—"a theory that situates individual's cognitions *within*, rather than just interacting *with,* social and cultural contexts of interaction and activity" (Salomon, 1993). One's self or complex identity emerges along with higher order, scientific concepts socially. How we think of ourselves and others is situated in human activity and participation in "communities of practice" (Lave & Wenger, 1991). Thus, for Vygotsky and the interdisciplinary developmental psychology he helped inspire, "All the higher functions originate as actual relations between human individuals" (p. 57). To that end, it is via meaningful and volitional social interaction that interpersonal processes are transformed into intrapersonal processes through the use of material tools and symbolic artifacts to mediate thinking.

Considering identities as inherent cultural-historical artifacts (see Vygotsky, 1978), cultural historical theory emphasizes the sociogentic nature of becoming individuals. That is, how we perceive ourselves and each other is an intergenerational and societal outcome. In considering the education of children from non-dominant communities, the bilingual aspects of multicultural development and the very process of ethnogenesis must be taken into account in schooling by those alien to the concept of cultural history and ethnicity. The latter include not only children, but also the adults who educate or shape policies for their education.

Identity as Activity

In Figure 3, Engeström (1987) uses a triangle to elaborate Vygotskian notions of mediated activity. Though somewhat limited in its one-dimensionality, the 1987 Engeström triangle illustrates the interactive nature of goal-directed, tool mediated, human activity such as the schooling of Latino children. Following Engestrom's heuristic below, the relation between the subject and community is regulated by rule; the relation between the subject and the object of activity is regulated by mediating artifacts or instruments; the relation between the community and the object of activity is regulated by the division of labor. In its ensemble, goal-oriented activity results in an outcome somehow related back to the object of the activity itself.

Figure 3
The Engeström (1987) Activity Triangle



Following the triangle, the (under) education of non-dominant populations is moderated by the (lack of) volition of its subject (here the dominant majority) to transform an object (Latino children) into something more or less than they already are. Thus, we contend that educational policies and practices comprise tools for shaping school-aged children into the adults that they eventually become and as future commodities and agents in an economic sense. The rules inherent in the majority host culture are also tools that largely define the division of labor and, indirectly, the resultant identities associated with the objects' and subjects' net outcomes. Over periods of cultural history, one group may develop and often sustain a superior pseudo-identity at the expense of another, less-empowered group.

Promoting CID+ as a Preventive Measure

The current state-driven education and normative practices for enhancing the learning potential of Latino school children is certainly instrumental in sustaining a massive proportional literacy gap ranging from three to five grade levels. It need not be so. Suppressing the inherent capacity for additive bilingualism and a broader identity development in new generations of children (non-immigrants included) in the current unresponsive system obsessed with sorting and testing is problematic at a number of levels. As Haberman (2003) notes, the achievement gap is not an aberration of American society. Instead, that gap is a potentially valued outcome resulting from how teaching and learning processes are managed politically. Capable new generations of potentially bilingual children are shortchanged and their academic learning and aspirations diminished through their labeling as Non/Limited English proficient and in the de facto "remediation" that follows.

While the development of and value drawn from a broader identity at the inter-cultural level founded on the principles of additive bilingualism (Peal & Lambert, 1962) is possible in our schools, the political economy of teacher education, TESOL training, certification, and practice all give the dominant group's workforce the impression that Latinos are a "problem". To that end, the linguistic and cultural identities of thousands of children in the K-12 pipeline are not used in timely fashion as a means to learn scientific concepts, vocabulary, and content in English. Much potential is wasted in chasing higher stakes testing results and ignoring the research evidence related to bilingualism, bilingual education and cultural psychology (Cole, 1996). Such institutionalized deprivation affects not only highly regarded gateway STEM areas, but also has secondary intercultural ramifications such as students' construction of oppositional identities to the same schools and schooling practices that marginalize their ethnic identities (Ogbu, 1974, 1978, 2003).

Rejecting the notion that human psychological development or a cultural historical phenomenon such as the achievement gap in Georgia schools or across the nation can be separated from the social, cultural, and historical contexts in which individuals participate either directly or peripherally (Lave & Wenger, 1991), cultural-historical theory frames human identity(ies) development as an activity shaped by the material tools and social and cultural psychological devices whereby communities have come to define themselves. Thus, from a cultural historical stance, the category of Latino non/limited English proficient can be understood as the product of social and historical activity, actions and intercultural encounters.

Furthermore, from a cultural historical perspective, *w*ho and what is (not) available routinely in activity settings over time influences how developing children and adolescents eventually perceive themselves. Specifically, when a school or school system symbolically or, even worse, literally restricts access to learning and children's cultural identities—a mediating tool that might otherwise propel their developmental trajectories—such restrictions impact academic self-concepts, motivation and often fulfill negative self- and others' prophecies. Restrictions translate into considerable challenges for some children year after year as U.S. public institutions remain committed to historical paradigms whereby that frame school as a means of producing a bona fide, God-fearing, monolingual citizenry (Gutierrez, Asato, Santos, & Gotanda, 2002; Gutiérrez, Baquedano-Lopez, & Alvarez, 2000; Moll & Ruiz, 2002; Olsen, 1997).

Group-based inequality is, in part, related to the (semiotic) role of discrimination as experienced during a critical stage by adolescents engaged in co-constructing their complex identities in multiple contexts. In the case of Georgia and the new borderlands, we speak more specifically of Mexican/Mexican-Americans and children of Central American heritage and/or origins. Such discrimination impacts motivation and self-esteem in ways predictive of school-centered learning and achievement. The suppression of CID through the category of non/limited English proficient often results, we suspect, in the loss of an otherwise crucial mediating tool of first language (L1) literacy.

At the margins of two or more worlds, many Latino newcomers are potentially placed more and more at risk by current policy and practices presupposing that developing literacy in English and meeting annual yearly progress (AYP) necessitates suppressing Latino children's primary cultural identities and learning potential (Gutiérrez et al., 2000; Moll & Ruiz, 2002; Valencia, 1997, 2002; Valenzuela, 2004). On the contrary, we argue for a positive and additive cultural identity option (CID+) as part of a framework for culturally democratic education, that as Ramirez and Castaneda (1974) noted;

> Recognizes that the way a person communicates, relates to others, seeks support and recognition from his environment (incentive motivation), and thinks and learns (cognition) is a product of the value system of his home and community. ... A culturally democratic learning environment is a setting in which a child can acquire knowledge about his own culture and the dominant culture; the learning, furthermore, is based upon communication, human-relational, incentive -motivational, and learning patterns that are culturally appropriate. p. 23

Thus, raising Latino achievement at the group level does not require sacrificing or destroying existing majority cultural identities. Rather, a socio-linguistic support system of inter-woven conditions for Latino children currently marginalized by U.S. schools needs to be in place to empower concept development in valued literacy or school content areas. Such a strategy would represent a departure from current "pull-out" or other remedial basic skill programs where ethnic groups are framed as deficient English speakers. The evidence from exceptional bilingual schools where the community supports dual-immersion practices is compelling. Generally at least one parent is bilingual and recognizes the advantages of additive bilingualism for their children. We argue that at least in high (Latino) density districts, such options become available to all students whose parents support the interrelated outcomes of a broader cultural (self) identity and ability to thrive with more than one sign system of language in a global society.

COUNTERACTING GROUP-BASED INEQUALITY

The dismantlement of group-based inequality as a complex policy restructuring cultural project requires more than policy makers' embracing multicultural diversity, but also emerging identities that children bring and negotiate in school. Likewise, simply believing that all children can achieve and learn at grade level in tests is not enough to redress the cumulative intergenerational and historical influences that produce lower levels of economic and other opportunities that define an achievement gap . While most voters believe the gap is something that originates and belongs to some minority groups, the gap is also produced in teaching and in how learning is assisted and guided by those who organize learning environments. Rigorous, challenging academic environments, designed for continuity, need to be supplemented with human resources comparable to the other more "established or institutionalized" programs that reinforce the child's development in valued areas with respect to school success in both formal and informal settings. Counter-action needs to begin at an early age. In addition to promoting CID+, coordinated and purposeful counteraction must also begin with responsive early childhood primary grades education marked by culturally mediated activities with high intensity and continuity of experience. The latter is crucial in sustaining positive identities during early and late adolescence; a critical period for identity development across all of its major components (physical, social, vocational, and political domains, among others). Counter-actions to those that today prevail after school and summers needs to continue in and out of schools in organized and additive layers of culturally responsive and ethical-activity settings.

We distinguish the Latino achievement differences in response to public education by attending to several subpopulations within this multi-ethnic label. The majority of Latinos in school today are not English learners any more than their peers are and both are U.S. born. Their lingua franca is English even as Spanish is salient in the home and community. The main difference is the distribution of poverty cycles where they are three to four times more likely to be poor than non Hispanic whites. So while our writing here focuses on EL's whose L1 is Spanish, they are fewer in number but growing in decades to come. And there is a third low SES subgroup that counts as Latino (non-English learner!) and once did participate in ESOL as an LEP student and it is generally lumped with the latter and middle and upper class Latinos; many of whom are identified with the dominant group through intermarriage.

In cultural historical theory, the cultural histories of groups as well as individuals must be considered in fully understanding how identities are forged under different historical periods.

The relevance of this partitioning is that while we describe Latinos here in relation to CID+ or who are poor, both children of immigrants who are learning English and those born here whose primary language is English and are not usually proficient in Spanish, the primary cultural identity is negotiated regardless of differing levels on language proficiency in either language and defines this community as a whole. Hence, we examine the problem of inequality in teaching and learning that defines the gaps. The key unit of analysis is the individual's sense of identity in the ethnic and social class domains that is derived from, defined by and informed through bilingualism and similar signs and symbols, including how dominant group reactions to both the individual and ethnic group are distributed. We argue that through the development of additive bilingualism, positive, wider identities can be more fully developed that benefit both the integration of a positive cultural identity at the individual level and also the societal best interests. We employ contrasting conditions of ELs who are poor with a predominantly bleak economic future with other U.S. born Latinos whose primary language is English to address the mediational role of CID+ in a holistic fashion .

While most children of poor immigrants whose L1 is not English are placed at risk to drop out or graduate 3-5 years behind grade level, the overrepresentation of Latinos in the poverty cycle and under representation in STEM and other professions concerns vulnerable children whose cultural identities, regardless of current language status, predict success in school and life. Because of historically and politically determined policies, the vast majority of this large and growing ethnic group are placed at risk by how they are taught directly and indirectly, regardless of the new borderlands as described here or of those where Latinos are a majority, learning and teaching outcomes are important but often moderated by how the social and ethnic identity formation process is negotiated or reacted to by youths who today are likely to encounter subtractive schooling conditions that more often than not impact negatively upon already marginalized identity. This two step reality (under-education by undervaluing additive bilingualism and undermining it until secondary school in ways that threaten CID+, can be re-mediated to mitigate alienation, discouragement and damaging stereotypes and self-fulfilling prophecies. Coordinated counteractions thus combine, for example, responsive early childhood dual immersion education for all students in diverse communities and non Latino; afterschool enrichment that enlists local undergraduates to assist schoolchildren at risk to forge CID+ in school. Counteraction strategies include the new ways we prepare monolingual and monocultural teachers to work with diverse learners and developmental histories. It requires presently systemic workforce development of bilingual, effective pedagogues at every level.

Raising Latino achievement through a sustained policy of counteraction that includes attention to CID+ is not, we imagine, a change that many Americans can easily conceive of or "believe in". In a time of economic upheaval, few can imagine allocating scarce resources for the explicit investment in "other people's children" (Delpit, 1995). As we see it, the prevailing view in Georgia, as in much of the U.S., is that children of non-dominant communities must wait their place in line, or more specifically, their place in a cultural historical time line. However, simply waiting for equity in learning and economic opportunity to come for those still subjected to group-based inequality has not been a reliable or productive strategy for them or the nation. However, from a cultural historical view, the societal changes needed to allow the institutionalization of bilingual schools and teachers is one that can be effected gradually (sociogenesis) by educating the dominant group in recognizing the ways the nation and their children will benefit in the long run. The road to equity in education is a precursor to excellence and might be won by focusing on collective economic gains for the nation as much as by the ethical argument that defines social justice. Both types of counteractions are needed for change both sides can believe in.

From a cultural historical analysis, unless dismantling educational inequality becomes a higher socio-cultural priority, the structural conditions have already been organized for a caste-like future relegating Latino children to intergenerational urban or rural poverty. Thus, in closing, we wonder about the extent to which the basic problem of group-based inequality is socio-genetic and an educational project that in some ways has already begun. That is we suspect that the historic and systemic (under education of non-dominant communities through subtractive paradigms of cultural identity/ies grows out from, is supported by, and is dependent on collective and volitional, goal directed, tool mediated, and social activity. To that end, we conclude with the observation that dismantling group-based inequality is to be determined by and hinges upon, among other things, more enlightened language/identity based educational policies and practices that might "mediate" or counter-act the unfortunate intercultural histories and identities that shape us all—in the shadow of Stone Mountain, Georgia and similar communities.

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HOW PRINCIPALS CONCEIVE OF THEIR LEADERSHIP ROLES IN THE IMPLEMENTATION OF A REFORM MATHEMATICS CURRICULUM IN PREDOMINANTLY HISPANIC SCHOOLS

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Only recently have researchers begun to consider the important role of principal leadership in reforming mathematics teaching and learning at scale. Without leadership to set the direction for the reforms, support teachers to develop the pedagogical and curricular understandings to implement the curriculum as intended, and redesign the organization to facilitate the reforms, we will continue to have only pockets of excellence. This paper presents a slice of a qualitative case study designed to learn how principals conceive of their leadership roles in implementing a district-adopted mathematics reform curriculum, especially related improving opportunities for Hispanic students. This paper focuses on principals' perspectives as they embark on the first two years of mathematics reform, including their ideas about teaching and learning of mathematics, about supporting teachers, and about holding teachers accountable for the reforms. Data from principal interviews and observations is analyzed in relation to the PRIME Leadership Framework and the research based-best practices in leadership.

INTRODUCTION

This paper presents a slice of a larger research study that explored principals' conceptions of their roles in leading mathematics reforms in predominantly Hispanic schools. While much has been written about how principals shape instructional decisions in their schools (Elmore, 2000; Marzano, 2005; Waters & Grubb, 2004), little is known about how they conceive of instructional reforms, particularly in mathematics and science (Hallinger & Heck, 1998; Spillane et al. 2001; Spillane, 2005). In this paper, I report on the ideas three principals have about their role in the implementation of a reform mathematics curriculum in that was adopted district-wide. These principals lead predominantly Hispanic-serving schools, where student proficiency in mathematics has been of particular concern. The Hispanic student population is of particular interest to this research, which is supported by the Center for Mathematics Education of Latino/as (CEMELA[1]). Central to the work of CEMELA is the question of how policies (e.g., curriculum, assessment, students' placement) impact mathematics learning and teaching for low-income Latinos.

The primary question that drove this study was: How do principals conceive of their leadership role in the implementation of a reform mathematics curriculum in predominantly Hispanic schools? Subquestions include:

- What is each principal's leadership content knowledge related to the reforms?
- How do they conceive of setting direction for curriculum implementation?
- How do they conceive of their role in developing teachers' abilities to learn new pedagogy to deliver the curriculum as intended?
- What do they believe about organizational redesign to support the reforms?

BACKGROUND

Of central concern to this study is what Cobb & Smith (2008) have referred to as "the problem of scale". The teaching and learning of mathematics has been the focus of national reform efforts for over twenty years. Recognizing the need for all students to learn about important mathematical concepts and processes with understanding, the National Council of Teachers of Mathematics (NCTM) enlisted professionals from diverse expert communities to develop curriculum standards for school mathematics in an attempt to "develop and articulate explicit and extensive goals for teachers and policy makers" (NCTM, 2000, p. ix). Yet "despite the concerted efforts of many classroom teachers, administrators, teacher-leaders, mathematicians and policy makers" (NCTM, 2000, p. 5) and despite the fact that most districts across the country have standards for the teaching and learning of mathematics that are based on the NCTM standards, many students still do not have access to the kind of instructional practices that allow them the opportunity to learn important mathematics (National Research Council, 2001). In fact, research has shown that the variation in teaching pedagogy and in learning outcomes for students is often greater *within* schools than *between* schools (Hannaway & Hamilton, 2009; Newmann, King & Youngs, 2001). In her examination of research on teachers' use of mathematics curricula, Remillard (2005) explains that districts often adopt reform curricula as a "means to regulate teaching practices" (p. 211) and as a means to improve student

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proficiency, particularly in response to the failure of schools to raise student achievement for students of color and from low-income communities. However, they do not take into account that the materials being adopted are “foreign in form and content to most teachers” (p. 212). They require intensive ongoing support to learn the content, goals, approaches, and underlying assumptions of the curriculum they are being asked to use. Interestingly, in her examination, she does not consider the role of principal leadership in advancing the reforms, other than to suggest that administrators and policymakers view reform curricula as potential vehicles for change.

Some researchers are beginning to look at what it will take to improve mathematics teaching and learning at scale (Carpenter, et al., 2004; Cobb & Smith, 2008; Spillane, et al., 2002). Central to the conversation is the role of principal leadership. This paper argues that principal leadership is key in interpreting the goals of the reforms and in supporting teachers to learn to enact the new curriculum as intended.

Recent research has shown administrators’ understandings of standards-based mathematics instruction and of how they can support it is tied to their own ideas about the nature mathematics teaching, learning, and assessment (Burch & Spillane, 1999; Nelson, 1998; Spillane, 2002; Stein & Nelson, 2003). While there is significant research on the changes in *teachers’* beliefs about the teaching and learning of mathematics as they shift their approach to a more problem-solving view (Fennema & Franke, 1992; Franke, et al., 1998), I have found only a small body of current research giving insights as to what principals believe about mathematics teaching and learning (Buonopane, 2006; Burch & Spillane, 2003; Nelson, 1998; Stein & Nelson, 2003; Nelson & Sassi, 2005). There is evidence in this research that administrators, too, need opportunities for “conceptual reconstruction” (Nelson & Sassi, 2005, p. 213), to construct new knowledge about the teaching and learning of mathematics in order understand the intent of the reforms and to support a new intellectual culture required to sustain them. Stein and Nelson (2003) term these understandings “leadership content knowledge” or LCK. Leadership content knowledge can “equip administrators to know strong instruction when they see it, to encourage it when they don’t, and to set the conditions for continuous academic learning among their professional staffs” (Stein & Nelson, 2003, p. 425). This study adds to the body of research on principal leadership in mathematics by considering how they make sense of district policy to implement a reform mathematics curriculum in the context of their individual schools.

LITERATURE

Three bodies of work have been particularly important in framing this study: educational leadership; recommended practices for mathematics leadership; and leadership content knowledge.

Educational Leadership

Principals are “street-level bureaucrats” (Lipsky, 1980, p. 13) who stand at the intersection of powerful mandates to improve student performance in mathematics and significant reforms in classroom teaching practices that often take time to change. Lipsky coined the term to suggest that the people in positions to implement policy and to determine how it is actually carried out are employees who possess a significant degree of discretion in the implementation. Principals, by virtue of their supervisory roles, have such discretion. Thus, principals “serve as critical links between the district and the school for developing and implementing solutions to identified problems” (Rorrer, Skrla, & Scheurich, 2008, p. 308). Brokering the NCLB (2002) accountability demands and transforming teaching pedagogy to comply with research-based reform practices in mathematics requires leaders to both "conceive of the reforms and act to support them" (Nelson & Sassi, 2005).

A comprehensive review of the research on school leadership, commissioned by the Wallace Foundation, found that "leadership is second only to classroom instruction among school related factors in its impact on student learning" (Leithwood, et al., 2004, p. 3). Further, researchers on school improvement have found that significant obstacles to improvement have been insurmountable without a strong leader present to guide and maintain high expectations in the difficult work of changing a school's culture (Cobb & Smith, 2008; Duke, 2004). To affect reform, the principal's role is situated in a transformational leadership praxis that is most effective when it includes three skills: 1) setting the direction for the reforms; 2) developing the people who will carry out the reforms, and 3) redesigning the organization, to facilitate the reforms (Leithwood, et al. 2004).

Setting Direction

Setting direction includes the ability to model best practices of inquiry, and the creation of an environment of intellectual stimulation where teachers are encouraged to explore and challenge ideas about teaching and learning (Burch & Spillane, 2003; Nelson & Sassi, 2005; Spillane & Zeuli, 1999; Stein & Nelson, 2003). Setting direction requires a focus on curriculum and pedagogy, understanding that the teacher plays a central role in classroom practices (NCTM, 2000; Remillard, 2005; Senk & Thompson, 2003). It also includes the effective use of data to inform teaching and guide change. The principal has the responsibility to advocate for curricular rigor and effective teaching practices that make relevant mathematics accessible to all students. The curricula used at schools in this study advise that principals must: become knowledgeable about the curriculum, actively participate in professional development with teachers; attend to the needs of teachers; and communicate the curricular goals to parents and community (Lappan, et al, 2004). However, studies have shown that too often principals neglect their role in monitoring classroom instructional practices and curriculum implementation, resulting in large variation in both instruction and student outcomes (Elmore, 2000; Leithwood, et al. 2004).

Developing People

Once the direction is set, a successful leader must ensure that those who are charged with implementing the reforms have the skills and incentives to do so. This means providing teachers with sustained collaborative professional development with targeted support and holding them accountable to apply new learning (Ball, 2004; Leithwood, et al. 2004; Nelson, 1998, Stein & Nelson, 2003). As Remillard (2005) notes in her review of research, "the process of using a mathematics curriculum guide is complex and dynamic and is mediated by teachers knowledge, beliefs, and dispositions" (p. 239). Teachers require substantial support in constructing new mathematical and pedagogical understandings, as well as in learning to use new curriculum materials.

Redesigning the Organization

Research has shown that mathematics reform is more than using a new curriculum or book. It requires a paradigm shift for the school community to accommodate and promote the reforms (Leithwood, et al., 2004). Redesign can include reallocation of resources, such as materials and supplies to emphasize a focus on the adopted curriculum and pedagogical changes. It can also mean reworking schedules to provide time problem-solving lessons and inquiry-based teaching. It means scheduling time for teacher collaboration and professional development. For example, it may mean the reallocation of resources to allow teachers to visit model classrooms and reflect on new learning (Stein & Nelson, 2005; Remillard, 2005).

Mathematics Leadership Framework

In 2008, seeking to provide actions for mathematics leaders, the National Council for Supervisors of Mathematics (NCSM) produced The PRIME Leadership Framework (NCSM, 2008). The Framework is designed to answer the question, "what are the leadership principles, indicators, and actions that NCSM should endorse and that all mathematics education leaders should aspire toward?" (p. xi). The PRIME Framework addresses four Leadership Principles: Teaching and Learning Leadership; Curriculum Leadership; Equity Leadership; and Assessment Leadership.

PRIME highlights the interconnectedness of the four leadership principles in a lattice of action indicators that, together, describe "the conditions that must exist and the leadership actions that must be taken to move toward a deep, sustained implementation of the principles" (NCSM, 2008, p. 4). Curriculum leadership is thus contingent on equity leadership, teaching and learning leadership, and assessment leadership. I detail the Curriculum Leadership Principle here to highlight significant best practices that relate to this paper.

The Curriculum Leadership Principle states that leaders must "ensure relevant and meaningful mathematics in every lesson" (NCSM, p. 33).

1. Every teacher implements the local curriculum and uses instructional resources that are coherent and reflect state standards and national curriculum recommendations.
2. Every teacher implements a curriculum that is focused on relevant and meaningful mathematics.
3. Every teacher implements the intended curriculum with needed interventions and makes certain it is attained by every student.
4. Engages in a coordinated articulation process to determine the current status and subsequent gaps in the intended curriculum, the implemented curriculum, and the attained curriculum.

Addressing each of these indicators of the curriculum leadership principle is a responsibility that requires content and pedagogical knowledge on the part of the leader. Supervising instruction to "ensure the curriculum is coherent, focused on important, relevant mathematics, and well articulated across the grades" (NCTM, 2000, p. 3) is no small challenge. This charge was further detailed in an NCTM News Bulletin (Gutierrez, Bay-Williams, & Kanold, 2008, p. 1), stating that effective teaching of mathematics goes beyond content knowledge and management skills to include "developing and nurturing student, family, and community relationships". Supervisors must ensure that teachers "infuse their instruction with culturally relevant and engaging mathematics tasks that are rigorous yet accessible" (p. 1). For this reason, it is important to consider issues of access to a rigorous curriculum, culturally relevant mathematics and opportunities for students to express their mathematical thinking within the curriculum.

What is not articulated in the Framework is what researchers have discovered to be a serious challenge for reformers. Remillard (2005) explored the research on mathematics curriculum implementation and found that even when two teachers enact the same curriculum, the pedagogy and emphases may differ greatly, resulting in significant variation in what students learn. That teachers are "significant and active participants" (Remillard, p. 238) with the curriculum is emphasized in the research on the significant supports teachers need for developing new content knowledge for teaching (Ball & Bass. 2001; Ball, Thames & Phelps, 2007).

Leadership Content Knowledge

A third body of work that informs this study is based on the premise that principals' ideas about mathematics, teaching and learning influence how they supervise (Nelson & Sassi, 2005; Stein & Nelson, 2003). These researchers place the essential work of the principal as a leader of adult learners, who must set the direction with a concrete vision of quality classroom instruction, and as such, "they must know something of the subject under their purview – pedagogy and content (Nelson & Sassi). Similar to Schulman's (1987) notion of pedagogical content knowledge, this leadership content knowledge (LCK) connects subject matter, learning, and teaching to acts of leadership. Without LCK, "leadership floats disconnected from the very processes it is designed to govern" (Stein & Nelson, 2003, p. 446). Principals' specific understandings of mathematics learning and teaching substantially affect the nature of the instructional leadership they exercise (Burch & Spillane, 2003; Nelson, 1999; Nelson & Sassi, 2005). Their ideas about mathematics teaching and learning impact what they value and how they choose to act with regard to professional development, supervision, staffing, and setting expectations for teachers and students (Hallinger & Heck, 1996, Reys, Chavez & Reys, 2003; Nelson, 1998). How principals support teachers, supervise for fidelity of implementation, consider language and cultural issues related to mathematics, and work with parents and community are critical to a strong mathematics program (Kitchen, DePree, Celedón-Pattichis, & Brinkerhoff, 2007; Reys, Robinson, Sconiers, & Mark, 1999; Nelson, 1998). Yet studies show that principals have little knowledge of the changes implicated by reform in mathematics teaching and learning (Price, Ball, & Luks, 1995) and often rely on their teachers or outside "experts" to take responsibility for mathematics reform efforts because they perceive their own knowledge of mathematics education to be limited (Larson & Howley, 2006).

Sense-making

The fourth body of research that frames this study considers how principals' instructional leadership in mathematics is influenced by how they make sense of policy representations (state and district accountability policy, curriculum policy, etc.) and their unique school social context to construct their leadership roles (Cobb & Smith, 2008; Spillane, Reiser & Reimer, 2002). Principals are in a position to interpret and broker policy, as they hold the unique position in schools of supervising teachers and holding them accountable. Spillane, et al. (2002) suggest that how policy is enacted depends on the interaction of three constructs: 1) the implementer's cognitive structures (beliefs, ideas, experiences and understandings) related to mathematics, teaching and learning; 2) how policy representations are interpreted; and 3) interaction with the social context. To get at principals' conceptions of their roles in implementation of the curriculum, this study sought to learn about each principal's knowledge of the curriculum and the reforms (LCK), as well as how each interpreted district policy related to the reforms and how they navigated their unique school contexts.

The existing research has contributed greatly to what we know about principals' beliefs and ideas related to mathematics, teaching and learning. However, I have found no research that extends the construct of leadership content knowledge to include the Curriculum Leadership Principle in mathematics reform efforts. The PRIME framework suggests that these are essential principles for leading mathematics reforms to improve teaching and learning for every child. To add to the body of knowledge on principal leadership in mathematics, this study included a focus principals' conceptions of their role in the implementation of a district-adopted reform mathematics curriculum. How do they use their leadership content knowledge to set a direction for curriculum, to develop the abilities of teachers to implement the curriculum as intended, and to redesign the organization to support the reforms?

RESEARCH DESIGN

Methodology and methods

This is a qualitative cross-case study (Creswell, 2007) of three school principals at predominantly Hispanic-serving schools. Through a series of semi-structured interviews, classroom observations in which I accompanied the principal, conversations with district personnel, and teacher surveys, I studied principals' conceptions of their roles in leading the mathematics reform efforts at their schools. Principal interviews were designed to elicit their ideas and beliefs about: the subject of mathematics; the pedagogy of mathematics, including experiences that contributed to their own understanding of mathematics teaching and learning; how to support their teaching staffs with implementation of the new curriculum; how to improve opportunities for their Hispanic and English Language Learner (ELL) students to succeed in mathematics; and how to broker between the accountability expectations and mandates from the district and state and the classroom practices of their teachers. Classroom observations accompanying the principal during mathematics lessons provided a basis for deeper discussions of teaching and learning, and helped to elicit their ideas about what they observe and how they supervise.

Research Site

The Las Palmas School District where this study takes place serves community of 23,867 with 8,528 students enrolled in the public schools. The district was selected for three reasons. First, for the 2007-2008 school year, the district adopted National Science Foundation (NSF) supported, research-based reform mathematics curriculum at each grade level: *Investigations in Number, Data and Space* (Investigations) for grades K-5 and *Connected Mathematics* (CMP) for grades 6-12. During the first year, the district provided initial training from the publisher for all teachers, and during that year, teachers had monthly professional development meetings focused on mathematics facilitated by academic coaches. The second year, the schools were on their own. It is important to note that elementary schools in the Las Palmas district served K-6th grades. Thus, principals were responsible for supervising both Investigations and CMP.

A second reason for choosing Las Palmas is the demographics of the student population, 63% of whom are Hispanic, compared to 53% statewide. Two of the schools where I situated my study were in poor neighborhoods located outside the town limits, with 100% of students receiving Free and Reduced Lunch (FRL). In addition, over 80% of the students served in these schools were Hispanic, many of them from Mexican immigrant families, which means many of the students were ELLs.

A third reason for selection of this district was its focus on data-driven decision-making. This was important to the study because assessment data had become such a big driver in the accountability movement (NCLB, 2001), with implications for how principals brokered the reforms. The district had implemented policies that included monitoring individual student performance in mathematics and performance by teacher related to students' mathematics achievement using the MAP short-cycle assessments (NWEA, 2008). As part of the focus on collecting data, there was also a newly enacted district requirement for principals to document classroom observations that included a place to record curriculum fidelity. This meant I was able to talk with principals about their conceptions of the importance of data vis-à-vis their leadership of mathematics teaching and learning, as well as gather data on their conceptions of the district expectations for implementation and fidelity of the newly adopted curricula.

Participants

The three elementary school principals were intentionally selected because of their sites and their varied administrative experience. Two principals had only been in their school leadership positions a year and a half when the study began. The third was a veteran principal who had been at her school since it was build eight years prior. These schools served some of the highest numbers of low-income Hispanic and ELL students in the district. One of the schools had not met AYP in mathematics of reading for 5 years, and was in "restructuring", meaning they had specific sanctions and state pressures to improve. Figure 1 displays the demographics and mathematics proficiency for each of the schools.

	Tomasita Elementary School	**Camino Real Elementary School**	**Sands Elementary School**
Principal/years at site	**Mr. Torres** – 2nd year (Previously AP Mid School	**Ms. Passos** – 2nd year (previously AP Tomasita)	**Ms. Rojas** – 8th year (18 years as principal)
Grades	Pre-K – 6th (was Pre-K – 4th until 2007)	K-6	K-6th (was 5th-6th until 2007)
# Students	509	658	480
% Hispanic	84%	66%	82%
% ELL	50%	1%	60%
% FRL	100%	65%	100%
AYP status 2009	Did not make AYP in mathematics	Have made AYP each year (Only made by lower confidence interval in 2009)	Restructuring 2 (Have not made AYP 4 years) Made AYP in math 2009
SBA math scores	All students: 38% proficient Hispanic students: 35% proficient ELL students – 37%	All students: 45% proficient Hispanic students: 42 % proficient ELL: 14% proficient	All students: 43% proficient Hispanic students: 42% proficient ELL students – 34%
Bilingual math teachers	One at each grade level, some math taught in Spanish, some in English	One at each grade – teaches in English with ESL support	One at each grade level, teach in Eng. with some Spanish support

Figure 1 The participants and sites

Mr. Torres

Mr. Torres, at Tomasita Elementary, had been a 5th and 6th grade classroom teacher for seven years. He had taught in predominantly Hispanic-serving schools, where some of his students were ELLs. Although exposed to Spanish as a child, he was not fluent. He enjoyed teaching mathematics and described himself as "the one who volunteered to test out the reform materials in his classroom." He had been an administrator for three years, first as an assistant principal. He was the assistant principal at the feeder middle school before coming to Tomasita as principal the year prior to this study, so he understood the challenges at the middle school. Tomasita was 100% FRL and 50% of the students were considered ELL. At the time of this study, Tomasita had not made adequate yearly progress (AYP) in mathematics for three years. However, he noted with pride that the students had made more than the expected progress in mathematics on the 2007-2008 Standards Based Assessment.

Ms. Passos

Ms. Passos, in her second year as principal at Camino Real Elementary when the study began, had been an assistant principal at Tomasita for two years prior. Her teaching experience was in Kindergarten, yet she had a strong mathematics and science background and felt "fairly comfortable – an 8 out of 10" with reform mathematics. She is Hispanic, herself, but she admitted that she spoke only conversational Spanish that she learned from her New Mexican grandmother. Although the student body was 66% Hispanic, only a handful of students were considered ELL, in fact, not enough to be considered a "sub-group" for AYP determination. They had made AYP each year at the time of this study, however their ELL students performed considerably below Anglo children in mathematics and reading.

Ms. Rojas

At the time of this study, Ms. Rojas had been a principal for 18 years. She opened Sands Elementary in 2001, as an intermediate 5th-6th school. Sands became a K-6 school in 2007 years ago, thus the principal's leadership role had changed to include supervision of the earlier grades. 100% of Sands students qualified for FRL, and sixty percent were considered ELL. Sands had not made AYP for 5 years, and at the time of this study was in Restructuring, meaning the school had strict requirements for their Educational Plan for Student Success, as well as requirements for research-based interventions for struggling students. Prior to becoming a principal, Ms. Rojas's teaching experience was in pre-school and Special Education. She described herself as "one of those kids who didn't get math". She is Anglo, and was not bilingual, but she attempted to place a bilingual teacher at every grade level.

Data Collection

I conducted five individual interviews with each principal, three focus group interviews with two to three of the principals present, and four classroom visits with each principal between September of 2007 and May of 2009. All interviews were taped and transcribed.

Additional data to triangulate information from the principal interviews and conversations came from a teacher survey that asked about their attitudes and beliefs about the curriculum, and about the involvement of their principal in their mathematics teaching. Also, I attended district teacher professional development related to mathematics, and participated as the "knowledgeable other" for a Lesson Study team from each of the three sites. This allowed me to hear teacher discussions of their thinking about student problem-solving and to observe their interaction with the curriculum materials. To gather information on the district policies and expectations, I conducted informal interviews with the district mathematics coordinator and the bilingual coordinator.

Data Analysis

In the first round of analysis, I employed "selective open-coding" (Emerson et al. 1995, p. 155) of each interview and set of fieldnotes. After open coding, I moved to "axial coding" (Creswell, 1998) to consolidate codes and look for themes. This was done on an EXCEL spreadsheet, which allowed me to sort and classify coded data. In the second round of analysis, I created a grid framework using both the *PRIME Leadership Framework's* four domains down a vertical axis, and the three leadership roles of setting direction, developing people and redesigning the organization across the horizontal axis. Particular quotes/coded comments were then placed into intersecting domains. For example, where the PRIME Curriculum domain intersects with the Setting Direction domain, I placed Ms. Passos' comment about focusing more on standards than on specific curriculum (see figure 2). When I moved again to open coding, the development of subsequent themes allowed me to align principals' conceptions with the literature.

Ms. Passos	Teaching and Learning	Curriculum	Assessment	Equity
Set Direction	• What standard are you teaching to? • More small groups	• Focus on standards over curriculum • Curriculum important for weak teachers • Ambivalent about fidelity for "successful" teachers	• Improve student proficiency on MAP	• Any student not proficient on MAP must have math intervention • Make teaching relevant
Develop People	• Collaboration • Lesson Study • Professional development Plan	• Want teachers to seek training opportunities • No models	• Train teachers to data-drill for gaps in teaching	
Redesign Org.	• Math blocks • "Balanced math" says 45 minutes for CMP lesson	• Begin to look at student work from SBA • No materials in Spanish	Begin to look at student work	Hire bilingual teacher at each grade level

Figure 2 Sample findings for Ms. Passos using PRIME Leadership categories

FINDINGS

In the larger study, analysis of the data suggested that principals' conceptions about curriculum reform were deeply connected to their conceptions about the subject of mathematics, the other Leadership Principles of teaching and learning, assessment and equity, and their sense making about both district policy and the social context of their schools. The analysis presented here focuses particularly on conceptions of leading curriculum reform, but is necessarily inclusive of the broader role.

I begin with a brief analysis of each principal's leadership content knowledge in mathematics. Then, I present findings on their conceptions of the implementation of the reform curricula. Cross case findings will be included in the discussion.

Mr. Torres – LCK

Mr. Torres was driven by a strong sense of urgency about student success. "We can't send only 17 out of 70 kids proficient to the middle school!" A theme that permeated Mr. Torres' ideas about effective teaching for optimum student learning of mathematic was his belief that mathematics must be relevant – connected to students' lives. This notion of relevance was deeply intertwined with his beliefs about equity and opportunity to learn. He spoke of connecting to students' known experiences as critical to helping students construct meaning from prior experiences.

> You've got to become aware of your students, what they know and reference your math back to that. You need to present problems in a way that relates to where they're from. That's my biggest challenge here is trying to get these teachers to understand where they are in reference to the kids. What do Tomasita kids know? Where are they from? You want to find their easiest reference point that's around them all the time. So cars are big out here. If it's geometry, you start talking about a car – 20-inch rims and how much gas they can get with 5 bucks (2-11-09)?

Mr. Torres leaned more toward a problem-solving philosophy. He wanted students to have access to manipulatives, to real world problems, and to working in groups to find reasonable solutions to those problems. He challenged teaching that was strictly procedural, as in the following excerpt:

> I had one teacher who was hard last year. He was strictly an engineer and became a teacher. He had major discipline issues because, as I kept telling him, "They don't understand your board work. They don't understand the overheads. You have to get them in groups and start explaining to them in small groups, 'cuz if you get up and lecture – these are seventh and eighth graders trying to figure out pre-algebra! (9/7/08)

While he believed that student learning was the result of high quality instruction, and that teachers needed to employ questioning techniques and make curriculum relevant for students, Mr. Torres acknowledged that teaching mathematics was complex and challenging for teachers. He clearly stated his own affinity for mathematics problem-solving in the teaching of mathematics, but Mr. Torres professed, "It's hard to teach math . . . to really teach a kid to understand why. They (all teachers) can teach kids how to memorize, but concepts are harder". He believed that teachers were challenged by not knowing enough mathematics themselves and by their lack of management skills required to effectively run small groups and give students autonomy for inquiry.

Mr. Torres recognized that the district support for teachers to learn the new curriculum and pedagogy was insufficient and that his academic coach did not have sufficient background in mathematics teaching to support his teachers. Relying on his own sense of success as a mathematics teacher, Mr. Torres felt it was up to him to emphasize math and “compensate for our coaches not having as much math training” (2/11/09). In this role, Mr. Torres usually worked one-on-one to direct teachers to try new strategies. “I told her to teach math in Spanish because her kids weren’t getting it in English,” and “I told her I wanted to see more small group work.” Thus he relied his own mathematics teaching experience in supervising teachers.

But above all else in supervising teachers mathematics instruction, Mr. Torres made decisions about their successful teaching and held them accountable for improvement through the use of short-cycle assessment data. The majority of his interview comments contained some reference to MAP data. Improved student proficiency on the MAP, coupled with the active engagement of students in the mathematics lessons, was sufficient indication to Mr. Torres that teaching was successful, and, as discussed below, it even overpowered curriculum fidelity. He exuded a strong sense of agency related to understanding and using data to set targets for improvement, a leadership skill that aligned well with the district and state accountability demands. Therefore, his role conception was that he train his teachers to look at their student data, to drill into the data to find areas and standards where there are “teaching gaps”, and address those gaps with their teaching. If the data suggests that the teaching is not producing the desired growth, Mr. Torres take on a sort of teaching role with the teachers by “telling” or “showing” or “asking” teachers to try a new strategy with their teaching.

Mr. Torres and Reform Curricula

While Mr. Torres believed in the need to improve mathematics teaching to address student learning through challenging and relevant curriculum, He admitted to little knowledge of the adopted curriculum. “On a scale of 10 I’d say I’m a 3.” Thus he received the district policy messages about the elementary mathematics curriculum (Investigations) and the middle school curriculum (CMP) with ambivalence. Throughout the eighteen months of interviews, Mr. Torres never committed to a belief in full implementation. On a number of occasions, he confirmed that teachers who were fully implementing the curriculum showed greater gains on short cycle assessment scores. But he also spoke of excellent teachers, getting high scores, who “did a little bit of everything to meet the needs of the kids.” He saw students and teachers getting excited and feeling successful with Dots and Grids and Number Literacy, the programs that the district had brought in to supplement the reform curricula. Some of his vacillation was evident in the following remarks:

> My fear becomes, if you let them go that direction [choosing their own materials], you can let good teachers do that and they can find their way, but the ones that aren’t strong, you lose everything that CMP or Investigations is really honing these kids to do. That’s my fear that you go back to that place where kids are getting 20% on a standardized test (2/11/09).

But he indicated that this differential treatment created some challenges in supervising teachers. He felt that a strong teacher could use any curriculum materials and be successful, while a weak teacher might benefit from the guidance of the reform curriculum. However, this left him with no models for the weaker teachers. In describing his ambivalence he stated,

> She [the strong teacher] does Investigations, but she does everything else, too, whatever she sees the kids need. Other teachers see her doing it and they want to do that, too. But they can't. They should just teach Investigations because they don't know how to teach math in the first place. They don't understand it conceptually in their heads (4/8/09).
>
> If you're looking at your teachers, it's like differentiated instruction for your kids. I mean, for each one of my teachers, I'm looking at implementation in a different way (6/2/09).

Ms. Passos LCK

To be successful in mathematics problem-solving, Ms. Passos believed that students needed to be able to think more conceptually. In fact, she held this belief about teachers as learners, too:

> The most rewarding experience for me was working with adults who were learning math. My educational assistant was going to school to get her teaching certificate, and she was struggling with math. She came in one day and said she just didn't get what all those little numbers meant (describing exponents). So I pulled out the Unifix Cubes and told her to pick a number – kind of small because I didn't have that many Unifix Cubes. She picked eight. So we made a square that was eight by eight. I asked her what it looked like. She said, "A square. . . Oh, that's why they call it eight squared!" Then I told her to make a shape that was stacked eight high. I asked her what shape that was. She said, "A cube. . .Oh, that's why they call it eight cubed!" It was so rewarding to see the learning for a grown person who's gone through life thinking she couldn't do math 9/14/07)

She could see the connections between Kindergarten pattern activities in mathematics and high school algebra. This episode with an adult learner demonstrates that she understood mathematical "big ideas" (Ball, 1999), and that she believed that all individuals construct meaning based on what they know. When explaining her beliefs about what students need to know and be able to do in mathematics, she stated, "They need to demonstrate their understanding with write, draw and discuss. They need to be able to reason and communicate." (9/24/07). When she observed in a mathematics classroom, Ms. Passos expected to see students working in groups, able to discuss their thinking, and writing about their mathematics journals. "I'm fine if the teacher is modeling to the whole class some of the time, but there should be a lot of group time and problem-solving work." She related a particular observation in a first grade classroom.

> The student math journals in this classroom were incredible! One little girl, who the teacher identified as having middle ability in math, had written, "My dad had 10 motorcycles. He bought some more and now he has 20 motorcycles." Then, below that, she had written the math sentence 10M + ___ = 20. Isn't that fantastic! The teacher shared that with her parent. (Interview 9/14/07)

The teacher's role, one that Ms. Passos looked for in observations, was to be clear in her mind about "what her objectives are and what she is focusing on. "That helps eliminate confusion for the students" (focus interview, 4/2/09). On one set of classroom walk-throughs where we visited two different sixth grade mathematics lessons, she pointed out differences she noted in student engagement and understanding. She noted how students in the second classroom were more engaged and were talking together about the mathematics "because the teacher had her objectives for student outcomes so the students were able to articulate to her what they were learning."

While a strong advocate of constructivist teaching in general, and a problem-solving or inquiry approach in mathematics in particular, over the two years I spent with Ms. Passos, she also revealed that she was not completely comfortable with letting students struggle through misconceptions with mathematical thinking. One example was when she explained in a March 2009 interview that she had actually taught a CMP lesson so one of her teachers could attend professional development. She noticed one boy did not seem to understand.

> "I've got to tell you, the hardest part that day – it was so hard – was not giving the kids the answer. I didn't anticipate that it would be so hard. We preach guiding questions, but when you look across the room and you see little Dillon with that look of confusion . . . so I have to admit. I cheated. I took him aside and I retaught number lines until he understood it" (focus interview, 2/11/09).

Ms. Passos and curriculum fidelity

"As a good Las Palmas [district] principal, I should say you have to have fidelity [to the adopted reform curricula]. But in the pit of my stomach, I don't really believe that. I think you have to have fidelity to the standards" (4/2/09). Ms. Passos rated herself an eight on a scale of one to ten in her knowledge of the CMP and Investigations curricula. She was able to recognize which teachers were doing a CMP or Investigations lesson during walk-throughs. She also was able to quote information off of the CMP and Investigations websites. She was able to use that information in arguing that one curricular program may not "hit all the areas." While she believed her teachers needed to use the adopted curricula, she also argued that one program may not "hit all the areas." In fact, in the following quote, we see how Ms. Passos combined her understanding of the district's guidance for a balanced mathematics program with her knowledge of the curriculum.

> If you go to their website it says that prior to using CMP it is expected that the kids have a broad base in numbers and operations before they start CMP. So the expectation is that they already have those skills firmly in place, which may not be the case. And the second thing is, like the triangle that [the math coordinator] gave us, it does have to be balanced math so we have to make sure, whether the program provides it or not, that we are hitting all those areas (focus interview, 2/11/09).

As these comments indicate, Ms. Passos believed her teachers should be using the curriculum, but she was not uncomfortable with the idea of modifying the lessons or supplementing with other materials. She felt Camino Real students needed more support. While she believed that CMP and Investigations were excellent curricula in the hands of a good teacher, "There are materials that are in better alignment with your standards-based system than others, but I haven't seen anything that 100% fills all the holes" (focus interview, 2/10/09). She also recognized the problems if teachers did not fully implement.

> You know what the deficiency is [when implementation isn't successful]? They're not playing those games. They're ignoring the games and the games are what they [the trainers] said are going to teach the basic math facts and give students that reinforcement and our teachers are not doing it (3/11/09).

Ms. Passos identified a need for "a strong district curriculum influence" in sustaining the momentum for the reforms because "there are still enough teachers that aren't comfortable with it and complain. But for some teachers, doing the curriculum by the book would be a huge step in the right direction" (6/2/09).

In these comments and others, there was a tentativeness reflected in Ms. Passos' conceptions of leading curriculum reform. This was due in large part to the social context she entered as principal two years prior. "Teachers here don't feel a sense of urgency to change because they met AYP" (2/7/09). The social context was complacent and there was not sufficient incentive for teachers to face challenge of learning a new curriculum and new pedagogy. Therefore, Ms. Passos was pleased that there were district messages about curriculum fidelity that were prompting at least some of her teachers to implement.

Ms. Rojas LCK

Important to understanding Ms. Rojas mathematics leadership conceptions was the fact that she never taught mathematics in a traditional education classroom. Her experience was in early childhood and special education. In fact, her mathematics history was limited. She expressed a lack of confidence in mathematics when she described herself as "that kid in the back of the room that couldn't see how math applied to anything" (interview 9/12/07). She struggled with how to support teachers in mathematics because she had little experience herself with teaching mathematics, and she was not familiar with the reform curricula. Yet she had some very specific ideas about what should happen in the mathematics classroom to engage students. After observing a Lesson Study lesson where students became stuck trying to convert feet to inches, she felt the teacher should have done more to support the students' thinking by activating prior knowledge about measurement and reviewing.

> I like to see teachers' ability to ask the right questions to get kids to get them thinking for themselves. Not just showing them how to do it or giving them the answer, but really being able to provide that guidance for them to actually investigate and learn. Before the lesson there needs to be pre-teaching for any new concept, and it has to be shown visually (4/3/09).

In her observations of mathematics lessons, she stated that she looked for student engagement, higher-level problems to challenge all students, manipulatives, and teacher modeling. She also expected teachers to be using mathematics vocabulary. The specific vocabulary she looked for was from "several states' standards based assessments" which she found on-line. She made grade level copies for all teachers and even for the librarian and the coach so everyone could be modeling the words.

Ms. Rojas believed that the strongest teachers of mathematics had a core of mathematics skills as a foundation and are able to make mathematics relevant for their students. She related a series of successful mathematics lessons a teacher had done using baseball scores, because her student were interested in baseball.

Ms. Rojas and curriculum fidelity

From the first interview in the fall of 2007 until the last interview in the spring of 2009, Ms. Rojas continued to express her lack of knowledge about the mathematics curricula.

> I know I don't know Investigations. That's why it was so important for me to get an academic coach for math that knows the program and knows how to work with the program to fill the gaps. I haven't been involved and practicing it, and you have to be practicing your craft if you're going to do it (4/3/09).

When asked how many of her teachers were teaching with the new curriculum, she suggested I ask her mathematics coach because "I sure wouldn't know" (4/3/09). According to the coach, only about 30% of the teachers were attempting to fully implement the program. Whether in spite of or because of her lack of familiarity with Investigations or CMP, Ms. Rojas felt that teachers who taught the summer intervention program should be asked to use Investigations, because "If they don't already use it, they're not comfortable with it, and they will become more comfortable as they practice" (4/3/09).

Although not familiar with the CMP and Investigations curricula, Ms. Rojas listened to her teachers' thoughts and concerns. She echoed their complaints about how difficult the programs were to use, "Teachers say it takes too much time to make materials and copies (4/3/08)". She was concerned because, "The teachers say Investigations doesn't align with the standards. For example, 'predictions' is on the third grade SBA but not in Investigations" (6/12/09). And she was convinced that the curricula had gaps. "Teachers have said that because they had training in Dots and Grids and the other program, it has filled in gaps in Investigations" (4/3/09). In sum, it appears two issues contributed to her inability to fully support the adopted reform curricula. The first was her lack of familiarity with them. The second was listening to her teachers concerns – the very teachers to whom she attributed little skill in teaching mathematics. It is also likely that the district's introduction and support of supplementary programs like Dots and Grids and Number Literacy served to dilute the policy representation of a "mandated" curriculum. Because the district trained teachers those supplemental programs and because they were easier for teachers to learn than the reform curriculum, they felt they had "permission" to choose what they taught and when.

CROSS-CASE ANALYSIS AND DISCUSSION

In keeping with the three characteristics of effective leadership identified in the literature (Leithwood, et al. 2004), cross case findings on leadership conceptions related to curriculum implementation are discussed here organized according to how principals conceive of their role in terms of setting direction, developing people, and redesigning the organization (figure 3). As discussed earlier, one of the intentions of this district, and many others, in adopting a reform curriculum was to regulate mathematics teaching practices in an effort to raise student achievement levels (Remillard, 2005). However, the way these principals brokered the reforms, at least in the first two years, has implications for how districts might consider developing leadership to support the reforms.

	Teaching and Learning	Curriculum	Assessment	**Equity**
Set Direction		Don't know curriculum. Hard to create vision.		
Develop People		If curriculum isn't implemented school-wide, there are no models		
Redesign Org.		Time for teachers to collaborate		

Figure 3: Sample of organization of data for Curriculum Leadership Principle

Setting the direction for the reforms

Setting direction for implementation of a mathematics reform curriculum includes identifying and articulating a vision for the reforms, fostering the acceptance of group goals for the implementation, creating high performance expectations for teaching and learning, and promoting effective communication to assist in the development of shared purpose. There were three key findings related to how principals conceive of their roles in setting the direction for the curriculum reforms.

The first key finding was that the principals had limited familiarity with the curriculum, and thus did not have a vision for either the intended or the enacted curriculum. Principals could not actively promote that "every teacher implements the intended curriculum" (NCSM, 2008, p. 33) in part because they lacked deep knowledge about the goals, the lessons and the pedagogy. They expressed ambivalence about asking "successful teachers" to change. Principals tended to mediate district curriculum policy through their own beliefs and understandings. Mr. Torres believed that there were many ways to successfully teach students relevant and rigorous mathematics, and the reform curricula were important supports for struggling teachers. Ms. Rojas did not know the curriculum at all, and thus relied on teachers' impressions and comments to inform her beliefs. She was not able to hold teachers accountable for implementing the curricula because she was not familiar with them. Ms. Passos knew the curricula fairly well, yet she expressed a different goal in setting direction for her teachers. She expected that teachers would teach to the mathematics standards, and that no curriculum could be comprehensive in that goal. In addition, she struggled with a school social context that was resistant to learning new pedagogy, which impeded her ability to promote the paradigm shift that was required to enact the curriculum as intended.

The second significant finding related to setting direction was that, in spite of district policy mandating the use of a single curriculum, principals had a great deal of discretion with the interpretation of the implementation policy. The district adoption of the reform curriculum, the initial training of all teachers in implementation, and the purchase of only the new curriculum materials sent a message to all that there was an expectation to move in the direction of the reforms. All of the principals articulated a vision to move in the direction of greater implementation, yet promoted curriculum implementation according to their own beliefs and abilities. Mr. Torres remarked that the superintendent had asked in a principals' meeting for each leader to indicate the degree of implementation of the reform curricula in their schools, and the general trend was 60%. One must also consider that what principals reported depended on their understanding of full implementation. Were teachers doing all of the lessons? Were the lessons done with fidelity? Were they supplementing the curricula? Each principal expressed that the district-wide focus was helpful in providing the press for teachers to reform, but, as we saw with Mr. Torres, if MAP scores were good, the principals did not generally press the teacher to be true to the curriculum.

The third key finding related to setting direction was that each principal tended to conceive of leading the reforms from their particular area of strength in leadership content knowledge. The reform curriculum was not an area of strength for any of them, and therefore none of them sent a strong message about curriculum fidelity. But each principal had had a particular expertise or focus that they emphasized with teachers. For Mr. Torres, the emphasis was on improving student proficiency on the short-cycle assessments. If a teacher used blended curricula and students became proficient, that was acceptable. He For Ms. Rojas, the emphasis was on student engagement. If students were engaged in the task, using manipulatives, the mathematics lesson was good. And for Ms. Passos, the goal was that students became proficient in the standards. She perceived the reform curricula as excellent for getting most of the job done, but lacking in areas that teachers would have to supplement. These beliefs had implications for how these principals conceived of their role in developing the skills of their teachers.

It is important to note that, although principals did not make direct statements to connect curriculum beliefs with equity beliefs, all of the principals emphasized the need for teachers to make mathematics lessons relevant for their students. Mr. Torres used many concrete examples in guiding his teachers, like referring to skateboarding when explaining 360 degrees in a circle. Ms. Passos was excited about a teacher whose lesson on tessellations integrated with art. Ms. Rojas looked for pre-teaching that "connected to students' lived experience". Also significant is the fact that none of the schools had student materials in Spanish. Only Mr. Torres indicated that he thought there were 20 student books in Spanish in the school. These findings are discussed in more detail in the larger research study.

Developing the teachers

Developing teachers means increasing teachers capacities to implement the vision and goals of the organization by offering intellectual stimulation, providing individual support and providing appropriate models of best practice (Leithwood, et al. 2004). Developing teachers also means holding teachers accountable for new learning (Nelson & Sassi, 2005).

The district had a vision that the reform curriculum would be implemented and thus did initial training of all teachers the first year. But within the three schools, the district's message was mediated by the principal, and ongoing teacher support and accountability were the responsibility of the principal. All three principals were committed to the belief that teachers improve their practice in professional learning communities and to bringing them out of the isolation of their own classrooms. They each ensured that teachers had weekly collaboration times and had their academic coaches facilitate the discussions. Their role in setting direction for curricular reforms influenced their ideas about how the reforms were advanced through these learning communities. Interestingly, the principals stated that they rarely attended collaboration, leaving the facilitation to their academic coaches, who were not strong in mathematics. Only Ms.

Rojas had a mathematics coach, who admitted that teachers often skipped her collaboration meetings.

During the second year of implementation, professional development was turned over to the individual schools. Ms. Rojas hired a mathematics coach for in-house support. Ms. Passos encouraged teachers to seek their own professional development when it was offered from outside experts. And MR. Torres took on the role of teacher of teachers, giving them suggestions and instructions on how to improve their mathematics teaching by using more small group work and making their instruction more relevant to students' lives. None of the in-house support was specific to Investigations or CMP unless teachers themselves chose to collaborate on that.

The principals also realized that teachers who struggled with teaching mathematics needed models. They needed opportunities to observe strong teachers in action (Ball, 2004; Spillane, et al, 2002). A significant finding related to supporting and training teachers to implement the reforms was that principals came to see that they had very few teachers who could model Investigations or CMP in its fully intended form. For Mr. Torres, this was so significant that near the end of the study, he began to rethink his ideas about requiring even the strong teachers to do full implementation. "If I send a weak teacher to observe Ms. M., she pulls from here, there and everywhere, and that weak teacher will never be able to follow her" (6/2/09). Ms. Passos felt that her teachers could only receive professional development in Investigations and CMP from outside trainers, and encouraged them to seek such training.

Redesigning the organization

Organizational conditions must support and sustain the vision for curricular reform. For mathematics, specific practices include strengthening a culture of inquiry, providing time and resources to support new ways of teaching, modifying scheduling and infrastructure to accommodate the reforms, and building collaborative processes. Three significant ideas: Inquiry practices were evident, but principals did not specifically address the reform curricula. Ms. Passos and Mr. Torres talked of leadership practices that modeled an investigative approach with her staff. Ms. Passos promoted looking together at student work in staff meetings. Mr. Torres had his teachers drill into their short-cycle assessment data to find teaching gaps and think about ways to address those gaps. All three principals worked to ensure that there was time in the schedule for weekly teacher collaboration, but none of the three attended those collaborations to promote new ways of teaching. Interestingly, over the two years of the study, both Mr. Torres and Ms. Passos came to conceive of the importance of their presence at teacher collaboration to set the direction of the discussions.

All three principals were very responsive to teachers' requests for materials needed for mathematics instruction. Because they had pockets of teachers who were implementing the reform curricula, they came to learn of a number of resources that they needed to budget for, including overhead projectors and transparencies, copies, and student white boards and markers.

In terms of modifying schedules, all three principals spoke of ensuring a 75-minute block for mathematics. Yet Ms. Rojas said teachers often could not "fit that into their schedules". Interestingly, all of the principals were happy with a lesson plan that included 45 minutes for the Investigations or CMP lesson, and the rest could be devoted to Number Literacy or other materials the teacher chose to use. Both Investigations and CMP call for a minimum of 60 minutes for the lesson (Lappan, 2004).

A significant element of the redesign across the three schools was the effort to focus on data. Because short cycle and SBA data served as policy to set such a clear and measurable direction for schools, the schools devoted tremendous resources to assessing student proficiency and interpreting the data. Principals provided teachers with graphs indicating their students' progress toward proficiency in mathematics. Mr. Torres spoke with every class and had students look at their data. Ms. Passos and Mr. Torres had teachers drill into the data to find teaching gaps.

CONCLUSION

The three principals in this study were all deeply invested in improving mathematics teaching and learning for all of their students. They looked for teaching that was rigorous and relevant, based on the rubric of their own beliefs, experiences and understandings about mathematics, teaching and learning. They varied in leadership content knowledge. Ms. Passos had a strong mathematics background, understood the "big ideas" of mathematics teaching and learning, and believed that teachers learned about pedagogy by looking at student work together. Ms. Rojas had very little leadership content knowledge in mathematics, and knew nothing of the curriculum, so she evaluated good teaching based on student engagement MAP scores and teacher comments. Mr. Torres expressed a strong sense of agency both in interpreting short cycle assessment data and as a mathematics teacher. He felt that good mathematics teachers could improve student proficiency with a variety of curricular materials and focused his supervisory comments on instructional strategies like small groups and giving students relevant examples. At the end of the study, almost two years into implementation, he was beginning to conceive of the need to implement the reform curriculum at scale because he needed teachers to be able to support one another.

This study gives greater insight into the challenges of moving the reforms to scale, even in individual schools. This window into principals' conceptions about the curriculum implementation indicates the need to consider principals' leadership content knowledge, including curriculum knowledge, in reforming mathematics teaching at scale. Too often, accountability mandates to raise student achievement levels drive districts to adopt reform curricula (Remillard, 2005) without consideration for what support principals need to develop their leadership content knowledge and to make sense of the reforms. This study suggests that districts that attempt to scale up reform through "mandating" the use of a single curriculum need to consider how to support principals to implement all of the PRIME Leadership Principles in order to set the direction for mathematics reform, develop their teachers to implement the reforms, and redesign their school organizations to facilitate the reforms.

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Policy-Reactor: Harold Asturias

WHAT: CEMELA-CPTM-TODOS Conference
WHO: Harold Asturias (University of California-Berkeley
PLACE: Tucson, AZ
DATE: March 6, 2010

TRANSCRIPTION

Harold Asturias

Pedro's (Portes) paper in the abstract has this text that talked about the policies. How the policies push Latinos here to "conform to local notions of literacy grounded in monolithic assumptions of what it means to be an 'American.' Their exceptionality and potential for additive bilingualism is ignored." So I'm here to say I'm an American and hopefully you recognize that as America because I was born right there, right? (Powerpoint slide of globe with arrow pointing to country in Central America.) So, our notions of American…?

What I want to do today is make an if, then statement. Simple, hopefully. Not that the problem is simple. Not that the issue is simple. I'd like you to spend thirty seconds of my precious time talking to each other. Talk to your neighbor. Define to each other what is policy? Just talk to each other for thirty seconds and think about and define for each other what is policy. (Pause as the group talks amongst themselves.) As you do this try make sure that you give each other a picture of what a policy is. Can you create a picture in your mind what policy is? (Pause)

Ok, let me grab your attention back. I know that I didn't give you enough time but I think that one of the first thoughts I had was that we all have different pictures of what we mean or different meanings when we say policy, right? And probably some of you have the picture like this. (Points to a picture of a mask on the PowerPoint presentation)(People Laugh) Because it's a bit scary. And I think that's what it reminds me of because it was Maria's idea that you have to be able to take risks and have courage to address the issues of policy. There are several kinds of policies. I'm not going to define them all but you can imagine from the words there (PowerPoint slide) the kinds of policies that there could be. Substantive versus administrative, vertical and horizontal, top down versus collaborative, reactive and proactive policy, and current and future policy. What I would like to talk about is this issue of a public policy. A policy that seeks to achieve a desired goal that is considered to be in the best interest of all members of society.

And therefore, the question I think that we need to think about or that I try to address is what's the goal of schooling children? What's the goal of educating Latino, Latina students? And do public policies support those goals of educating and teaching Latino, Latina students. So the presentations today help us think about that and it's so good that we have some answers.

Pedro said that the solution is the policies that dismantle group-based inequalities, that we need policies that are more enlightened. Many that we have to pay attention to additive bilingualism and are based on the education and practices that might mediate or counteract the unfortunate intellectual histories and identities that shapes us all. Because the policies that, as they are, shapes us, and shapes our Latino/Latina students. Right? So that's easy. Make policies like this, that take, that work out from an (un-deficit?), move from an un-deficit model to this and then we'll have better policies. Better, better then. Barbara highlighted for us the issues that principals face in what she, helps us define as the leadership content knowledge. And that the policies that focus on accountability in a way that pushes for higher scores ignoring what it means to implement good curriculum, or curriculum that promote the learning of mathematics of Latino/Latina students, then may not support the goals of (incomprehensible) Latino/Latina students, particularly in mathematics. Makes sense?

Then she said we have to think about leadership as setting direction, developing people, and redesigning the organization. And redesigning the organization was a thing that we heard throughout in Debbie's (Valadez) presentation, in Maria's (Santos) presentation, and in Norma's (Torres Martinez) presentation. Because many, most of what they were dealing with is within the constraints of their mandates and the policies, they're trying to redesign the organizations in order to provide the support that English learners need, or, Latinos need. Yeah?

Then Debbie talked to us about the big challenge that she faces, laws that mandate that constrain her, what she knows is good content knowledge, leadership content knowledge. And how she has to make decisions and have to put in place structures in her school that within those constraints are provide the best opportunity to put her best for her students. And she highlighted for us the very limited policies, one of the end results of the policies that is very limited for mathematics in the school.

Then Maria talked about implementing policies, or putting in place policies that are guided by good principles, right? Remember, I'm not going to repeat all of the principles that she showed but if you remember that her slides had very good principles. And if you start with good principles then those policies, if they're policies having formed and based on those principles and what research says, then they're going to support those goals. Again this theme of (incomprehensible), working from an un-deficit model, there is, I think, consistent all across is what we need to, one piece that we need to pay attention to, and I like when she said, we must begin with the end in mind. What's the end? What's the goal?

Norma then described some of the policies, some of the pieces that are being put in place in Texas. She talked about the funding resources, funding being put in place so that the resources can be developed and implemented. She talked about every professional development must include the ELPS (English Language Proficiency Standards). Did I get it right? Okay. Then, instructional materials are being developed with the ELPS, the standard for English language proficiency in mind. And how the state's trying to think about using online tools. Sharing and develop (incomprehensible). And talked about the

Texas Curriculum Focal Points as a version of the NCTM Focal Points. And trying to use or make it, at least make accessible, all those resources that NCTM provides.

So I want to modify the questions a little bit to say the questions, I think that you may think that I'm wrong but I think that one question we have to ask ourselves is what's the goal of schooling children? Because I think we all have different ideas of that but we need to think about, and particularly for us, what is the goal of teaching mathematics to Latino/Latina students, and then speak to the last questions a little better for what we're going to do in the small groups to, so what do we do to influence policies that support the goals of educating in mathematics, (incomprehensible) Latino/Latina students.

So I'm hoping that a little bit of stimulating to thinking about the next step. Because we have a lot of knowledge, right? We are now through all of the sessions that we've gone and all the presentations, we know a lot of things but we need to think now. How do we take the next step? And how do we create conversations that go beyond the choir, because I call up the choir and we need to engage others who are making decisions also.

And then think about the nested constructs about policies being in place in, at the school level, at the district level, at the city level, at the state level. Which is kind of what Debbie and Maria and Norma were giving us is about all those contexts, without forgetting that the real work, if you want to call it real, happens at the classroom level, inside there, right? And you can also expand on the other end to go to the national level, but I didn't include both of those but we have to think about it in that kind of nested version of contexts.

So context and content knowledge apply through good guided principles to answer the question about what are our next steps, and that's what I'm hoping that we can together continue to discuss.

Thank you.

Policy Discussion

Following the research presentations, the practitioner panel and the reactor comments, the participants met in small groups of six to eight for discussions. The groups included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The connections question became embedded in the discussions of the other questions. This summary represents common themes identified within and across the working groups.

What do we know?

From the presentations by the practitioners panel it is striking to note that there exists a wide range of how ELLs' education is legislated from state to state. The question of whether bilingualism is valued seems to be the determining factor on whether there are additive or subtractive language policies. What becomes quite clear is that current research on best practices for educating ELLs is not reflected in the legislation in many states.

> It is crucial that educators provide a socioculturally supportive school environment for language minority students that allows natural language, academic, and cognitive development to flourish in both L1 and L2, comparable to the sociocultural support for ongoing language, academic, and cognitive development that native-English speakers are provided in school (Thomas, W. & Collier, V.).

The committees that are formed to work on state legislation often have little representation by members with ELL expertise and are motivated by political agendas driven by populace beliefs. A National Study of School Effectiveness for Language Minority Students' Long-Term Academic Achievement by Thomas and Collier clearly identify the success of bilingual programs. However, mainstream beliefs hold that bilingual programs are substandard and do not teach ELL students English. The underachievement of Latinos is often attributed to deficits in English language proficiency. However, many Latinos/as are not ELLs but are behind academically because of poverty. The same can be said for African Americans and Native Americans (Portes, 2010). A continuation of these current practices has no benefit to our nation and will perpetuate a historical system that results in a population of unskilled labor and unemployment.

Adding to an inequitable education system is the current competitive system for federal funding. Funding is not being allocated according to need. How funding is distributed is determined by proposals meeting constraints set by the federal government. However, whether these are best practices as defined in research are questionable. Evaluation of students continues to be tied to single tests as the only measure of success.

Current policies have often focused on teachers' failure and placed more expectations and responsibility on teachers but usually lack support for professional development and additional time needed for teaching, planning, etc. Some states not receiving federally funded programs are also placing the same requirements on their teachers without the needed supports. In addition, teachers in some states are in a quandary as how to best instruct ELL students given legal restrictions.

Fair identification of ELLs is also a concern (Portes, 2010). Some states' language inventories identify English only speaking students as ELLs because there may be another language spoken by someone in the home. The identified ELL students are segregated into special classes to learn English. These students are then required to "test out" of the ELL categorization by demonstrating English proficiency in reading, writing, listening and speaking. Other English speakers are not required to take these tests and, dependent on the age level, may not be able to pass them.

Also, there are very well organized movements in the country that are influencing our legislative and judicial systems with studies that are based on insufficient or erroneous data regarding the education of ELLs as well as mathematics curriculum and education. ELL data in particular needs to be disaggregated into levels of English language proficiency, special needs ELLs, and ELLs with interruptive formal education rather than producing over-generalizations from the data that describe all Latinos or all ELLs. Educators and researchers have not been well organized to counter these groups, however, the influence of these movements are resulting in the denial of civil rights to students and teachers.

What are the implications for practice and research?

There is much work to be done in order to change the current political climate. It is imperative that proactive steps be taken immediately to stop current trends in the Southwest from spreading. Legislators and decision-makers must be educated about: misconceptions of language acquisition; the complexity of language (dialects, etc.) and culture; bilingual education and economic impact; and mathematics language and education. There needs to be well-organized groups in every state that take on the mission of educating the Council of Chief State School Officers (CCSSO), STEM caucuses, and legislative staffs that are responsible for writing these policies. These groups need to involve all stakeholders affected by education policies.

Policy briefs based on research studies need to be written that describe best practices for mathematics education, bilingual education, and ELL education. (See more information and examples at: http://ucaccord.gseis.ucla.edu/publications.) These briefs need to be specifically written for different audiences (policymakers, educators, parents, researchers, and general population). These briefs need to be written, organized, and distributed through a network of organizations working together.

The guiding principles of quality bilingual education and effective mathematics curriculum, instruction and assessment need to be popularized with the general public through a focused campaign. This will inform and empower families and communities to demand these programs for their children. Data regarding harmful practices that result in lack of success among Latinos, e.g. low graduation rates, need to be shared with communities.

Meanwhile, it is essential that teachers and administrators be well informed on current legislated policies in order to identify what is allowed and not allowed under the law. Administrators should be required to collaboratively develop an in-depth understanding of these policies and design steps to effectively support the implementation of these policies. Families and communities need to be thoroughly informed of their legal rights regarding these policies and given information on programmatic choices for their children. The business community needs to also be more informed on the effect of current policies on graduation rates and their resources tapped to support litigation.

Current laws in some states should be challenged on the basis of civil rights, equity and discrimination. National groups such as the National Latino Education Research Agenda Project (NLERAP) and Mexican American Legal Defense and Education Fund (MALDEF) have the expertise, experience and manpower to lead an effort in this regard. Due to challenges of state education policies related to teaching English learners, Texas has systematically addressed the educational needs of English learners by including English language acquisition standards into their core subjects and all professional development in the core subjects include ELL standards. Teachers of ELLs are supported through summer academies and have follow-up professional development through online platforms that are organized into professional learning communities. Texas has purchased state licenses to give teachers access to ELL and NCTM materials on-line.

With strong leadership at different levels, policies can be formed and supported to result in the educational success of Latinos. For example, in the New York City Schools, Maria Santos, Executive Director, New York City Department of Education, used the following guiding principles to lay the foundation of the educational experiences of students:

- high expectations and deliberate theory of action
- keen focus on academic content
- design and enact practices based on the characteristics of the students and their communities
- monitor student progress and adjust or modify action for results
- hone the capacity of staff to enrich academic language development and mathematics concepts and skills

- practice purposeful collaboration among multiple levels, including teachers, parents and students.

After implementing these principles that guided many aspects of students' experiences, the ELL population has made steady gains and significant improvement in meeting the standards on both state English language arts and mathematics tests over a period from 2001 to 2008. Former ELLs are actually outperforming students were not ELLs (Santos, 2010).

In today's political climate, qualitative data is often discounted as being too complex or having too small of a sample size and failing to make connections to larger situations or possible causes. Quantitative data on the other hand may over generalize because it forces people into categories that might not fit. We need to find a way to incorporate elements of both quantitative and qualitative data in order to maintain the essential results identified by both methodologies to better inform educational decisions and policies for Latinos.

Education currently is under an assimilation model for success. ELLs are perceived from a deficit view. The education system needs to change to address a sociocultural system.

> We argue that through the development of additive bilingualism, positive wider identities can be more fully developed that benefit both the integration of a positive cultural identity at the individual level and also the societal best interests. (Portes, 2010)

Currently, marginalized students' are silenced or ignored. A Children's Bill of Rights would be appropriate for all children, especially for Latino children.

In order to meet the requirements of No Child Left Behind state departments currently screen and approve programs. However, the criteria are not transparent. Transparent criteria for review of research and programs need to be developed through TODOS: Mathematics for ALL, and then TODOS could issue a seal of approval, becoming a clearinghouse for research and programs benefiting Latinos.

Changes need to be undertaken in our higher education institutions. Mathematics and education departments need to be more closely connected. Our higher education institutions need to develop agency in youth and college-aged students. Goals of diversity education and developing a deeper understanding of cultures through required service learning should be implemented. Preservice and inservice teachers need to have more community experiences where they are in the position of being the learner, gaining insights into the culture and recognizing the funds of knowledge of the community (Moll, et.al., 1992). Teachers need to receive needed support for this involvement.

Misconceptions of how mathematics is learned are also widespread in our society. Resources need to be invested in public relations beyond the education circle for the purpose of building an understanding that mathematics is experientially learned, and that there is utility in good mathematical thinking skills and mathematical ways of looking at the world. The myth that the language of mathematics is universal needs to be discredited as learning mathematics requires communication beyond the use of symbols. Mathematics has its own academic language and discourse.

Partnerships with organizations that share similar concerns need to be built. Partnerships must also be formed with businesses that have common interests in the need for changing current educational practices and can offer support through resources. Schools with high success rates for Latinos need to be identified and their practices examined. Current systemic supports for schools need to be identified and utilized.

What questions do we need to research further?

We identified the following questions as needing further research and investigation:

- What is the impact of subtractive language policies on ELL students? What impact does segregating ELL students into English Language Development classes have on the ELL student—how are their content scores affected? What are the long-term consequences of these policies?

- How are teachers who are working under legislated restrictive language conditions impacted?

- How can we transmit to the general populace a positive attitude towards bilinguals?

- What is the impact on special education students who are mis-labeled as ELL and placed in ELL programs?

- What factors influence how Latinos/as identities are formed? (This study needs to focus on the diversity within the Latino/a population.)

- What are the effects of current policies on mathematics education of Latinos?

- What is the impact of policies and practices that affect Latinos' graduation rates as well as reports indicating the educational successes and failures of Latinos?

- How do we identify, enact, and support good policies at multiple levels (school, district, state and nation)?

- What can we learn from disaggregated quantitative studies on the educational achievement of Latinos/as that attend to and analyze the achievement of the diverse Latino population?

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CEMELA-CPTM-TODOS

Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Transforming Mathematical Identities Through After School Settings

Section 8 of 9

Chair: Aria Razfar, University of Illinois - Chicago

Tucson, Arizona March 4 -6, 2010

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Mathematics for ALL

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A SITUATED VIEW AT "SCALING UP" IN CULTURALLY AND LINGUISTICALLY DIVERSE COMMUNITIES: THE NEED FOR MUTUAL ADADAPTATION

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"Al llegar a otra cultura, quitate las sandalias porque la tierra que pisas es sagrada" ("On approaching a new culture, take off your sandals because the ground you step on is sacred")

(Quote embroidered on a Peruvian tapestry)

INTRODUCTION

This paper describes the long and deliberate process of weaving a community's intellectual resources into a theoretically informed bilingual/bicultural after-school activity that shares much of same reverence for culture as depicted in the Peruvian tapestry above. Recent accounts of what constitutes successful efforts at scaling up emphasize the importance of taking into account the ecology of the new setting (Coburn, 2003; Hubbard & Mehan, 2002). Unlike more conventional perspectives that conceive "going to scale" as a matter of replicating an innovation in a variety of new settings, this new perspective considers the sociocultural resources of ethnic minority communities as foundational to its goals and conceives scaling up as a complex and multifaceted process of mutual adaptation in which participants "adapt reform models to the needs of their local contexts" (p.4).

Below, we provide an account of a small-scale approach to this new type of "scaling up." We focus on *La Clase Mágica* (Vásquez, 2003), an educational innovation that is responsive to the histories, cultures, goals, and interests of diverse communities by utilizing their intellectual resources as tools for learning and development (e.g., Moll et al., 1992). Specifically, we describe our experience adapting its bilingual/bicultural, intergenerational, and collaborative approach to learning and development, to a new cultural community within the American Indian Reservation of San Pasqual in Southern California. By highlighting instances in which the adaptation process took on an entirely different developmental trajectory than the earlier efforts at Mexican origin communities, we hope to provide similar educational endeavors with valuable insights into "scaling up" in culturally and linguistically diverse communities.

In 2009, *La Clase Mágica* (LCM) was comprised of a cross-system collaborative effort involving five local organizations jointly coordinated through a partnership between the Center for Academic and Social Advancement (CASA) and the University of California, San Diego (UCSD). These five community-based systems, distributed across San Diego County--four in predominantly Mexican origin communities and one in an American Indian reservation—target the academic performance of minority youth by building on the knowledge and experience they bring to the learning setting (Vásquez, 2003). Recognizing that minority students often struggle in school because of a mismatch between the cultures of the home and school (Au & Kawakami, 1994), *La Clase Mágica* sought to improve school success of minority youth by developing a participatory pedagogy that capitalizes on the socio-historical context of households and other community resources (e.g., Moll et al., 1992). Such community and family-based "funds of knowledge," reflecting different sociocultural environments, mold the learning dispositions of children from diverse ethnolinguistic backgrounds and, when integrated into curricular and pedagogical innovations, support indigenous students' literacy development (McCarty & Watahomigie, 1998). Thus, the "processes of everyday life of the target populations rather than on observable markers of folklore that assume that all members of a particular group share a normative, bounded, and integrated view of their own culture" (González, 2002) forms the foundation of *La Clase Mágica*. That is, *La Clase Mágica* integrates into its multicultural education aspects of the lived culture of the community.

Despite these concerns, we also understand that promoting substantive change is vital to the productive progression of education, and we therefore must assist in its process. For example, one relevant finding of our efforts demonstrates that elements that are resistant to change in the school context show great malleability in the context of our situated approach to serving minority communities. In our projects, "teaching and learning," what Elmore (1996) describes as the aspects of schooling that are the most resistant to change, are most easily expanded to include alternative forms of knowledge, instructional strategies, and contexts for learning (e.g., the Internet, the playground, peer/peer interactions) in after school settings. We argue that if the host institution and the community take an active part in the design, implementation, and evaluation of the innovation, buy-in, the most critical factor of successful sustainability and transplantation, is possible. According to McLaughlin (quoted in NCREL, 2003), "a 'mutual adaptation' process where the need for fidelity is balanced with the ability of local actors to adopt specifics of the model to local circumstances is the most likely to succeed" (p.3).

In this short article, we highlight the accomplishments and challenges we faced during two distinct stages of the adaptation process and the richness these provided in understanding our approach to educational reform from a situated perspective: 1) The first phase focuses on negotiating entrée and adapting the *La Clase Mágica* model to serve First Nation peoples, a process that involved developing reciprocal relations of exchange that equitably met the goals and expectations of the three partnering institutions—UCSD, CASA[1] and the San Pasqual Educational Center; 2) The second phase began in 2007, when the research team, building on the strong partnership that had developed across the past eight years, introduced a new curriculum centered on the acquisition of 21st-century skills. This second phase shifted the adaptation efforts on the development of a new globally relevant curriculum, and, most importantly, elevated the project to a new level of expression as a situated approach to educational reform. We begin with a brief sketch of *La Clase Mágica* and then discuss the challenges and accomplishments we experienced in developing a strong and mature collaboration across the two distinct phases.

LA CLASE MAGICA: A DESIGN EXPERIMENT IN SOCIAL ACTION

Theoretically and organizationally, *La Clase Mágica* is a social and cultural adaptation of an after-school educational activity called the Fifth Dimension. Like its predecessor, it links university and community institutions in a partnership to funnel resources to local children at a community-based computer club. In the case of *La Clase Mágica,* the focus is on meeting the social and intellectual needs of Spanish-speaking children from Mexicano communities in Southern California (Vásquez, 2003).[2]

Following Vygotsky's theories on the role of play in learning and development (Vygotsky, 1978; Griffin & Cole, 1984), the Fifth Dimension artfully mixes play and education to promote the literacy and cognitive development of elementary school-aged children. The combination of fantasy and skill-building activities give life to a culture of collaborative learning in which learners direct their own development through the guided assistance of adults—typically, UCSD undergraduate students, more knowledgeable peers, and a magical electronic entity known as the Wizard (Nicolopoulou & Cole, 1993). Successful with Anglo, English-speaking children, the Fifth Dimension proved unable to recruit or retain local bilingual children to participate in the

[1] The Center for Academic and Social Advancement/Centro de Avance Social y Academico (CASA) grew out of the efforts of the community to continue support for the initial site of *La Clase Mágica* in 2000. Today, it supports the implementation side of the initiative in consort with the University of California that provides the research and educational expertise.

program, despite team members efforts to overcome the linguistic and cultural barriers these children faced (Vásquez, 2003).

Bilingualism and biculturalism became the norm of *La Clase Mágica*, the fledgling innovation of the Fifth Dimension, and the remediation of widespread and persistent under-achievement of minority youth in K-12 education and their under-representation in higher education (Stanton-Salazar et al., 1995; Vásquez, 1996) became its mission. To accomplish this, *La Clase Mágica* focused on enhancing the academic and social development of local learners by building on their multiplicities in language, culture, and intellectual resources. It partnered directly with community members with the goal of developing an integrative collaboration that would give rise to a new consciousness among all participants-- a consciousness that grew out of mutual relations of exchange in designing a community-based program that was as much a product of the university as it was of the local community. Language and identity were approached from a constructivist position which views language choice and identity as socially constructed in the interactions among participants within the social sphere and/or community. Thus, close attention was given to why, when, and how child participants not only draw on particular languages and cultures in the learning process but also how they use the cognitive, social, and cultural resources provided by the adults to accomplish intellectual tasks. As the adaptation process unfolded, local and outside knowledges were carefully and systematically woven into the design, content, and symbolic structure of the project, and how it impacted the developmental trajectories of the participants and supporting systems alike (i.e., the family, the community, and public institutions) (Vásquez, 2003; Relaño & Vásquez, 2007).

On a typical day at LCM, minority children from low performing schools gather during after-school hours at their community/educational center to use computer and telecommunication technologies to play computer games and to enjoy the company of undergraduate students enrolled in a UCSD practicum course and other adults including parents and community volunteers. Child participants take the role of “actors” (Bruner, 1990), making choices and completing tasks in a bilingual fantasy world. Bilingual “task cards” or guide sheets that detail the goals and objectives of each segment of the activity facilitate communication between the primarily Spanish-speaking children and the mostly English-speaking undergraduates. Following the code-switching styles of the community, the two target languages are intermingled in the task cards to allow children to use their prior experiences as links to new knowledge and to support the university students to scaffold their meaning-making processes.

El Maga, the magical electronic entity of *La Clase Mágica*, and its pesty friend, *La Mosca Cosmica* (The Cosmic Fly), also build on the literacy practices of the children by engaging them in real-time, computer-based online chats after each activity or when they need to share their personal concerns. *El Maga’s* identity and gender remain secret to add to its allure and mystery as it represents the only “authority” in the system (Vásquez, 2003). Supported by a bilingual environment where either English or Spanish are used interchangeably, *El Maga* helps promote a fantasy world in which the children can imagine themselves as masters of their own destiny, and in particular masters of their own development (Vygotsky, 1978). In this constructed bilingual world, children, as Vygotsky asserts, “are able to see beyond themselves in play,” a significant experience for minority students who often ascribe, voluntarily or by imposition, to problematic identities rising out of the clashes of mainstream and minority values. Their electronic pal, thus, mediates their sense of self as well as language choice and engagement (Vásquez, 2003).

To create such a world in the day-to-day life of each of the sites, *La Clase Mágica* draws on five principles that reflect the situated and participatory nature of our innovation: 1) a recognition perspective in research and practice that draws on the cultural resources of the local community; 2) a focus on multiplicity and diversity; 3) a commitment to deliberate and continual change; 4) a direct partnership with community members themselves rather than the institution; and 5) a multi-generational approach to teaching and learning (see Vásquez, 2003, for greater detail). Over the last 20 years of research, we have found that these principles serve as the framework from which the adaptation of *La Clase Mágica* is negotiated to respect the cultural values and richness of our partnering communities, engendering a uniquely culturally relevant project that recognizes and reflects the needs and resources of the target community.

In stepping onto a "sacred ground"—i.e., entering into collaboration with a new cultural community such as San Pasqual —*La Clase Mágica* gains new meaning and a new purpose. A new research/implementation team is assembled to streamline the philosophical and pedagogical foundations of the program to the new cultural context and, together with local participants, a new program with its own unique cultural perspective and institutional goals is established. Every segment of the local system is designed to work together to reinforce the ongoing orientation and commitment to diversity and to the academic advancement of ethnic minority children. Ideally, community members who share a common interest in the well-being and education of the community and its children participate in this new formulation. They are given opportunities to assume leadership roles in designing and implementing the program on equal par with their university and institutional counterparts. Long-term sustainability is accomplished when individual members of the community take up the project's goals and objectives, and the community and the university come to accept each one another as valued and respected partners.

THE CHALLENGES AND ACCOMPLISHMENTS OF STEPPING INTO SACRED GROUND: PHASE I

The partnership with San Pasqual Reservation Educational Center, which serves a population of about 750 San Pasqual Band of Mission (Kumeyaay) Indians and is located 50 miles from the university, began in 2001 when Vásquez joined a UCSD team involved in a collaborative initiative to connect the 19 San Diego tribal communities with high-speed Internet. The San Paqual Educational Center is considered the heart of the community. Eight years hence, collaboration has been slow, deliberate, and evolving but highly instructive in promoting our growing expertise in designing culturally and linguistically relevant educational environments. Our growing knowledge of indigenous Kummeyaay culture and the extent to which the native language, *Ipaii* was present—and surprisingly absent—in the community, fueled a delicate relationship-building that worked at stripping away historical tensions between the Kumeyaay people and both the University of California, San Diego and the Mexican origin community—two entities Vásquez represented.

When the new adaptation of *La Clase Mágica* program opened in the Spring of 2002, it did so as TACKLE, an acronym developed by the Center staff to express the fundamental characteristics of the developing program: "technology," "culture," "Kumeyaay," "literacy," and "education." For our San Paqual partners, the name TACKLE signified a willingness to succeed in the midst of a history of oppression and marginalization. The name also represented their self-determination for tackling the challenges facing a threatened language and culture and a desire to recover an ancestral repository of knowledge and identity.

As TACKLE settled into its own physical and symbolic position within the reservation's existing after-school program and began to reflect the needs and resources of its constituency and the constraints of its host institution, challenges to the core principles of *La Clase Mágica* became evident. Crossing cultural borders to partner with community members, for example, became a prominent challenge not only because of the long distance to the site and the insularity of the Center during site hours but because the cultural borders were wide and historically grounded in broken promises and forced displacement. These factors made it difficult to work directly with interested parents and community leaders not associated with the educational center and even harder to develop an adequate adaptation of *La Clase Mágica* that reflected the cultural and linguistic realities and objectives of the community. Several obstacles surfaced in meeting the community's need for multiplicity and diversity in the curriculum: 1) the linguistic reality of the San Pasqual Reservation called for a tri-lingual approach involving English, Spanish, and Ipaii languages, 2) the approach called for language *recovery* rather than language *revitalization* as anticipated by the research team, and 3) full linguistic resources were typically unavailable and when present were part of ritual language or ornamentation. Consequently, the adaptation process aspects of *La Clase Mágica* that had been useful among Mexican origin communities became ineffective and oftentimes, obstructive.

Although challenging, these new understandings highlighted two other theoretical constructs— *participatory collaborative activity* and *context embedded change*—that guide the adaptation process. Both of these constructs support our cultural relevant approach and, we believe, have led to establishing a sustainable collaboration with the Educational Center at the San Pasqual Reservation. Both have been critical for crossing cultural borders between the research team and target communities, , but in the context of the linguistic and cultural complexity of San Pasqual, helped re-direct the goals and objectives of the adaptation process to focusing on the "intangible cultural heritage" of the Kumeyaay. The focus on language, which has important implications for communication, identity, social integration, education, and development, help set the stage to initiate strategies to safeguard a living heritage of cultural expressions and practices (UNESCO, 2009). Thus, these two guiding principles of the adaptation process opened the possibility by which the Kumeyaay cultural heritage could be protected and kept alive if given time and attention.

Participatory Collaborative Activity

In the context of the original *La Clase Mágica,* participatory collaborative activity meant that the research team was basically conducting research in its own cultural community. Although not all members of the research staff were of Mexican descent or even residents of the community in which *La Clase Mágica* emerged, they shared a close affinity with the local language and culture. This sense of kinship facilitated access to the families and to high-profile members of the service areas, making it possible for community members to become integral to the design and implementation of the project early in the developmental process.

At San Pasqual, this approach to adaptation was met with seemingly insurmountable obstacles of distance and lack of access to the physical and cultural resources of the community. Building a robust collaborative relationship without sufficient knowledge of the community's "practices, representations, expressions, as well as the knowledge and skills, that communities, groups and, in some cases, individuals recognize as part of their cultural heritage" (UNESCO, 2009) was a tremendous challenge for the research team. The cultural practice of holding back from readily entering into binding relationships with outsiders held up the acceptance of the project . Parents or other possibly interested individuals were also out of reach during TACKLE sessions, leaving the site coordinator, who was a member of the research team, to keep our presence alive, albeit if only for three-four hours a week. Thus for years, collaboration across the partnering institutions settled on the participation of one to two UCSD undergraduates every academic quarter and on adapting the curriculum when resources were available, as was the case when a *La Clase Mágica* coordinator and former UCSD student joined the staff at the Educational Center.

From the perspective of the research team, our initial efforts can rightfully be considered culturally naïve, at best, and our curriculum material far from being easily adaptable into the Kumeyaay language and culture. These obstacles prompted the team to become informed on the community's practices and traditions and to establish a continuous presence on the reservation. Thus, it was decided that Marcello, at the time a team member and graduate student in Anthropology, would spend six months attending cultural classes and community celebrations at the Educational Center. During this time, she collected Ipaii vocabulary commonly found in storytelling sessions and found posted on the walls throughout the center. She became familiar with iconographic representations of Kumeyaay culture, while at the same time giving the project visibility in tribal community. Signs of progress towards this goal became evident in small but significant expressions of acceptance. Marcello, for example, began to receive spirited welcomes at the cultural classes and, when the site opened almost a year later, the Center staff made a suitable location available for the program. The staff also permitted greater flexibility in the use of the technology and gave the children greater freedom of expression and movement during the TACKLE session. This meant amending the structured environment in which Center activities were held—i.e., the policy of using headphones to avoid disrupting others during computer play sessions was discontinued during session to allow collaboration between the children and the undergraduates. Children were also allowed to wonder freely within the computer room to communicate with other "friends" [undergraduate student helpers] , an important element of problem-solving and self-directed learning. Another sign of acceptance was the award of extramural funds by the Tribal Council to develop and maintain a close working relationship with community representatives. The UCSD team reciprocated by committing time and effort to accommodate the Center's concerns, in particular the development of a "*culturally sensitive module*" to raise awareness among UCSD personnel of the proper protocol for entering a sovereign nation. Additionally, readings on American Indian educational values and pedagogical approaches were integrated into the course curriculum, and students who volunteered to participate at TACKLE were given separate training on the module.

Thus, in spite of the many challenges, success of our participatory collaboration was evident in the measure of "ownership" that took root at the San Pasqual Educational Center. The first sign was when a *La Clase Mágica* coordinator, who was married to a tribal member, moved to the reservation and was hired to manage TACKLE. The "maze", the organizing mechanism that structures the path of the children's activities at site (see Vásquez 2003 for details), was re-designed to one of the most iconographic tribal representations—a woven flat basket. Later, more fundamental tribal knowledge and culture were integrated into the curriculum material (e.g., the task cards) when an American Indian student, formerly a student of Vásquez's, was hired as an assistant to the coordinator. These developments exhibited budding success towards participatory collaboration, although, a fully grounded collaborative participation was still years away.

Context Embedded Change

We also found early in the adaptation process that to serve non-Spanish-speaking learners required a distinct sociocultural perspective that was unfamiliar to the research/implementation team whose knowledge of American Indians was, for all practical purposes, based on narrow and stereotypic depictions found in literature and media. This inexperience was particularly evident in the attempt to interlace the symbolism of *La Clase Mágia* with the local language and culture. Although community representatives expressed a desire to have their native language integrated into the program's materials, access to the linguistic resources of the community required new understandings. First, the team was not readily given access to "the legacy of the tribe"—i.e., their full linguistic resources—to learn the language and conduct the adaptation. This was a moot issue because few if any native speakers of Ipaii lived within the reservation; symbolically, however, it meant that the charge of language recovery was a tribal right and responsibility.

The Ipaii language, like almost all of the remaining 50 Indian languages in California, is close to total extinction, counting only on a few isolated elderly speakers who guard it as a birthright. As a consequence, Ipaii is not available at the reservation as background knowledge for the children, the team, or the community itself. However, given the concern of the potential impact of language loss on traditional culture and that "learners who are not taught their language and culture often face problems integrating into the more positive aspects of mainstream culture" (Fishman, 1996), educational leaders allowed members of the research team to incorporate the Ipaii language and Kumeyaay culture into TACKLE after-school activities. Team members thus assumed the responsibility of designing the curriculum by both integrating the language that was readily available, as well as seeking translations of terms tribal youth could easily adopt. It became obvious that rather than revitalizing a threatened language, the team found itself collecting discrete bits and pieces of decontextualized terminology and cultural references.

Curricular adaptation no longer entailed using the native culture and language as bridges to new understanding; rather it meant using English to bridge to the community's native resources. It meant re-translating the Spanish-English materials used in *La Clase Mágica* back to English only and then incorporating a sprinkling of Ipaii terminology commonly found throughout the environs of the reservation. Incorporating the Ipaii language in the taskcards was done on a trial-and-error basis until the right amount was found not to hinder those individuals with little or no contact with the tribal language.

The *sprinkling method* of the adaptation process began by including "basic words" in Ipaii, such as colors, numbers, and short phrases within an English-based text. Although these words were commonly found posted throughout the community, even this small amount of native language, was too cumbersome for the children to absorb at one time. This forced the re-translation of many of the terms back into English and integrating Ipaii in a more progressive fashion outside the norms of standard code-switching. The objective was for children to master a set of vocabulary, then few more names of objects in Ipaii and native symbols, enclosed in parenthesis, were to be gradually incorporated throughout the guide sheets written primarily in English. The introductory paragraph of a task card (Sticky Bear, a computer game), the segment that situates the game within the child's cultural background experience, illustrates the sprinkling approach:

> "Remember when you were little *('estik)* you *(maa)* would ask your mom *('ememaa)* and dad *('epepaa)* … "How do you say…"? Well, now you (*maa)* are in school! You're learning a lot of words. Teach your friend (*unyeway*) all the words that you (*maa*) know by repeating the words in this game."

In the same fashion, terminology of the reservation's landscape and American Indian culture and history were integrated wherever possible in the curriculum materials (i.e., the maze, task cards, the electronic entity, etc.). The Center staff also helped to rename elements of the *La Clase Mágica* model according to their own mythology and iconography. For instance, *El Maga* and *La Mosca Cosmica* were renamed *Shuulluw-Shuuluk* (Thunder and Lightning).

Despite the challenges, the adaptation process underscored two important insights: 1) support of the Ipaii language entailed recovery efforts rather than revitalization as had been at the original *La Clase Mágica,* and 2) there is no substitute for the critical role that community members play in the success of the project. Both insights signaled a need for greater involvement by both partners and greater funding for the site, conditions not immediately available. Thus, the adaptation basically came to a halt, and collaboration across the two partnering institutions settled on an administrative relationship of keeping the site open and assigning one to two undergraduates per academic quarter. More intense collaboration did not begin again until the fall of 2007 when *La Clase Mágica* embarked in a new phase of adaptation and renovation of the whole program to address the social and educational realities of the 21st century, and TACKLE was suggested as the language and culture site.

TACKLING GLOBAL ISSUES: PHASE II

In the fall of the 2007-2008 academic year, the concern that minority youth were suffering the double jeopardy of not being fully prepared to ascend the educational ladder or engage in the social and economic realities of the 21st century prompted *La Clase Mágica's* research team to reconsider its emphasis from success in school to success in a globalized world. The new information age and ubiquitous advanced technologies called for new competencies and new responsibilities (Suarez-Orozco & Baollan Qin-Hilliard, 2004); however, minority youth were not only suffering a digital divide in terms of access to communication technology, they were also experiencing a cognitive gap in the way in which computer technologies were used for management rather than to enhance their intellectual capacities (Warschauer, 2002). Explorations into what type of pedagogical interventions could be feasible and attractive in an after-school setting occupied the team for much of that year. In collaboration with Vásquez's undergraduate course, "Education and Global Citizenship," the team settled on five 21st-century skills" (www.21stcenturyskills.org) involving digital and financial literacy, language and culture, environmental protection, and health and nutrition as vital assets for effective 21st-century citizenship (see www.sdcasa.org).

The following academic year, a team composed of UCSD undergraduate students, coordinators, and post-doctoral researchers with different levels of expertise in globally relevant areas of knowledge was formed to develop a new global curriculum to meet the needs of the sites and target communities. Readings on globalization and education were integrated into the undergraduate practicum course and, at site, task cards began to incorporate activities focusing on the five targeted skills required of an effective global citizen—i.e., the ability to think analytically about issues that span across disciplinary contexts, to synthesize knowledge from multiple disciplines, and to cultivate understanding and appreciation of other cultures and traditions.

The introductory paragraph of a newly developed task card on an Internet game related to health and nutrition (Food for Thought website) illustrates the weaving of the founding principles of *La Clase Mágica* with those of the new 21st-century curriculum;

> "Did you know **que hay comida muy rica** but at the same time very healthy! Did you know **que en tu plato de pescado, arroz y frijoles que hace tu mami hay muchas proteinas** that will make you super smart and as strong as superman? Enter into the world of the food pyramid **y discubre** what YUMMY foods **son buenos para ti**. **¡¡¡Que rico, ya empezamos!!!"**

By year's end, the team had crafted a more elaborate conception of the curriculum under the umbrella of a larger initiative called "The Hubs of Innovation (HOI)." Theoretically, the adaptation was conceived to transform the goals and objectives of *La Clase Mágica* from a horizontal dissemination of academic knowledge and skills that prepares minority youth to succeed in school to a horizontal and vertical interconnection of knowledge and skills that prepares future citizens for the exigencies of a globalized world. In close collaboration with our community partners, each of the after-school sites was "globalized" into a hub of expertise and knowledge that focused on one of the five essential 21st-century skills—i.e., Language & Culture, Digital Literacy, Financial Literacy, Health & Nutrition, and Environmental Protection (for details, see www.21stcenturyskills.org). To increase the buy-in and involvement of the host institution, each skill was carefully matched with the interests and resources of the local community—e.g., the host institution of the site that became the Financial Literacy Hub was part of an extensive network of financial power elite within the community, while another became the Health and Nutrition site because the recreation director was majoring in nursing and homeopathy at the nearby state university. Each hub was conceptualized to locally generate knowledge and expertise on its target skill and then, using new information technologies and a series of social networking activities, share it with the other four hubs in the system. Participants would have opportunities to gain an in-depth understanding of an area of knowledge while at the same time they learned to use multimedia technologies to share their knowledge with the other sites on the HOI Online Community Network (demo available at www.casasd.org/hoi/index.html). A select portion of the subjective and locally generated knowledge would also be disseminated, consumed, and reformulated outside of the HOI environment.

Given the desire of the San Pasqual's Educational Center staff to hold onto some part of the *Ipaii* language, TACKLE became the host of the Language and Culture Hub. Giving greater prominence and coherence to the scattered bits of language found on the walls of the Center and the guide sheets was a very attractive proposition for the reservation's educational staff. They welcomed a deeper adaptation of the curriculum materials that could bridge the intergenerational gap found among many of the residents of the reservation and instill in their younger members inquiry skills, community involvement, and the concomitant academic skills of reading, writing, and bilingual development. They had no illusion of anyone gaining fluency in the almost extinct indigenous language; their goal was to expose it to Native youth so that they could maintain it in memory. They welcomed the task of designing new activities to recover as much as possible of their own ancestral repository of knowledge and identity and the possibility of learning from and possibly informing the recovery efforts of other threatened languages. They were also eager to interweave the knowledge of the native vegetation of the area—i.e., the names of the plants (in Ipaii) and their use in Kumeyaay tradition —they had acquired in a collaborative effort with researchers from the conservation branch of the San Diego Zoological Society, an initiative the research team was also interested in re-instating.

The trusting relationship that developed over the eight years of collaboration with the San Pasqual Educational Center staff strengthened our culturally relevant approach to scaling up, and during the second phase of our collaboration, its two building blocks—participatory collaborative activity and context embedded change—gained further viability. Participatory collaborative activity expanded beyond the close relationship that was maturing between the research staff and the Educational Center staff to be more closely involved with the developments that were taking place in the UCSD practicum course, and context embedded change deepened with the shared goal of uncovering new cultural heritage and integrating it into the TACKLE curriculum. Both of these principles were invigorated when a reservation-native coordinator of the after-school activities was hired and given extra time to collaborate with the UCSD team on revising the TACKLE curriculum. At UCSD, the new curriculum focus was well-received by the undergraduate students who were already immersed in digitally driven lifestyles. Undergraduate students who had taken the practicum course in previous quarters and had developed strong bonds with the children and community members at TACKLE were recruited as part of the Innovation team as salaried staff or for course credit.

The continuous presence of the students at the reservation was fruitful in many ways. It allowed the team the confidence and the time to engage in long and in-depth conversations with reservation staff members and thus gain a frame of reference for the community's "processual approaches to culture" (González, 2002). Access to daily life activities provided access to the processes involving historically accumulated "funds of knowledge" (Moll et al., 1992) that households possess. For example, in a conversation with an elder and cultural facilitator at the Educational Center, the team learned that a respect for the land, the veneration of elders, and the importance of gift-giving to new acquaintances whom one is hosting, among others, were core values the community wanted to pass on to future generations in the after-school curriculum.

The more engaged participation of the research staff at the community also highlighted the impact that the historical oppression of the Kumeyaay and the diaspora that followed after the eviction of the Kumeyaay people from their ancestral lands had on language and culture loss. As put by a reservation elder, "When they kicked all the people off their land, they scattered everywhere; some to the cities and some to other places" [Fieldnote, BD 05/07/09]. To gain more substantial knowledge of the linguistic reality of the reservation community, a "parent-guardian" survey was developed by team members in collaboration with the San Pasqual staff. [3] The survey investigated the linguistic and cultural resources at all generational levels of the home environments and also had the added benefit of effectively strengthening the participatory collaborative progress that had been made in Phase I of the initiative. For the first time in eight years, access to parents who provided vital information was made possible. Interestingly, a preliminary analysis of the survey found that parents wanted more Ipaii language incorporated into TACKLE activities, a desire that complemented the goals and objectives of the research team. In response to this request, team members along with the new coordinator experimented with integrating the parents' suggestions in the activities and curricular adaptation of the TACKLE materials. Additionally, a rich source of ethnographic detail collected by informed students who were formally trained in ethnographic field methods added important information on the cultural values of the Kumeyaay community.

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[3] The title of the survey was changed from "parent survey" to "parent-guardian survey" because many children are raised by the extended family and not necessarily their parents.

Context Embedded Change

The findings of the parent-guardian survey and an elder's comment, "We [only] have two people who speak enough [Kumeyaay] to form sentences" [Fieldnote, BD 05/07/2009], led to weaving the Ipaii terminology into culturally relevant games that were popular among the children, such as the "Indian Taco" game. A version of musical chairs, the game entails calling out food items that form part of an Indian taco—i.e., bread, tomato, lettuce, beans, etc.—initially all in English. The team added as many Ipaii words available for its ingredients and also added names of colors, animals, seasons, and fruit items. To reinforce the same terminology by means of different educational/play activities, the same word categories are used in the bilingual Ipaii-English "warm-up" activity sheets that were developed for TACKLE sessions. The adaptation of the materials reached a new level of cultural relevance with the inclusion of Shuulluw-Shuuluk as the counterpart to *El Maga* and the re-writing of its origin myth to align with the community's cultural values of the Shuulluw-Shuuluk.[4]

Our preliminary findings show that the latest joint efforts between the new *Innovation* Team and San Pasqual's reservation staff are bearing fruit. The curriculum is becoming more culturally sensitive and enriched, and we believe that our efforts have gained a greater level of trust and inclusion. Our data and the everyday life of the program show many signs that we are addressing UNESCO's concern that "when languages fade, so does the world's rich tapestry of cultural diversity. Opportunities, traditions, memory, unique modes of thinking and expression are valuable resources for ensuring a better future - are also lost". In particular, we are seeing greater excitement over the new curriculum expressed by both participating children and the undergraduates. For example, when asked what she thought about the new activities and the use of Ipaii at TACKLE, a seven-year-old child from the reservation exclaimed: "I really like to learn Kumeyaay because I don't get to learn it at home. It's really fun getting to play ice breaker games, but I really like the handouts [activity sheets] because I get to learn something new" [Interview, MD 05/06/09]. The excerpt below, from a longer field note written by a practicum student, also illustrates the successful elements of the adaptation process and of the new warm-up activities:

> "The different activities that we are using to adapt the Kumeyaay language and culture are really working and it's definitely a big step to recovering the language […] I feel that there was definitely more interactions when you have these different warms up for the kids to do, it allows them to speak and actually practice the [Ipaii] language […] Today with the handouts, both [girls] would ask me questions when they encounter a problem with finding the right Kumeyaay word or figuring out the number or season" [MD, 05/29/09].

At present the process of cultural/linguistic adaptation of the curriculum continues at the same time that the San Pasqual site takes on the role of a Language and Culture Hub of expertise. In the coming year, the site participants will gather more expertise on language preservation and will develop and manage a wiki and/or blog to share their expertise with the

[4] A practicum student with special knowledge and expertise of the Kumeyaay culture, transculturated El Maga's origin myth in the Spring of 2009.

larger HOI community. They will establish a digital Help Desk on the HOI online networking site to share helpful insights they have acquired on their specific area of emphasis. If developments go as planned, these digital channels will connect Native youth with other children and families of Mexican origin across the other four sites, nationally with two *La Clase Magica* sites springing up in the University of Texas system and internationally as part of a tri-national exploration on learning involving Colombia, Spain, and the United States.

CLOSING STATEMENT

When we juxtapose our experience at San Pasqual in disseminating our efforts to new cultural contexts, we are struck by how our work relates to recent perspectives on this topic. We agree with Coburn (2003) that this process is multifaceted and complex. However, we worry about the implications of scaling up as simply the spread of any feature of an innovation. From our vantage point, scaling up is a holistic effort that involves considerations of long-term sustainability and community ownership. Otherwise, it becomes a reform that is either sustained by only a few individuals, exists strictly as set of commitments without actions, or is unconnected to the larger project of bringing about substantive change in how we approach education [how we educate children]. Our concern is that such understandings of reform can easily constitute yet another failed attempt to enhance the educational opportunities of individuals who have traditionally been underserved all the way up the educational ladder.

We are also concerned how the current education reform context will affect our situated approach to scaling up. Given the federal government's interest in replication approaches to scaling up, we worry at multiple levels. Although we have found that the context of after-school projects has provided a supportive venue for drawing on practices and perspectives that contrast with those of schools, we know that the current push for standardization and scripted curricula promoted by the globalization processes is making its way into after-school programming as well. Indeed, the current trend of using standardized measures as indicators of a program's success is contributing to this tendency. Given this trend, we are concerned about funding and sustainability of successful educational innovations grounded in the histories, cultures, goals, and interests of diverse communities rather than the strategy of replication.

While it is too soon to tell what the full impact of TACKLE will be on the individual participants or host institutions' ability to serve their respective constituencies, there is much promise for up-take and further dissemination. A careful review of ethnographic field notes written by staff and students enrolled in the undergraduate courses over the last nine years of our scaling up efforts demonstrates that the process is labor intensive and slow, but it is possible and noteworthy. Through resolve and continuity, the research shows that scaling up can be accomplished without major compromise to the fidelity of the program. Across our system, children are offered a culture of learning that is often not available in other areas of their life, school included. At *La Clase Mágica*/TACKLE they learn academic content and skills in a socially constructed world of play that is created in their image. They get access to computer and telecommunication technology without giving up their native resources and, in the case of San Pasqual, have at their disposal mainstream knowledge that allows them to seek out native resources. This environment nurtures children to effectively learn to learn and to be prepared to enter the culture of schooling using their community's intellectual resources as a basis for new learning (Vásquez, 2003). In playful adult-child interactions, child learners are guided towards higher levels of academic achievement and university admission while parents are guided towards a greater understanding of the learning process and schooling as well as the possibility of university training themselves. The undergraduate students on the other hand receive a meaningful undergraduate education that prepares them to enter into a diverse and complex world.

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MARGARITA'S LIVING THEORY INFORMING A FEMINIST, CRITICAL MATHEMATICS EDUCATION

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Previous work in critical mathematics education argues for the importance of integrating social change in mathematics learning settings as a way to address inequity in society. Feminist perspectives of education highlight the need to integrate the perspectives of women of color in critical or transformative education. This paper provides a case study of one young Latina's experience of such an educational setting in which she engaged in a community movement to save her school from being closed. This paper explores Margarita's experiences, through the lens of identity and agency, as an illustrative case study of critical mathematics education. The unique "living theories" generated from this analysis provide for a nuanced understanding of the potentials of a feminist, critical math education.

INTRODUCTION

Having worked in after-school programs for three years in the small, neighborhood school of 250 students, Agave Elementary School[1], I have watched the progressive infiltration of federal and state educational policy within the walls of a school whose heart is its sense of family and community. This feeling of community welcomes you upon entering the school where you will see colorful student artwork and notices to parents in English and Spanish, for example. The collective sense of the school is that it is genuinely invested in its students, it welcomes and values families, and it acts as a central part of the community. Unlike many schools that serve similar low-income, majority-Latino communities, Agave embodies the "caring" Valenzuela (1999) advocates for. Despite this, as evidenced by the conversion of a brand new gymnasium to a remedial learning lab, the loss of the Physical Education teacher and the prominent display in the professional development area of each student's test scores, this school has not escaped state and federal policy influences. In fact they are obligated to respond in ways that often result in a move away from the centralization of the students, families and community within the school.

In spite of and despite these influences, seven young Latinas and I engaged in educational practices along the lines espoused by educators such as Freire, where the purpose of education is to engage in a process of freedom from societal oppression (Freire, 1970/1993). The influences of societal structures related to gender, race, class, ethnicity and language worked to shape and were shaped by the interactions in this setting. Coming from a perspective that education can be a site for societal transformation and work towards equitable experiences and outcomes for students and society, critical education espouses the potential of experiences that at once seek to understand, confront and change these oppressive structures. This educational experience forms the basis of my dissertation study, from which this paper emerges. This particular paper seeks to provide a nuanced understanding of the importance of integrating a feminist perspective with critical mathematics education through a case study of Margarita.

Rationale: Why Latinas/os? Why Mathematics?

Critical or transformative education is even more urgent for schools that serve low income students of color because of the persistent marginalization and more aggressive implementation in these schools of increasingly oppressive educational influences such as remedial reading and mathematics programs (Oakes, 2005). As a result, educational statistics and qualitative accounts of schooling experiences of Latina/o students indicate that the current structures of schooling reflect the present political landscape that marginalizes students of color and students living in poverty (Moll & Ruiz, 2002; Trueba & Bartolomé, 1997; Valenzuela, 1999). For example, as evidenced in English-only legislation that is spreading across the country and in place in Arizona, or in attempts to eliminate a Raza (Mexicano/ Chicano) Studies Department from a school district here in Arizona, these students' bilingualism and community knowledge and practices are seen not as assets that are integral to their schooling but as deficits or barriers to their success (González, Moll, & Amanti, 1995; Licón Khisty & Viego, 1999).

In particular, mathematics is a subject in which access and subsequent opportunities for empowerment are limited or often absent in schools attended by students of color and/or students living in poverty (Boaler, 2006; Oakes, 2005). Given that mathematics is increasingly being seen as a gatekeeper to upward social mobility and access to higher education and jobs (Moses, 2001;

[1] All names are changed to protect anonymity of research participants.
[2] School Principal, name changed to protect anonymity.
[3] District Superintendent who initiated the school closure process.

Noddings, 1994; U.S. Department of Education, 2006), it plays an important role in our society, whether deservedly so or not. Skovsmose (1990) argues that mathematics underlies much of the organization of our increasingly technological society, and that it is particularly powerful in validating systems that serve to oppress and support capitalism. As an example from our educational system, the "unquestioned" power of mathematics is used in the current accountability system because it is based on assessment through testing. In this case, mathematics is used to create what is perceived as a neutral system to classify and sort students and schools. There are several such societal beliefs with serious consequences for students.

Regimes of Truth in Relation to Mathematics, Gender and Race

The multilevel complexity of factors that influenced and were subsequently influenced by the practices the young Latinas in this study engaged in must be considered in an account of their learning experiences. In particular, there are societal beliefs embedded within social structures or what Foucault (1980) called "regimes of truth" which surround us daily, although often not visible. These "truths" are socially constructed ideas that become institutionalized over time until they become "obvious" facts and therefore remain unchallenged or in some cases go unnoticed. Often, institutional structures such as schools serve to create the conditions that then validate a "truth" as evidenced, for example, in the intense emphasis on test results alleged to be a "true" and neutral measure of student ability, while in reality they are supporting current stratification (Lipman, 2004). For the purposes of this study, I will address interacting regimes of truth in relation to mathematics, race and gender embedded within social structures that influence the learning experiences of Latina/o youth in order to better situate the case study of Margarita and argue for the integration of a feminist perspective with critical mathematics education.

Because mathematics is often seen as a neutral endeavor (Ernest, 1991), blame for low achievement or low participation in higher level courses or mathematical careers is often placed on students, their gender or their cultures. A lack of critical analysis of the origins of societal practices leads to little understanding of the ways in which mathematics emerges from a White, middle class, male tradition of mathematical knowledge production (Gutiérrez, 2007; Walkerdine, 1998) and how it organizes society in sometimes oppressive ways (Baez, 2000; Skovsmose, 1994).

Because mathematics is often perceived as a gendered subject better suited to boys, it creates further marginalization for female students (Boaler, 2007; Ernest, 2007; Mendick, 2005; Walkerdine, 1989). Fennema (1995) says, "We should be open to the possibility that we have been so enculturated by the masculine dominated society we live in that our belief about the neutrality of mathematics as a discipline may be wrong, or at the very least, incomplete" (pp. 33-34). Feminist perspectives of mathematics education interrogate the "regime of truth" in relation to the neutrality and unconditional acceptance of the discipline of mathematics (Walkerdine, 1998) and women's ability to participate in mathematics (Boaler, 2007). Without interrogating and reformulating these regimes according to a feminist perspective, our educational system will continue to marginalize girls from mathematics and validate beliefs such as girls cannot do mathematics or are not as good at mathematics as boys. Rather than accepting and exploring differences between girls and boys as inherent, Boaler (2007) suggests a perspective where practices in relation to mathematics and gender are seen to embody social production and reproduction of power relations (Varenne & McDermott, 1998). As of yet, little research speaks to the interactions of socially constructed categories such as gender with critical mathematics education, and particularly the importance of a feminist perspective of what it means to do mathematics.

Although less directly and openly "talked about" than the relation of gender to mathematics (Ernest, 2007; Lubienski & Bowen, 2000), is the fact that similar "regimes of truth" exist regarding deficit perspectives of who can do mathematics based on their race and class (Diversity in Mathematics Education Center for Learning and Teaching, 2007). These beliefs, in this case regarding the aptitude and societal location of students of color, are an accepted and assumed part of our educational institutions. Research should speak to countering these and regimes of truth and at the same time write a new truth, one that does not necessarily use the same measures or structures that are the manifestations of these oppressive beliefs.

Countering Regimes of Truth through Critical Education

Recognizing that learning happens in a socio-historical and political context, there exist research and practices that seek to disrupt current societal structures through an explicitly political educational and research agenda. This critical educational paradigm, whose explicit purpose is an education that results in individuals making apparent and challenging oppressive social structures, suggests a way to disrupt and confront "regimes of truth," particularly for students most marginalized and oppressed. When this form of critical education emphasizes utilizing mathematics as a tool for societal transformation, it has been termed critical mathematics education (Frankenstein, 1983; Skovsmose, 1994). The potential for transformative mathematics educational experiences has a rich and growing tradition within the mathematics education community. Recent research points to the need to gain a more nuanced understanding of what critical mathematics education would look like in an elementary setting (Gutstein, 2006; Simic-Muller, Turner, & Varley, in press; Turner, 2003; Turner & Varley, 2008; Turner, Varley, Simic-Muller, & Díez-Palomar, in press)

RACE AND GENDER IN CRITICAL MATHEMATICS EDUCATION

Feminists of Color and Feminista Perspectives on Education

Feminist scholars of color have highlighted the need to include the perspectives of women of color in envisioning critical, transformative education, arguing that their experiences of oppression and survival at the intersections of race and gender uniquely position them to transform education and society to achieve equity (Henry, 1998; Hill Collins, 2006; hooks, 1994, 2000; Villenas et al., 2006). And so, while work in critical mathematics education highlights the need to understand how younger, elementary-aged students experience critical mathematics education, I would also argue for the need to examine these experiences from perspectives which foreground the perspectives of women of color, in this case Latinas.

Lather (1991), speaking to what critical education theory can learn from feminist praxis, says: "Feminism's grassroots, 'no more experts' credo is premised on the sturdy sureness that, given enabling conditions, every woman has something important to say about the disjunctures in her own life and the means necessary for change" (p. xviii). However, as detailed by Hurtado (1989), examining the experiences of white women and women of color reveals the persistence of race in maintaining the current structures of inequality. Critical race and Latina/o critical theory argue for the "[foregrounding of] race as an explanatory tool for the persistence of inequality" (Ladson-Billings, 1997, p. 132) and the primary tool for this is countering the dominance of a "normalized" white, middle-class perspective (parallel to Foucault's (1980) notion of regimes of truth which are societally produced and institutionalized) with the (counter)narratives of people of color (Delgado Bernal, 2002; Villenas & Deyhle, 1999). In the case where gender is also a salient influence, I argue we must foreground the (counter)narratives of women of color, which this paper sets out to do.

Feminists of color (Anzaldúa, 1999; Delgado Bernal, 2006; Henry, 1998; Hill Collins, 2006; hooks, 1994, 2000; Villenas, Godinez, Delgado Bernal, & Elenes, 2006; Zavella & Takash, 1993) advocate for starting with and looking to the margins- that is, to the locations of marginalization and to those most marginalized- in order to understand how social structures are shaped and also transformed to be more equitable. They advocate for an explicit focus on the experiences of women of color in understanding educational environments. It is within these spaces, or what Anzaldúa (1999) calls borderlands, that women of color have learned to deal with oppression and it is also where change happens in society. hooks (1990) describes these margins as a site of resistance and empowerment rather than deprivation or domination, explicitly countering the deficit-based perspective often used to understand the educational experiences and practices of students of color. In practice, I see these margins as not necessarily physical spaces, but interactions or environments in which individuals are working to make change or where they are enacting practices that contribute to change. In essence, this work argues for us to look to spaces where women are confronting oppression and transforming social structures to be more equitable. In the context of this study, the co-constructed spaces where the girls participated in a community movement to save their school are an example of these borderlands. The community engaged in confronting district policies which they saw as oppressive and sought to change these structures.

Anzaldúa (1999) eloquently describes the unique knowledge generated by women of color in these "borderlands" spaces:

> When we are up against the wall, when we have all sorts of oppressions coming at us, we are forced to develop this faculty [la facultad] so that we'll know when the next person is going to slap us or lock us away. …It's a kind of survival tactic that people, caught between the worlds, unknowingly cultivate (p. 61).

Anzaldúa argues that a new mestiza consciousness is being cultivated among those who live in the borderlands, these borderlands that I see as the spaces between human agency and oppressive societal structures. Feminista perspectives of education foreground the experiences of Latinas and Chicanas, arguing that their unique schooling experiences and the methods they have developed for survival are redefined as education, and that these "living" theories are essential to feminist and critical educational theories, among others (Villenas, et al., 2006).

This framing of education informs my perspective on the educational setting of the study, namely with the experiences and perspectives of the Latina participants being central. For example, I see the Latina participants' stories about and reflections on being involved in mathematics and a community movement as living theory about the potential of centralizing activism in school (mathematics) educational experiences. With this in mind, I focus on the identity, beliefs and practices of one young Latina in my study in relation to her engagement in mathematical investigations and community activism and the resulting "living" theories that were generated, which I outline in the findings. In turn, these theories shape and inform understandings of a critical, feminist mathematics education.

Connections to Mathematics and Science Education

In describing how society resists thinking that seeks to disrupt dominant societal narratives, Lather (1991) states, "The 'crisis' these high priests of Western culture perceive is to their continued hegemony over the production and legitimation of knowledge in the name of national heritage and the legacy of Western civilization" (p. xvi). This same argument applies to the current perceived "crisis" in the United States regarding the status of mathematics education and its ties to U.S. global economic competitiveness (see National Academy of Sciences, 2007; National Commission on Excellence in Education, 1983; National Research Council, 1989). Related to a neo-liberal, market driven society (Apple, 1992; Lipman, 2006) in which education is organized along the needs of a two-tiered society of intellectuals and laborers, there is a push to increase mathematics performance of U.S. students on comparative international tests solely to maintain our economic standing (equated to power) in the world, rather than being motivated to do so because of inequitable educational opportunities. Anzaldúa (1999) describes, "In trying to become 'objective,' Western culture made 'objects' of things and people when it distanced itself from them, thereby losing 'touch' with them. This dichotomy is the root of all violence" (p. 59). Although not directed specifically at mathematics, her words speak to the dangerous, dominant notion that mathematics is seen as an objective truth or an 'object' distant from humanity and the people that created it. Utilizing Skovsmose's (1994) argument about the (often oppressive) formatting power of mathematics in society, there is an urgency to include the voices of women of color in re-envisioning mathematics education so it is a part of reducing this "violence" and is used as a tool to transform society to be more just. In addition, this re-envisioning could involve recognizing when mathematics is not the most appropriate tool, challenging the high status of mathematics in society.

A feminist perspective, most elaborated in the field of feminist science education (Calabrese Barton, 1998), highlights the need to challenge the role of the teacher as holder of knowledge and the traditional paradigm of science education that excludes. I argue, similar to Walkerdine (1998) that this is equally relevant in mathematics education. Similar to mathematics education, a feminist perspective on science education describes learning as engaging in the practices of science (or mathematics), rather than as memorizing an outside, pre-determined body of scientific knowledge (Brickhouse & Potter, 2001), and in which the experiences of the students, in this case girls, are then centralized.

Although several of the above researchers focus on the need to include the voices of females of color in particular because of the effect of race and racism on educational experiences, little (if any) mathematics education research speaks specifically to girls of color or to a feminist of color perspective in relation to mathematics. Several studies that discuss differential achievement and the effects of enrichment programs for girls of color focus on science rather than mathematics, and do not discuss the interplay of race and gender explicitly (Ferreira, 2002; Hammrich, Richardson, & Livingston, 2000).

Civil's (2003) work with "Girls in the System", an after-school and summer program to involve Mexican American and Native American girls in STEM fields, reveals insight into the interplay of race and gender in mathematics education. For example, preliminary findings indicate participants came in with a perception that scientists and mathematicians were White men, while at the end of the program they had shifted this perception to be people like themselves. There is a need for further research that includes feminist and critical race theories in relation to critical mathematics education. As Delgado Bernal (2002) advocates for the need to recognize students of color as holders and creators of knowledge, I advocate for the need to recognize Latinas as holders and creators of mathematical practices, as well as knowledge about social transformation, and the interplay between the two.

Informed by feminist notions of challenging and re-envisioning the status quo in mathematics education, and feminist of color perspectives that centralize the experiences and ideas of those most marginalized, I consider how my research works towards equity through mathematics education. More specifically, I seek ways to foreground the experiences of young Latinas in an attempt to understand what the field of mathematics education and broader society might learn from them and from their experiences confronting injustice and working towards societal transformation. I draw upon identity and agency as analytic tools that examine the interplay between the micro and macro contexts of learning, because it is within these borderlands that change occurs (Anzaldúa, 1999).

Identity and Agency

Why identity? Gutiérrez (2008) advocates for mathematics education research to focus on advancement [understood through the successful educational experience of students of color] and the context of learning because they serve as tools for humanizing education. Gutiérrez is referring to Bartolomé's (1994) description of a humanizing pedagogy "that values the students' background knowledge, culture, and life experiences, and creates learning contexts where power is shared by students and teachers" (p. 189). From this perspective, understanding students' *unique* identities and incorporating them into educational experiences is central. In addition, the context of learning is important, recognizing that there are social, historical and political influences on educational experiences. In this sense, the notion of identity serves as a useful tool to understand students' unique learning experiences, given that the social, historical and political contexts of learning impact individuals in unique ways. This is particularly important if we are concerned with a humanizing educational experience that builds upon the knowledge, culture and life experiences of students traditionally marginalized based on deficit assumptions of who they are in terms of race, gender or other social constructions.

Identity in Mathematics Education

Within the field of mathematics education, several researchers describe the role of identity in relation to learning mathematics (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Martin, 2000; Nasir, 2007; Sfard & Prusak, 2005). From the participatory perspective, mathematical identity is seen as enacted through participation in mathematical practices with attention to the beliefs about mathematics and doing mathematics that underlie these practices (Boaler & Greeno, 2000; Cobb, et al., 2009; Nasir, 2007). From the narrative perspective, mathematical identity is also evident in the stories students tell about their participation in mathematics and their beliefs about themselves as doers and creators of mathematics (Martin, 2000; Sfard & Prusak, 2005). Because participants wove in stories from their lives and about their participation in making change during the research process and I was able to capture how this was enacted and their own descriptions of this participation, I focus on the beliefs and practices of the case study in conceptualizing of identity.

Identity and agency

Martin (2000) recognizes the flexible and fluid nature of identity being influenced by the mutually constitutive nature of agency and structures. Cobb, Gresalfi, and Hodge (2009) examine how classrooms impact "what it means to know and do mathematics" for students. Their findings highlight the "productive" mathematical agency students develop within a classroom that emphasizes the collective generation of ideas (conceptual agency), rather than agency in mastering the discipline where authority resides within a domain of mathematical knowledge passed on by the teacher (disciplinary agency). Placed within an understanding of the sociopolitical context of these classroom experiences, attention must also be given to research that indicates productive mathematical beliefs may not be enough to ensure participation, particularly for marginalized students (Lubienski, 2000). Therefore, I expand upon the idea of developing students' mathematical agency to include a focus on the sociopolitical context of that activity as well, which I believe is essential for marginalized students.

Baez (2000), recognizing that agency both constitutes and is constituted by social structures, recommends looking at the margins of power, the locations where oppressive structures are being confronted by the oppressed, in order to understand how these structures can be changed. These margins of power are akin to the borderlands or margins where change happens, according to feminist scholars of color (Anzaldúa, 1999; hooks, 1990). An individual's enactment of a sense of self as one who can make change (influenced by the social structures in a given context) is termed one's sense of *critical* agency (Baez, 2000). Pruyn (1999) differentiates *critical* agency from agency because it is built from an awareness of how oppression operates in society, similar to the idea of informed critique from critical race theorists (Solorzano & Delgado Bernal, 2001). This focus also validates and centralizes the experiences of students of color, as put forth by critical race and Latina/o critical scholars (Brayboy, 2005; Solorzano & Delgado Bernal, 2001; Villenas & Deyhle, 1999). In describing his experiences with teaching mathematics for social justice, Gutstein (2007) argues that his students developed a sense of agency, or "a sense of themselves as subjects in the world" (p. 420), and as people who can also make change in their worlds, evidenced in their critical stance towards society. Drawing from the work of social justice educators (Freire, 1970/1993; Gutstein, 2006a; Turner, 2003) and critical race theorists (Brayboy, 2005; Solorzano & Delgado Bernal, 2001; Villenas & Deyhle, 1999), I conceive of a sense of *critical* agency as how individuals see themselves as people who can make changes in their lives and society. Agency becomes a useful lens with which to understand how students experience a critical mathematics education environment, as well to understand the transformative potentials of this work.

In documenting these Latinas' stories about themselves (Sfard & Prusak, 2005) and their forms of participation as evidence of identities (Boaler & Greeno, 2000; Lave & Wenger, 1991), I seek to understand the historical and political context of their identities and agency. As such, I focus on their critical agency in particular, which is evidenced in their statements about their ability to make change, in their patterns of participation, and by actually engaging in practices and forms of activism toward making change. I see it as important to note both a participant's *sense* and *enactment* of critical agency because it reflects the interactions between structures and agency. For example, when participants prepared a presentation to convince the school board to keep their school open, their sense was that they would be able to make change. However, we were initially told that the students were not going to be given a venue with which to share their presentation. There was a distinction between their sense of critical agency and how it was received by the surrounding social structures.

Considering the above conceptual framework, the purpose of this paper is to provide an in-depth case study of the living theories from one of the Latina participants in this study, Margarita. This paper primarily provides a narrative of Margarita's mathematical and sociopolitical identity as evidenced in her participation in and narrative about the Save our School Project, weaving Margarita's own words and my analysis. These resultant living theories provide for a more nuanced understanding of the importance of a feminist, critical mathematics education.

METHODS AND SETTING

I believe that both educational and research experiences should be locations for societal change, which influenced my decision to draw upon critical ethnographic, feminist, and PAR methodologies in this study. These methods are particularly appropriate because they push and blur the relationship between research and praxis, which Freire (1970/1993) describes as the commitment to activism and the struggle for universal human rights. This perspective guided both the approach to research and the approach to facilitation of the after-school setting of this work.

Data Collection and Analysis

For the purposes of this paper, analysis relies heavily on interviews with Margarita, as well as transcribed video data of her participation in after-school sessions and her student work. Analysis of the multiple sources of data consisted of several stages of review of data for open coding, beginning during data collection and transcription, and including several sessions of joint analysis with the participants (Strauss & Corbin, 1990). I employed grounded theory (Charmaz, 2008) techniques to data analysis, which involved a processual approach to analysis and collection of data. In this approach, reflection throughout data collection allowed me to refine the research process, making adjustments in how I facilitated the after-school group and collected data. This dialectic between theory construction and reality allowed for analysis of "participants worlds and of the processes constituting how these worlds are constructed" (Charmaz, 2008, p. 204).

After-school Setting as a Site for Research

The after-school setting of this research deserves attention because it both afforded the unique learning setting that was central to this research and significantly shaped the nature of the findings. I recognize that there are several unique affordances of the after-school setting (e.g., Vásquez, 2003) that would be more difficult to negotiate during the school day and which influenced my choice of working outside of a classroom. At Agave, after-school programs are allowed considerable curricular flexibility; I was not required to address particular standards or benchmarks at specific points in time as a classroom teacher would be. Additionally, similar to many after-school programs that have proven successful for low-income, non-dominant students (e.g. Rahm, Martel-Reny & Moore, 2005; Vásquez, 2003), the math club learning environment was characterized by informal adult-child interactions, learning in the context of activities that spanned multiple settings (school, neighborhood, city), and increased level of student choice. Although I ultimately feel that the impact of critical mathematics education would be most significant in the classroom setting, the after-school math club proved to be an appropriate setting for this study.

The Math Club and the Save our School Project

Beginning in the fall of 2007, seven fifth grade Latinas met once or twice a week after-school with me to engage in problem-solving based mathematics connected to community contexts. In addition, the structure of the math club allowed for significant community-building time often where the participants shared their lives with each other. It was during one of those times in which the subject of the impending closure of their school arose and became the focus of our work in the club. The following case study details Margarita's mathematical and sociopolitical identity through analysis of her participation and reflection on the project.

MARGARITA

"If you had confidence and perseverance, then I think, personally, you could do anything."

12/07/07

Background

This quote illustrates Margarita's agentive stance toward the world, including the mathematical world. Margarita is a child of parents who grew up on the United States-Mexico border; a father who shows exceeding pride in his three daughter's academic accomplishments, and a mother who believes in grassroots activism and care for others. Margarita shared a similar commitment to academic success, a critical stance toward the world, and a commitment to care for others (expressed in her desire for a future career as a veterinarian or a child care provider). Margarita speaks Spanish at home with her parents and grandmother, yet she clearly described her struggle with Spanish as an academic language:

Margarita: If I'm doing multiplication or take away, then I would probably speak in Spanish or English, but if I was doing division, like the harder ones, division or fractions, I would have to speak English 'cause fractions right now, doing the minus and stuff like that and times and stuff like that with fractions gets to your head and stuff like that … so I would have to probably speak in, in English for that. … Because like sometimes in Spanish I, like *menos* is take away so if I were to talk I would know how to say take away in Spanish so if I were to explain it to somebody I would tell them, 'this *menos* this equals what?' And if I were to say division, I have no idea how to say division in Spanish so I wouldn't know how to explain to somebody who just knew how to speak Spanish.

Maura: And is explaining part of the math class, learning math?

Margarita: Yeah, it's also if my teacher asks me questions and I can only speak Spanish then that would be, that would be hard for me because I don't know how to say fractions or division in Spanish. I know how to say math and all that stuff, but I just have no idea how to say division and fractions.

Interview 1: 12/07/07

Although Margarita focused explicitly on vocabulary, she referred to specific instances of communicating mathematical ideas in Spanish and the struggle she anticipated if she did not have the Spanish vocabulary to communicate her thinking effectively, reflective of Arizona policies for English-only instruction (Proposition 203, 2000).

Beliefs about Mathematics and Herself as a Mathematics Learner

Margarita attributed her success in mathematics to her ability to communicate her ideas and persist with problems, and to her desire to develop understanding of mathematical concepts. When asked how she sees herself as a math student, Margarita responded:

> Some kids hate math but at this point I'm acing math because math is really easy for me. I like math. It's one of my favorite subjects. It's just something that I click right away. … Probably 'cause I pay attention so I understand. I find all the details that help you find it. My teacher, he finds a way of doing it like finding division but I have a different way of

finding it so for me it'd be easier.

Interview 1: 12/07/07

This excerpt reflects Margarita's confidence in her mathematical abilities and her problem-solving orientation toward the discipline (ex: finding out what strategy makes sense to you). While Margarita did describe problem-solving based mathematical practices, she also spoke about more discrete activities and an emphasis on speed, such as memorizing your times tables and feeling that she might "get in trouble" if she did not "finish her work." Although she did report that math "clicks right away" for her, and that it is important for her to be the first student done with her work, she also described the need to "take your time" in order to find the strategies that make the most sense to you in order to really understand. I see these somewhat contradictory ideas (needing to finish quickly vs. taking your time) as evidence of the dual presence in many U.S. schools of an emphasis on a reform curriculum intended to develop student understanding and the pressures from a timed, standardized-testing and tracking-based school culture.

Beliefs about Mathematics Learning

Margarita shared ideas about the benefits of problem solving-based mathematics, and she also expressed a related, critical stance toward knowledge generation. Margarita explained that investigations, such as the ones involved in the Save Our School project, helped her to learn math, "Because you don't know, first of all with the investigation you don't know what type of math you're doing. That's what you have to find out, 'Oh I have to do multiplication for this, or division, or addition, or subtraction.'" Her words describe a perspective of mathematics that encourages engaging in mathematical practices as the focus of learning, rather than in learning isolated skills.

Margarita spoke about the benefits of working with others, which she believes is particularly useful when engaged in investigations. She described one such investigation as follows:

Margarita: But I was with Maribel and Vanessa and we were working on it- they were marking the tally marks I was getting the streets, counting them and then multiplying them and they would mark it. And it was a really fast process.

Maura: What do you think made it fast?

Margarita: All of us with teamwork and we were all putting the effort to it and we were all really trying.

Interview 2: 04/17/08

Margarita spoke not only about how she and two other girls in the math club each took on a role in converting survey results into data points, but she also commented that this resulted in a "fast process."

She described the learning environment in the math club in contrast to her regular classroom as facilitating this kind of collaboration:

Maggie: Instead of – you can't get up on your feet in the normal class, like you have to stay with the person you are working with, you can't go around and check what they're doing to see if you or your answer to see if whoever you are working with to see if you got the answer right with another pair. But when you're at the after-school you can move around and ask them, 'Oh, what did you get? Because I got this.' And then we look at

> each other's work and we see if one of us got it wrong. And it's kind of better than in class.

Echoing her previous ideas about coming to your own solution strategy rather than just accepting one provided to you, Margarita said she thinks it is "better" when she can use her peers as a resource in evaluating her own solution, a practice that evidences a more agentive stance toward mathematical knowledge generation.
In elaborating on the benefits of productive mathematical conversations with peers, Margarita stated:

> You talk with several people and then you either agree or disagree and then you tell them why you agree and then they could probably say, 'Oh yeah, it's pretty true.' And it's kind of- it's easier to talk about because during class it's different because you don't have that much people to talk to. You only talk to one person and then what if you get the answer wrong and you two are like, 'What? That's the same- we have the same answer.' It's right but you can't go to the other person and say, 'Oh yeah, I disagree because this is what that is.'
>
> *Interview 3, 05/08/08*

Margarita described the agentive mathematical practice of argumentation as a beneficial practice facilitated by the math club learning environment.

Margarita also spoke about her belief in the importance of being able to facilitate your own learning in mathematics, stating, "There [are] different strategies and you can find your own strategy." This perspective reflects her critical stance toward knowledge generation in the sense that Margarita believes you should not just accept a strategy that someone else tells you. She elaborated on this idea, saying, "If you want to learn something and somebody *told* you, you should never believe it. You should figure it out for yourself." Margarita's sense of agency in this statement reflects what Pickering (1995) calls *conceptual* agency because it involves a student choosing methods and developing meaning in contrast to a student following established methods of arriving at a solution (*disciplinary* agency). Margarita's agentive stance toward learning was also paralleled by her critical stance toward making change.

Beliefs about Making Change

On several occasions Margarita referred to a deep concern for, "how the environment's changing and stuff and how it's changing worse 'cause people just don't care, and human carelessness and how it's destroying the ozone layer; it's man-made." She also felt it was something she could change. Margarita went on to connect mathematics to her desire to do something about this concern stating, "Math is something that I like a lot and I think [environmental degradation] has to do with math. Probably to see how much, like if I were to count how many people throw away stuff into the ocean, like how much percent a day, then I would see, we'd try to lower it." In the beginning of the year, when asked if she felt like she could do anything about environmental degradation, she responded, "I think that one person could make a big change, you just have to have confidence and be able to find stuff and information about it and have resources." She also described specific steps she would take, such as doing online research about the problem, collecting trash on beaches, and connecting with others concerned about the issue, because, as she explained, "Two different smarts can make a big difference." Toward the end of the school year, Margarita again identified the environment as a primary concern of hers; and when asked if she could do something about it, she responded, "I think I could if I had the extra people, right? Like people to help me, the material and stuff." Margarita's presence and participation in and her analysis of the Save our School Project in the following section is reflective of her confident sense of self mathematically, her critical stance toward the world, and her thoughtful and willing contributions to the girls' group.

MARGARITA'S PERSPECTIVE ON THE SAVE OUR SCHOOL PROJECT

Margarita regularly provided unsolicited reflection on the community movement to save the school, the girls' group participation in this movement, and her own learning. This emerged in the form of reflective writing, letters to school board members who would ultimately vote on the school closure, contributions to group discussions, and as solicited in interviews. For the purposes of this paper, I have collected the various themes that emerged from analysis of Margarita's participation and reflections in particular. These themes highlight Margarita's unique contributions as living theory for the importance of a feminist perspective of critical mathematics education.

The Importance of Drawing upon the Past and Considering the Future

Learning from the past. Early in the year, I asked all of the girls to take a disposable camera home and document what was important to them. Margarita chose to take a photo of the front of Agave. The photo was taken from an alley behind Margarita's house, where it empties onto the street that runs the front length of the school. Barely visible in this photo is the name of the school etched above the door frame of the original school building, founded in 1921. The fact that the school was established in 1921 evolved into one of the arguments made by the girls in terms of the long presence of the school in the community. In fact, the idea of considering history and past and future generations of students at the school became a central argument to the girls' rationale for keeping the school open.

Margarita worked with Vanessa, another participant, to include the following in their letter to the board members: "Vanessa's grandmother is 80 years old and she went to Agave School. She has many memories and she is also concerned about the school closing down." This quote illustrate a recurring rationale used in arguments by participants and other community members against closing the school, involving generations of family members attending Agave and the sociohistorical importance of the school in the community.

Margarita returned to and expanded upon this idea, recognizing that the girls could tap into the knowledge of older generations as a resource for saving their school:

> You know how she knows the history and probably, if they did try to close it when she was, if she was in the school and they tried to close it, and then probably, she probably helped, or maybe she didn't. But she could probably say we can, we can stop it from happening again.

Sharing that several of the girls had family members who had attended Agave prompted this recognition that there could be critical funds of knowledge in their own communities and families about what to do in a situation such as a school closure. Along with the girls' own ideas for action regarding the school closures, parents of the Mathematigals, school staff, and other community members generated a wealth of ideas for making change.

Through conversations with several parents, or in the testimonios (González, 2001) or stories of the girls, it is evident that these girls come from families who are aware of structures of oppression, which informs social critique and even action to make change. Examples of such social critique included references to structural oppression such as racism. Similarly, Margarita's critiques sometimes included references to an understanding of the role of racism and sexism in society. One specific example of familial critical funds of knowledge was when Margarita's mother spoke passionately and at length at a parent meeting to organize against the school closings. She presented ideas such as creating a human chain around the entire school as a public statement of the community support for the school, advocating for the needs of the low-income community and the impact the closings would have on the families. She also urged other parents to attend meetings despite it being a challenge for them, recognizing that many worked several jobs, and taking time off would be a sacrifice. Margarita's own words often echoed those of her mother, advocating for the active participation of others in the movement. This example highlights the importance of family and community influence on the nature of Margarita's sociopolitical identity and on the sort of knowledge that might be overlooked and undervalued in classrooms.

Preparing for the future. Another aspect of integrating critical funds of knowledge into learning environments is recognizing that communities are continually generating knowledge for future generations. In this study, the girls recognized that their own participation in the movement would lay a foundation for future generations confronted with the same possibility of school closure. Margarita spoke of the importance of speaking up against the closures, stating:

> It's like you're being a role model for so many people. And you're showing them, you're showing them that if you, if you speak up you can make a difference. And then by doing this, we can actually make a difference. And then if it ever comes across when we're not in this school anymore and it happens again, they know what to do.

Margarita's statement captures the idea that the critical funds of knowledge generated by their experiences with making change are a potential source of informed critique and activism for future generations. From a feminista perspective (Villenas, et al., 2006), Margarita described the "living theory" regarding confronting oppressive social structures as they play out more locally in policies and subsequent practices (such as cutting public education funding) that the girls would generate and could form the basis for the education of future generations. This recognition of their own role in generating important knowledge for future generations supported their sense of critical mathematical agency to engage in the process of using mathematics to make change.

Voice within a Community Movement

Margarita repeatedly emphasized the collective nature of the girls' club activity as contributing to a meaningful part of the community movement. Margarita described what she learned from the project in the following way: "I learned that if you work together you can do something- you can make a difference. … I would say teamwork is important because people together make such a big difference, not just one person." The fact that the girls were a part of a collective was significant and facilitated their sense of the importance of their voice within the movement, furthered by the fact that they were the oldest students at the school.

Throughout her reflections and conversations, Margarita expressed a sense of responsibility to give voice to younger students who might not otherwise have a voice in the process. She felt that the girls' math club members had a responsibility to represent the younger students and that proving or showcasing this concern for others actually contributed to the strength of their argument. Margarita explained, "'Cause as fifth graders they say we're role models and [as] role models, we should be able to help and speak up." Margarita continued to argue for the importance of their involvement when reflecting on the makeup of the community members who attended the first board meeting regarding the closures:

> It's because the school's important, not just- cause whenever you look around, when you were watching who was helping, it would only be parents and employees. And like if the parents, usually like the ones that are older, that don't go to this school, like that used to come to this school, they were helping too. Yeah, but if you look around, it wasn't any of the kids from our school, like younger ones. So that's why. That's why I think we should help. To see that it's not just important to the parents and the employees and stuff. To see that it's also important to us.

Not only did the concern for others permeate arguments related to the importance of the girls' group participating in the larger community movement, but also in interpreting various mathematical analyses of the closures. Margarita brought up a concern with the impact of the closure on the community in a mathematical analysis of the walk to the proposed receiving school (if Agave were to close, students would go to a school 1.3 miles down the road). Margarita argued that the walk to the new school would adversely impact the younger students because the walk would be difficult and dangerous for them. I argue that because Margarita was from the community from which the project emerged, her sense of critical mathematical agency was strengthened by a desire to consider the needs of community members and the importance of the group's participation in the movement.

Connections to Previous Work

These examples highlight the wealth of critical knowledge present and being generated within communities who confront various forms of inequity on a regular basis and the potential value of integrating this knowledge in feminist, critical educational settings. From the standpoint of critical race and feminist theory, experiences of low income communities of color in confronting inequity are essential sources of knowledge about social change. Critical race theory purports that resistance to ascribed social structures (such as school district policies) is transformative when informed by critique (Brayboy, 2005; Solorzano & Delgado Bernal, 2001; Villenas & Deyhle, 1999), which positions a community's critical understanding of social structures as an important source of knowledge. These critical funds of knowledge were essential to Margarita's participation in the community movement. In the SOS project, Margarita repeatedly drew upon her own critique of social structures (such as test scores as a measure of a school's worth) in order to enact critical and mathematical agency. Margarita's unique experience, providing the perspective of a young woman of color, clearly highlights the often untapped critical knowledge present in communities of color and the possibilities of critical education.

CONCLUSION

The living theory generated from this case study highlights the importance of a feminist perspective of critical mathematics education. This perspective speaks to mathematics, social change and engaging in both within educational experiences.

Through analysis of her interview responses, Margarita's case highlights the importance of collective activity within the mathematical practices she engaged in during the girls' group. She also described her agentive stance toward mathematical knowledge generation, one which reflected the importance of conceptual agency, where power resides within the students' conceptual understanding rather than within an outside discipline of mathematics, akin to an outside body of knowledge one must acquire through banking education. This also aligns with her description of the mathematical activity within the club in which the problems the girls encountered invited their agentive approach to determining an approach to the problem, rather than a predetermined method they were expected to follow. These insights provide a perspective on mathematics education that relies on collective knowledge generation around real problems that matter to students.

Margarita's case also highlighted several themes related to social change. Margarita spoke of the importance of considering the sociohistorical context of policy-making, with her critique of the school district proposal to close a neighborhood school where generations of family members have attended. She also acknowledged the importance of the critical funds of knowledge present in the experiences of past generations in confronting oppression. Analysis of her participation also uncovered the importance of her own families' funds of knowledge, as evidenced in her mother's contribution to the movement. Margarita also highlighted the knowledge she gained through participation in the movement, namely the importance of collectivity in activism and the responsibility she has for giving voice to those who are younger than her. These often overlooked critical funds of knowledge are essential to critical education within classrooms in order to honor, value and incorporate the experiences of students of color into learning experiences.

Reflections on Feminist, Critical Mathematics

I close this paper with a reflection written by Margarita that captures the essence of a feminist perspective within critical or transformative education and the role of mathematics within such education. When Margarita asked me if she could write a reflection, saying "Miss, I just need to get it all out," she wrote the following:

> **My reflection on our Journey through possible Closure** Margarita
> The journey through possible school closure has been horrible by that I mean I can't believe we are going through this possible school closure we never had expected to come!!!!!!!!!!!!!!!!!!!!!!!
> This reflection makes me feel like I have changed on how I felt about my voice. Well because I used to think I have never gotten a say on particular things, and not this year I'm in math club for the first time. I actually feel like I have a say on many things now. I've thrived in this club this club makes me feel appreciated, loved, important, and also makes me feel like my voice is making a change. By all of this, if you don't understand me I mean I finally am in a group that gives you a chance to speak your mind when something very important is happening (which everyone should get or feel like they spoke their mind but not everyone has experienced that yet. I finally have after 9 yrs.) I thought this U.S place was supposed to be about freedom and people speaking what they felt. Maybe Mrs. Arías[2] was right not everyone is as nice as you want him or her to be. That's how I feel about you [district superintendent][3], Yes I am calling you out Mr. This whole journey makes me feel so strong at least I think stronger than ever before. I am just wondering why our school why now!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! Now I am starting my conclusion. Math was a big part of this journey it helped me find percentages. By the way when the news paper came out saying they were going to save $4 million we found out

[2] School Principal, name changed to protect anonymity.
[3] District Superintendent who initiated the school closure process.

> that same day that you would only save 1%. I'm not lying our video says that and our video came out before the newspaper saying you would only save 1%. SEE I CAN PROUDLY SAY AGAVE'S MATH CLUB (GIRL'S ONLY) HAS BEEN DOING THEIR MATH ALRIGHT!!! ☺ ☺ ☺ ☺ ☺ ☺ ☺ COMMENT: We are going to be survivors Agave it will survive yes yes. I HOPE SOME ONE GETS MY MESSAGE IN A BOTTLE![4]

Margarita's letter captures several themes related to the findings of this case study and the possibilities of transformative education such as feminist, critical mathematics education. She described the feeling of having a voice to make a change about something that matters to her. She also believes that all young people should have this chance to make change and spoke to the importance of and her pride in including mathematics in that voice, the ultimate goal of feminist, critical mathematics education.

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[4] Margarita is referring to the Police song entitled *Message in a Bottle*, which we included as the background music for the digital story.

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Juxtaposing Mathematical Identities: Same students with different contexts, perspectives, and languages[1]

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In mathematics, the intricate relationship between learning and identity is predicated upon students' quality of social engagement with mathematical practices. In this paper, I explore five Latino/a students' actions in and narratives about a mathematics afterschool program over a three-year period. Students juxtaposed their experiences within the program and those in the regular mathematics classroom, claiming a difference in the nature of mathematics and in the type of interactions with the participants. Their mathematical experiences, however, were mediated by both their language preference and their reified positions in mathematics, which, in turn, informed their perspectives on what counts as mathematics.

In this paper, I explore bilingual Latina/o students‘ interactions in a mathematics afterschool program and their narratives about these experiences in which they contrasted them with the experiences they had in their regular mathematics classroom. Simultaneously, I explore how these experiences and narratives mediate their mathematical and bilingual identities. Thus, I investigate participants‘ sense of self and self-world relations (Holland, Lachicotte, Skinner, & Cain, 2003), both with respect to mathematics and to being bilingual speakers, and to how their learning and identity evolve dialectically (Martin, 2006; Lave & Wenger, 1991) through social and linguistic processes (Ochs, 1996). In other words, the question is how the quality of students‘ social and linguistic experiences in specific environments mediate how they situate themselves—and are situated by others—as mathematics doers and bilingual speakers. This focus on students‘ narratives and actions considers critically their opportunities, access to resources, and ―the mirrors and windows provided to a student" (Gutiérrez, 2008, p. 360).

Previous studies inform us that students‘ ideas about mathematics and its uses are impacted by the different ways they engage with others around mathematical practices (Lave & Wenger, 1991; Nasir, 2002), appropriating mathematical discourse and identity (Boaler, 2002; Moschkovich, 2004; Nasir, 2002), and developing mathematics *dispositions,* i.e., ―ideas about, values of, and ways of participating with a discipline" (Gresalfi & Cobb, 2006, p. 50). Language

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during interaction not only has a symbolic content, but also represents a socially-organized praxis whereby participants develop an integrated and situated understanding of events, knowledge, emotions, status, and relationships among people; thus, language is a socialization tool, and simultaneously, its use is socialized (Ochs, 1996; Jackson, 2009). Research in and out of school settings has similarly demonstrated that afterschools with flexible structures (blurring categories such as student/teacher, play/learning, and school/home) promote interactions that capitalize on children‘s interests and choices, providing opportunities to enact multiple social identities (Cole & the Distributed Literacy Consortium, 2006; Vásquez, 2003).

Being the largest minority at 15.4% (US census, 2007), Latinas/os‘ social situation in the US is often aggravated by deficit model approaches in terms of language, race, or ability (Cashman, 2008). Arguments against language and cultural diversity are based on the belief that diversity brings divisiveness. These ideas commonly transfer into students‘ negative attitudes toward their own heritage language, often to the point of their refusing to speak it (Brisk, Burgos, & Harmela, 2004). Over one half of Latina/o students speak mostly English at home (Llagas & Snyder, 2003). Latinas/os are less likely to have access to experienced and qualified teachers and to receive equitable funding per student, and more likely to face low expectations. Thus, Latinas/os‘ achievement gap problem could be reframed as an opportunity gap (Flores, 2007).

From this standpoint, the mathematics learning process closely relates to what students do, their opportunities, use of language, and identity development. The exploration of Latina/o students‘ voices regarding their mathematical and bilingual socialization processes may help us better understand how ecologies mediate their experience as bilingual learners of mathematics.

Setting and Cases

Los Rayos de CEMELA, a non-remedial afterschool mathematics program, was an adaptation of the *Fifth Dimension* (Cole, 2006) and *La Clase Mágica* (Vásquez, 2003). *Los Rayos* provided a hybrid space promoting both languages; with a playful approach, but not play; with a school subject, but not school (Khisty & Willey, in press). Its problem-solving curriculum included probability, proportions, geometry, and pre-algebra. Students met twice weekly for ninety minutes and worked in small, self-selected groups made up of 2-3 students, 1-2 facilitators (undergraduate pre-service teachers called ―UGs"), and occasionally some students‘ mothers.

Participants, mostly US-born and from a Mexican background, are all bilingual (Spanish and English). Thirty-one students participated over a three-and-a-half-year period, having an average of seventeen students per semester, though twelve consistently attended the program. It started with their 3rd and ended with their 6th grade. Students self-selected to be part of *Los Rayos.* The hosting public school in Chicago—a dual-language school—had an almost entirely Latino, low-income student population, with over 60% English language learners (Office of Research, Evaluation, and Accountability, 2008). For this study, I selected five student cases: Candy, Graciela, Patty, Ramón, and Mario, who respectively had average math grades of: B, A, C, A, and C. These cases were selected for comparative purposes (Miles & Huberman, 1994).

Framework

Rooted in a sociocultural approach, I consider identity development as a socialization process which involves the assignment of meanings to specific situations through *language* and *actions* by both the target subjects and others (peers or adults); these meanings in turn inform participants' dispositions, goals, and actions about being a kind of person, a math doer or a bilingual person (Ochs, 1996). Thus, I define *mathematical identity* as socially negotiated dispositions and beliefs about one's ability to participate and perform effectively in mathematical contexts (Martin, 2006). Correspondingly, I envision *bilingual identity* as a set of recollections of individual and collective experiences and consequent beliefs about one's language skills which mediate linguistic preferences and values. As participants are socialized, their identity evolves from *peripheral* to more *central* participation in a community (Lave & Wenger, 1991).

As I explored students' experiences and voices reflecting their bilingual and mathematics identities, I observed that students' narratives juxtaposed their experiences based on the setting (regular classrooms or the afterschool program). Consequently, I used the term *context*, i.e., the relationship between a setting and how participants interpret it (Moschkovich & Brenner, 2000). Similarly, students contrasted their *perspectives*, i.e., personal relationships with and definitions of mathematics mediated by their status and opportunities in the subject. Finally, they discussed their *language preference*. Thus, identities are produced and changed in social spaces with *sociomathematical norms* (Yackel & Cobb, 1996) in a multi-layered, complex process involving different *ideologies* and cultures, a political process that evolves in the classroom, the world, and in students' heads (Hirst & Renshaw, 2004). This ideological *contestation* driving social actions informs students' identities as math doers, learners, and bilinguals. Thus, power dynamics or *distribution of authority* in their mathematical learning (to whom and about what) (Cobb, Gresalfi, & Hodge, 2009) privilege certain voices, thus reifying conventional (*script*) and unofficial (*counterscript*) ways of speaking and doing mathematics (Hirst & Renshaw, 2004). Together, these processes may promote either *conceptual agency* (processes support students' already available resources) or *disciplinary agency* (processes linked to structures of a discipline) (Boaler, 2002), or even better, combine both. This frame presents mathematical and bilingual identity as a socialization process indexed both by participants' narratives (about their experiences) and their interactions with others in contexts with certain types of relationships (social, conceptual, and ideological) that afford opportunities and mediate the alignment and appropriation (or not) of local practices.

Procedures and Analysis
This study of students' bilingual and mathematical identities explores both *narrative* and *discursive* dimensions of students' *identities*. *Narrative identity* refers to relevant, reifying, and endorsable collections of stories that students tell of themselves and which are collectively co-constructed practices (Sfard & Prusak, 2005). Here, students express their rationales about the kind of persons (math doers or bilingual speakers) they think they are and why. For this, I analyzed interviews conducted at different points during the program. These include two individual student interviews per case in years two and three (forty minutes each); a collective debriefing session at the end of year one (twenty minutes); and three focus-group sessions during the last semester (thirty minutes). Interview questions explored the purposes of the program, including students' experiences in the afterschool, their experiences learning mathematics in and out of school, and their language use (Spanish, English, or both). Questions ranged from semi-structured to open-ended formats, and students' ideas were elaborated by the interviewer. Facilitators conducted interviews either in Spanish or English. The second dimension, *discursive identity*, refers to the participants' actions and reactions during interactions with others that signal meaning about ―who" the participants are (Brown, 2004). Here, I analyzed teachers' interviews, UGs' field notes, student-developed artifacts, and three years of videotaped interactions at *Los Rayos* that signaled the discursive identity of student cases. Specifically, I explored and selected episodes that triangulated students' narrative identities.

This analytical process consisted of the identification of general themes drawn from a pool of memos focusing on two questions: a) *What meaning do students make from their own participation in mathematical practices?* and b) *What does it mean to these students to do mathematics as a bilingual person?* From this pool, I separated episodes referring either to mathematical or bilingual identity and developed a corresponding open coding system. As I noticed students were differentiating between the quality of mathematics they experienced in regular school and in the afterschool program, I decided to develop themes that would highlight these differences. Second, I revisited the selected episodes using videos and transcriptions, first to develop conceptual themes identified through narratives, and then to triangulate, support, or contrast them with episodes of discursive actions depicted in the rest of data sources.

Key Themes and Results

I present results concerning students' mathematical identities in three main sections: 1) contexts, 2) perspectives, and 3) languages. Only the last section deals specifically with bilingual identity. Throughout, I describe and contrast students' mathematical identities indexed by their *narrative identities* (what they tell) and *discursive identities* (what they do).

1. Development of Mathematical Identities in Two Contexts

Students' narratives distinguished between two mathematics contexts (afterschool and regular classroom) in three main variables: **(a)** the quality of interactions, **(b)** the access to resources, and **(c)** the quality of mathematics.

(a) Students asserted that the nature of interactions they had in each context mediated the meaning and quality they perceived for each context. Partly, this relates to the number and kind of adults they worked with in each setting. Graciela (G) asserted having enjoyed working with older people—that is, the UGs (undergraduate facilitators)—because she perceived them not only as ―cool and popular" but also as understanding of the students and able to speak about ―outside school stuff":

> F (facilitator): So you just think that with the UGs you can talk about things at another level?
> G: Yeah, it is like having an older sister that I never had. *Se siente bonito porque nos tienen confianza*.

It seems that the level of trust with the UGs is connected to a greater level of intersubjectivity with students, maybe due to a similar age (younger than teachers),while with teachers the gap was linked to school structure. Patty (P) and a friend (A) observed a different type of attention from these adult groups:

> A: Because teachers do not really…, okay when they give us a problem, they just say to do it. They do not really ask us, they ask us if we need help when we are doing the work. We usually need more time, I do not know more information about what is it going to be about. You are usually there and if you understand fine, and maybe it is going to take a different way.
> P: Yeah, they say you can ask them, but then when you ask they say: ―Why weren't you paying attention!!!" yeah…But here the UGs see if I really get it.

Patty interprets the teacher's attention and actions as less caring and effective in mathematics as those of the UGs. Students also reported that in the formal mathematics classroom environment, they had less of a central role and less support during the meaning-making process in mathematics. These aspects seem connected to the type of negotiation that students had access to as well as to the level of authority exercised by the teacher, which in turn transferred to unproductive dispositions about the way they did mathematics in their classroom. Contrarily, they seemed to identify with the normative identity at *Los Rayos* and noticed more genuine mathematical support, generating different dispositions about the context and themselves. Mario and Ramón described themselves interacting differently at *Los Rayos* because mathematics becomes part of their social interaction: –Mario: [At Los Rayos] we can like have fun and then think, and then have fun and then think about the problem and talk." Similarly, Patty said: –At school we really get to speak to each other only during lunch time." Contrasting with their classroom practices, here students‘ interactions involved a different distribution of authority, collaboration, problem solving, discourse, meaning making, and fun—i.e., their own sociomathematical norms. Students engaged in various games that nurtured relationships and interest in mathematical concepts. Students enhanced their understanding of the underlying mathematical concepts by manipulating and playing with them. Thus interaction patterns in the afterschool nurtured networks across and within generations around mathematics. These results are consistent with Strobel, Kirshner, O‘Donoghue, & McLaughlin (2008).

(b) Students described accessing resources (e.g., manipulatives) differently in each setting. Ramón and Mario contrasted their experiences based on their opportunity for choices:

R: In class the teacher picks the groups, but in CEMELA we pick our own.
M: Yeah, in CEMELA we can choose our friends and whatever we want to use.
R: Yeah, in class we just have paper and stuff.
C (facilitator): Are you saying that you cannot use stuff like blocks and calculators in your class?
R: Yeah, but the teacher decides when, then we just got pens and paper.

Students emphasized not being able to access materials and friends as resources or to choose when to access them. *Los Rayos* capitalized on students‘ choices by not only having them select their resources, but also the tasks they wanted to work on. Thus, students asserted a different distribution of authority and negotiation in each setting: facing passive/active roles in decision making and open/limited access to resources. Schoenfeld (1985) declares that selection of heuristics during problem solving nurtures metacognitive development.

(c) Students also reported engaging in a different kind of mathematics in the afterschool program. One difference concerns the role of control in each setting, especially to whom students were accountable. Ramón and Mario described having a critical role by being accountable to each other in the afterschool program, portraying a collaborative process and exercise of conceptual and disciplinary agency in mathematics. By contrast, in the classroom the teacher possesses the authority and control. Mario characterized his mathematics class as ―boring,” thus limiting the development of productive dispositions towards engaging and becoming a problem solver in that context. In a way, he negotiated these practices by resisting them. During an interview, Mario role-played what he meant by ―boring” by standing in front of the blackboard and asking me to copy what he wrote (i.e., 5 x 1 = 5, 5 x 2, etc.). When he role-played CEMELA mathematics, he constructed a small clay dinosaur and compared it with a much taller toy dinosaur and asked me: ―How many times it [small one] needs to grow to become as big as this one [tall one]?” He solved it by developing a multiplicative structure. He said it was ―fun” math, not only contextualized, but purposeful: ―you have to do something, not only numbers and reading; you talk, and you do, and then make, and then we learn,” i.e., with what Boaler (2002) calls a ―dance of agency.”

The other cases resembled Mario‘s argument. Graciela, who was consistently a high-performing math student in her class, argued that math started becoming different to her when she joined *Los Rayos*. What doing mathematics meant was transformed; for her it became experiencing mathematics with comfort and enjoyment: ―I saw the fun side of the math.” Similarly, Candy claimed that her relation with mathematics reached a turning point: ―When I came to CEMELA, I started doing better in my class because I felt better and I knew how to respond to the teacher.” Candy expanded her conception of mathematics with an experience that not only filtered the level of anxiety, making it more accessible, but also helped her acquire a practice, a type of discourse (not necessarily vocabulary), that increased her comfort enough to engage in mathematical conversations in her classroom. In fact, one day at the afterschool Candy decided to play with manipulatives. She and a friend created various designs with blocks to find perimeter and area. Although she brought the topic from class, she played with it by deconstructing and recreating it:

> Candy created ―a flying snake.” She did not want to find out the perimeter because it would be too long, so she only found the area. She used a strategy of grouping and counting cubes by color; though she said, this was still a process for finding the area, though it was not like the previously learned formula: it‘s a new way. Alma suggested getting the area by multiplying chains of blocks that were the same length (CL field note).

Summarizing the two contexts of mathematical identity, students developed particular relationships with mathematics (Boaler, 2002) and with each other at *Los Rayos* which differed from mathematical practices and relations associated with the classroom. Table 1 displays students' narratives and actions regarding the three variables they used to differentiate contexts. As I explored their discursive identity, I noticed that in the afterschool students at times also faced these variables in limiting ways. I assumed, therefore, that this could also be the case in the classroom. Nonetheless, students' narratives clearly depicted two ways of doing and relating to mathematics depending on the context, two parallel rather than intersecting mathematical identities, which were informed by social dynamics in each context.

Table 1: Mathematical Identities in Two Contexts

Regular Classroom				**Afterschool Program**			
Perception of Practice	Interactions	Use of Resources	Quality of Mathematics	Perception of Practice	Interactions	Use of Resources	Quality of Mathematics
Limiting	D/N	D/N	D/N	Limiting	D	D	D
Enriching	D	D	D	Enriching	D/N	D/N	D/N

Key: D = portrayed by students' Discursive Identity; N = depicted by students' Narrative Identity

A regular teacher from the hosting school commented: ―CEMELA helped kids to strengthen their self-esteem […], these students take leadership roles in the classroom. I think the UGs provided lots of strategies, it's like what Beth Warren says about the hidden curriculum." Thus, she noticed CEMELA students having appropriated practices, discourses, and tools necessary for learning in school, but not necessarily ―taught" in regular classrooms. These realizations come from students' voices after having the opportunity to experience mathematics in a new setting. Paying attention to the practices they consider enriching may help regular classrooms promote stronger relations with mathematics and mathematical identities.

2. Development of Mathematical Identities from Two Perspectives

Although all five students described having productive mathematics dispositions and leadership roles in the afterschool program, some still reported having marginal participation:

> C (facilitator): How do you share your strategies and solutions in class?
> M: I don't know, the teacher just asks… I just ask Graciela, she always knows, then she tells us.
> C: But you can have your own answer, right?
> M: The thing is that we ask them [pointing to Ramón] because they are so smart. They have like a big, big, big brain. They are like the fastest. And us [pointing to another male peer], ours are like dead!

As Mario (M) detached himself from the practices promoted by his teacher, he also declared having certain difficulties understanding the task. Consequently, he willingly relied on others (Graciela and Ramón or the teacher) whose reified positions promoted a more central role in the mathematical practices. However, various facilitators reported that Mario successfully engaged in math, for example: “Mario was counting nickels and said the total was 180. I asked ‘what, 180 nickels or money?‘ Then, he paired nickels, counting by tens and got 180. I asked the same question, and he said, ‘180… un dólar con 80 centavos.‘” Mario commonly performed mathematics in unconventional ways, but when interested, he made an extra effort and succeeded, while others with more conventional, effective means completed the task sooner. This seemed to inform his own and others‘ beliefs about himself having a peripheral role and less disciplinary agency. Mario may have appropriated and used certain roles that had been socialized and reified in the classroom to identify his own and others‘ status. He claimed that this social construction of status transcended settings, privileging some and excluding others—like him—in mathematics (Cohen, 2000): “Ramón gets to do all the work.” Although Mario identified these patterns across contexts, he seemed bothered by that privilege in the afterschool program. In fact, at *Los Rayos* Mario took up leadership roles. When he—a 6th grader—tutored 3rd graders, he created tasks what he knew and considered as “fun” math.

These conflicting narrative and discursive perspectives in Mario‘s case are similar to Patty‘s, who also took active roles doing mathematics in the afterschool program. These episodes of ownership for both did not, however, seem to produce a productive perspective. So, in a way, they engaged in these tasks, but they did not perceive themselves as engaging in math per se. Rather, they saw themselves in an activity where mathematics was simply a functional tool. Thus, they associated their “belonging” to the activity, but not necessarily to mathematical practices. The following write-ups (Figure 1) illustrate these divergent perspectives:

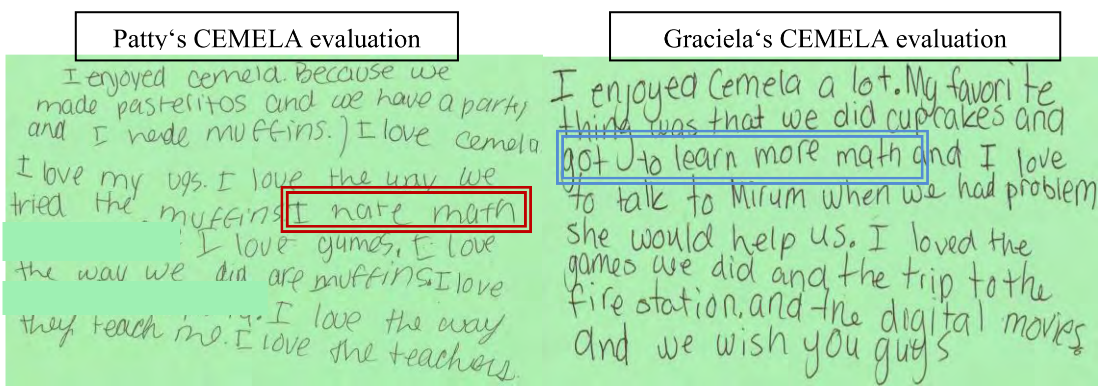

Figure 1: Students‘ evaluation of CEMELA‘s activities

Despite these students‘ continuous collaboration, Patty contrasted every practice as either “loving” or “hating” and located math in the latter category. Graciela‘s write-up partly resembles Patty‘s; however, she cited “to learn more math” through

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the activities. As Graciela loved the projects, she also loved the math. Participants—though in the same task and context—developed subjectivities and connections to mathematics from different perspectives. Cohen (2000) informs us that status carries over from setting to setting; here status seems to mediate the meaning of and belonging to math, a fixed experiential identity that, in Patty‘s case, transfers and maintains a deficit perspective. This perspective might relate to students‘ mathematics definition and/or to their experiences in their relationship with math. Perhaps Patty‘s comprises what she hates: a decontextualized, formal subject demanding disciplinary agency. In fact, a facilitator reported in her fieldnotes: ―Patty gives up too quickly; she says she isn‘t smart. I think she needs a little push. Perhaps, she doesn‘t try to do certain things because she is afraid that she might go wrong.” Thus, it seems that overemphasis on procedures and demands on disciplinary agency may limit Patty‘s attempts to engage and succeed in mathematics. In situations that promote her conceptual agency, she is more willing to take chances and do mathematics. Oppositely, Graciela includes a broader definition: ―math with a fun side.”

Table 2 summarizes students‘ perspectives portrayed through their narratives and actions as math doers in class and at *Los Rayos*:

Table 2: Mathematical Identities in Two Perspectives

Regular Classroom			**Afterschool Program**		
Student Cases	Math Doers	Not Doers of Math	Student Cases	Math Doers	Not Doers of Math
High / Regular Performing	D/N	D/N	High / Regular Performing	D/N	D
Low Performing	D	D/N	Low Performing	D	D/N

Key: D = portrayed by students‘ Discursive Identity; N = depicted by students‘ Narrative Identity

Students with higher achievement seem to have greater narrative and discursive resources that enrich and mediate their concept of and perspective regarding mathematics. These situations ask us to reconsider and re-mediate students‘ experiences, not only as how they imagine their participation (Nasir, 2002), but from what perspective.

3. Development of Mathematical Identities in Two Languages

Although students spoke either language at *Los Rayos*, English dominated. Students used more Spanish in only a few projects (e.g. recipes), perhaps because those projects drew more from home-related experiences. Regardless of their language preference in discursive practices, overt negotiation of language usage seemed tensional. In Mario‘s interview, although he initially chose Spanish, he seemed troubled supporting his preference and claimed liking English better. Students‘ language choices appeared to give students different identity perspectives aligning—or not—to powerful (English) or less powerful (Spanish) paradigms. For example:

R: [reading] *that are common to the circle and the pentagon, but not in the triangle or the rectangle*[2]
A: Do it [read] in English!
J (facilitator): He can do it however he wants. He can do it in English or Spanish.
A: I can't understand him!
J: [to R] Can you tell him what you mean?
J: [to R] Ok, let's take a look at it. *That are common to the circle and to the…, what is it?*
A: Read it in English!
J: [to A] I will. [to R] *But not in the triangle or the rectangle. Does it mean that it is…*

When students demanded the use English, it often resulted with facilitators switching to English, sometimes for the rest of the activity. Conversely, instances demanding Spanish never emerged. The argument here is not whether students choose one language or the other, but that the process represents a highly contested space in which the dynamics are likely to privilege English. In this regard, all students reported having to speak Spanish at home because their parents do not speak English. Their statements reflected a need to justify using a language other than English at home. Students used verbs implying an obligation rather than an option or asset. Twelve out of fifteen students from the afterschool self-reported higher fluency in English, though Spanish is their primary language. Table 3 depicts how language preference was indistinctively distributed as students discursively interacted in mathematics without difficulties. However, in their narratives, their choices seemed connected to achievement level. Students with higher disciplinary agency and more central mathematics participation identified themselves with the dominant language, perhaps reinforcing their centrality.

Table 3: Mathematical Identities in Two Languages

Student Cases	**English**	**Spanish**	**Both**
High/ Regular Performing	D/N	D	D
Low Performing	D	D	D/N

Key: D = portrayed by students' Discursive Identity; N = depicted by students' Narrative Identity

It is pertinent to further explore whether low performing students chose the bilingual option in connection to their peripherality, or whether their low achievement was mediated by their language preference, thus affecting their social status. Perhaps society's political language issues infiltrate, affecting students' performance (Cashman, 2008; Hirst & Renshaw, 2004).

[2] Italicized text represents translations from Spanish to English.

II. Discussion and Implications

Students‘ narrations and actions reflected experiencing relationships of different quality in two contexts: *Los Rayos* and the regular classroom. These relationships reveal that students developed distinct ways of committing to mathematics. Correspondingly, students described how these contexts, with different levels of authority and capitalization on students‘ conceptual and disciplinary agency (Boaler, 2002), mediated a different quality of interactions that adults had with students and that students had among themselves (Strobel et al., 2008). At the same time, these ecologies accordingly provided different access to resources and mathematical experiences and practices that turned into two different relations with mathematics. These juxtapositions made in students‘ narratives inform us about what students appreciate and how these processes provide opportunities to develop stronger and richer relations with mathematics. Students developed a counterscript against authoritarian and teacher-centered approaches and described how these social and linguistic processes constrained their relationship with mathematics, which is not exclusively academic. Thus, the social ecology and the manner of engaging in mathematics impacts how students narrate their relation with the subject or what becomes to them their mirrors and windows to it (Gutiérrez, 2008). Schools and teachers interested in developing stronger student relationships with mathematics need to reconsider the social quality of opportunities, practices, and ―windows" for students to access mathematics. Perhaps, school power structures would benefit by re-structuring its power dynamics between teacher and students. Perhaps, regular schools could expand their practices by integrating some of what students have argued here as beneficial in their encounter with mathematics, which accounts not only about the subject matter itself. Perhaps, afterschool programs and regular school practices could work together and be implemented to provide students different opportunities and approaches to learning and doing mathematics.

However, other challenges still remain since students‘ identity as doers of mathematics was mediated by the different ways they interpreted or perceived the mathematical practices and themselves, that is, their own perspectives. Students with regular higher math classroom performance seemed to have a wider scope of resources that allowed them to re-interpret and expand, in their perspective, their concept of mathematics and their relationship to it, which added new values to their mathematical repertoire. Nonetheless, despite their active discursive engagement with mathematics in the afterschool program, students with lower mathematics performance reported a continuous peripheral role—not always consented to—in mathematics across settings. This might have translated into separating their identities from ―mathematics." Interestingly, these positions were recognized and reinforced by most peers and adults. When these students faced problem solving tasks they understood, they often engaged in mathematics in unconventional ways; they took risks and solved the tasks. But when more standard forms of mathematics were demanded, they withdrew, claiming they either hated or disliked it. Perhaps what they meant is that they feel successful when they use their own familiar resources. Thus, these students ended up distancing themselves from mathematics, one where they were continuously unsuccessful, thus, developing a non-mathematical status and identity. As a result, their engagement in new practices at *Los Rayos*, although mathematical, these practices or activities were not deemed as entirely mathematical. Their previous mathematics engagement of reiterative, powerless social positions and low expectations in their classroom shaped their mathematical perspectives, narrowing their sources and definition of mathematics, consequently causing them to overlook themselves as doers of mathematics. Cohen (2000) argues for the promotion of equitable participation. I further claim that, as teachers and researchers, we also need to explore from what perspective students are seeing their participation in mathematics, so that in cases such of those in the afterschool, we not only promote situations and task where students with usual low-performing achievement are successfully engaging in mathematical activities; but also, at the same time we may need to overtly and explicitly reinforce the fact that those activities relate to actions of doers of mathematics. The main idea is to become proactive, so that these students may also have a chance to see themselves, become aware, that they, too, are successfully connecting with and doing mathematics.

Finally, students' bilingual mathematical identities emerged with different kinds of tensions. Whereas students fluently and discursively utilized both languages (Spanish and English) in mathematics as needed, any overt, apparent negotiation of language subordinated Spanish. In fact, all interviewed students but three (including the two students with low mathematics performance) self-identified as English-dominant speakers. Interestingly, students who selected bilingual resources also evoked peripheral mathematical participation. I wonder whether their reified peripheral status might have led to or given them the chance to choose linguistic practices peripheral in the US. This pattern calls for deeper study of the interconnection between language preference and mathematics performance and their relationship with social status and learning. I point this out, since students surrendered bilingual identities in the face of dominant paradigms (use of English). As schools may open different windows for students informing their mathematical perspectives, I think that our societies, with their demands, open windows to students that inform them of their language status. For example, a current controversy such as immigration law SB1070 in Arizona informs students in various ways about attitudes from mainstream society against and criminalization of immigrant people often related to Latinos/as. Consequently aware of such ideologies, students take stances to help protect themselves from pervasive racist language ideologies. Perhaps this acquired disposition regarding language differences eventually translates into an intentional form of social assimilation in the form of a pruning process of their bilingual discursive capital, and students thus willingly acquiesce to dominant language ideologies, becoming English monolinguals (Brisk et al., 2004; Hirst & Renshaw, 2004; Valenzuela, 1999). In fact, students' narratives and actions portrayed negotiation processes connected to elements beyond their immediate experiences and subordinated their conceptual agency to master-narratives on language (Brisk et al. 2004; Martin, 2006; Valenzuela, 1999). Therefore, as schools support bilingual education, perhaps we may need to include processes that go beyond providing translations to students or simply bringing bilingual people together into an educational space. Perhaps, what we may need is an overt decolonizing approach for teachers and students challenging appropriated ideologies on power differentials of language (Tejeda, Espinoza, & Gutierrez, 2003), so students may see the whole set of their bilingual resources as an asset in their education and their mathematical practices.

Schooling appears to influence the socialization of students in mathematics as a specific type (high- or low-performing) by either promoting or inhibiting their disciplinary or conceptual agency. Thus, equity promotion calls for both the development of intersubjectivity and students' positioning as valuable and competent in mathematics (Esmonde, 2009). Latino/a students deserve access to high-quality mathematics education and to their personal life experiences, that is, an education that honors their diversity and is constructed upon a genuine knowledge of their real identities and not those projected onto them, on opportunities for mathematical discussions in their preferred language, and on mathematical tasks that account for and further students' personal and mathematical knowledge and resources (Gutiérrez, 2002; Moschkovich, 2004), an education where goals, identity, and learning intersect (Esmonde, 2009; Nasir, 2002).

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After School-Reactor: Shelley Goldman

WHAT: CEMELA-CPTM-TODOS Conference
WHO: Shelley Goldman (Stanford University)
PLACE: Tucson, AZ
DATE: March 6, 2010

TRANSCRIPTION

Shelley Goldman

I'm very excited and inspired by all the wonderful work we have heard about from the practitioners and the researchers. Our collective task is to think about what we're learning from research, and what we can do with those learnings. The papers we have heard indicate that we have made some important progress and also point to the great amount of work we still have to do.

I tried to focus on what we could learn from the papers and the experiences they reference. Encouragingly, there were many positive leanings that we need to review. Even though this comes as a list of points, it can help us discuss and continue to move forward on an agenda for how after-school can be powerful learning settings. Let me qualify that I will not treat every item on the list equally. My job is to point out the many topics of interests as a way to seed further discussion. The list will give us a picture of where research has been and what we're learning about the informal after-school space. My list from the papers is very consistent with what we heard from our practitioners, so I'm very enthusiastic about where this might lead us.

The commentary takes up what was learned in three main ways:

1–review of main points from the papers
2–synthesis of after-school
3–synthesis of math learning
4–How we might seed our discussion of how to move forward

1. Implications from the papers.

The paper presented by Olga Vásquez and co-authored by Angelica Marcello, *A Situated View at "Scaling Up" in Culturally and Linguistically Diverse Communities: The Need for Mutual Adaptation*, examined the scaling up process for after-school initiatives. The paper represents a critical part of the conversation that we need to be in at this time. Olga defined scale in an interesting way that identifies what is important: adapting reform to the needs of the local context and engaging in a complex and multi-faceted process of mutual adaptation. The story of TACKLE was a story of “stepping foot into the other culture," and it was a big step for a program to take. I was reminded of a story that is told of a woman who arrived in Alaska from the one of the lower 48 states to take a new job. In talking about it, she said something like, "Oh, I finally made this big trip out here and I'm ready to work.” One of the Alaskans put up her hand and said, "Did you walk here?"

(Laughter) The point was that in walking there, she might have learned enough to be ready to start the work that she needed to do. Walking to and through Alaska might have given her the experience, knowledge and perspective with which to start the "on the ground" work. Olga's paper tells the story about walking together in the new culture, taking it very seriously, and taking the bumps—the, "walk-don't-fly" kinds of comments. The walking in the culture causes you to recoil, rethink, reflect, learn, and ultimately, end up with a much better informed version for going to scale with the work.

There were also five principles in Olga's paper that help with stepping into the other culture. (1) Draw on the cultural resources of the local community. This is a central aspect of CEMELA, as well. (2) Focus on multiplicity and diversity. (3) Make a commitment to change. (4) Build a direct partnership with community members. And, (5), take a multigenerational approach. Even with these principles as a guide, Olga tells of how trying to scale the program in another cultural community caused them to hit speed bumps. They discovered many tensions moving the model, but ultimately they had success from maintaining the principles.

Carlos A. López Leiva's paper, *Juxtaposing Mathematical Identities: Same Students with Different Contexts, Perspectives, and Languages*, picked up on one of the principles, which was the multidimensionality, the "multi," in all these culturally centered learning experiences. "Multi" is important in all its guises. It may be about being multilingual; it may be having your people moving in multiple settings. There are all kinds of multidimensionality around, and in studying mathematical and the bilingual identities and making a comparison from school to CEMELA after-school experiences, much was learned including that identity is complex and not one thing. We're still trying to discover exactly how to understand identity theoretically and practically. Carlos's paper brings up that a multiplicity of identities are being negotiated simultaneously by individuals and groups as they're moving through different institutions where they are coming in contact with different and new kinds of experiences. Mathematics is one of those new and different kinds of experiences. Carlos has identified some factors in his papers that led to transformations in identity and what math meant to the kids. Let's examine them.

Factors that played a big role (with some being more critical than others)

- Choice and control over the activities engaged
- Connections between real life mathematics and applied mathematics
- Relevance to important critical issues, problems and skills
- Giving kids authority, agency, control, and helping to develop them
- Different kinds of accountability and locus for it across institutions
- The idea that learning can be fun yet purposeful
- There were more, and maybe even better learning experiences (than what is usual in school) to be had in the different settings, especially in the after-school setting.

Besides identity, Carlos pointed to some important factors that revealed that were specific to math learning. The kids had opportunities for problem creation, for reasoning, for symbolizing, and for engagement in problem-solving tasks regularly. These opportunities, that kids had with math, also provided them opportunities for developing their language expertise in both of their languages. That last point about language development is crucial.

Maura Varley Gutiérrez' paper, *Margarita's Living Theory Informing a Feminist, Critical Mathematics Education*, reiterated and reinforced some of the analyses of the earlier papers. Her story of Margarita examines confrontational, conflict and liberation aspects of education that were available.

The paper considered how math could be the space where the margins of power are confronted. The math identities that were developed by the kids were examined along with and in relation to the Agave school closing. The "Save our school" movement that was engaged became the important nexus for liberation. This authenticity of experience in the community as a driver of identity and learning was present in the other papers as well. Olga, for example, told how moving to a real-life, global issue and problem had a big impact, making an education-for liberation stance possible. Together the papers drive home the point that critical education stances are crucial to be taken on with the children. Liberation education may even be the perfect lever for content learning. Maura's analysis reveals the important aspects and kinds of experiences that after-school environments can foster that help or transform math identities and thus productive math agency.

Let's take a look at the big list of factors culled from Maura's and Carlos's paper that have to do with math, and in Olga's paper specifically about after-school. As a thought experiment, let's treat them as "facts" instead of fact-ors or principles that diminish or qualify their certainty. The language of "factors" keeps us very tied to being careful with our research and how we govern our treatment of data. Let's be bold for today, allowing their qualified findings to be treated as facts so we can take them up in discussion.

2. So, what positive messages about learning do we know from these papers about after-school?

We know that identity can be exercised, and that in after-school, there is some freedom from school standards. This may be a temporary freedom. In after-school we still have the ability to have more informal and friendly social arrangements among learners and more gentle status relationships between adults and children. We can access various physical and community settings in after-school. Control of program and activities can be shared and negotiated, with kids and community members having some power over the spaces and social conditions of their learning. There are still choices possible in after-school.

The informality of after-school lends itself to natural ways for language to be used and transacted in ways that make sense to the participants. Olga's group recognizes that there is a language revitalization problem, yet the paper points to a wonderful real-world relevant need around language issues and development that could be its hallmark. There is the ability for kids, adults, and community members to naturally code switch with access to the language choices.

Last, after-school offers opportunities for community-involved, multiage, and multigenerational work. This enables the ability to meaningfully share the work with people that are important to all in the after-school centers.

3. Now, let's move to some of the "facts" we learned about math.

Math can be a space for confrontation. Math can be about work on problems that are of immediate significance to kids, their families, and their communities.

Math can be a place where the learners are organized for fun and engagement as they're working on math. There is a drive towards participation and there is access to problem solving and reasoning. Learners have the ability to generate, test out and reflect on different kinds of strategies for math work and math problem solving.

There is a drive toward these crucial math participation structures—and without the pressure of accountability to which most kids are accustomed in school math. The papers document that learners have the ability to evaluate their own solutions in the after-school space. Taken together, we see an inverse relationship that defines math learning in after-school: accountability demands are decreasing while abilities to learn and self-assess are increasing. These are conditions that encourage math learning.

4. How do we summarize what we can do to further our goals for learners with this list of facts in ways that can move our conversations forward?

We need to continue developing critical education settings. Olga shared aims for an agenda addressing educational, social, economic, and global issues. We need to keep confronting inequality as a regular part of what's getting done in after-school, and do that with a sense of past, present and future, and through engaging language, culture, and community. My emotional response is to implore that we should proceed by keeping school out of after-school as much as possible. Or more boldly, after hearing from our practitioners, Sagrario Guadalupe Moreno, Susan Foree, and Grace Dávila Coates, we should be working to bring the successful facts of after-school into school, and try to steer the conversation in that direction. I know that we generally experience the opposite pressure. I work at a charter school and I see our teachers are dissatisfied with the after-school program because it's not supporting what they're trying to accomplish in school to the extent they would like. They would like the after-school program to be more of an appendage to their school program. In short, they'd like after-school to reinforce school learning objectives. School is constantly trying to push into the afterschool space.

Let's not yield to that pressure. After-school learning needs to keep its identity and maybe even push into school. We heard today how math learning could be a vital part of the after-school story. It's extremely important that math that matters and is of relevance and consequence to the kids is considered a staple in their learning. It should be present at all costs. After-school needs to stay free of a failure imperative as a counter-balance to what exists in schools. It needs to draw on the resources of the local community, focus on diversity and multiplicity of experience, and embrace participation structures that make math a part of the life of the community. All together, the papers and testimonies generated today give us this fountain of facts with which to think, discover, and create successful learning in the culturally relevant after-school space.

Transforming Mathematical Identities Through After-School Settings Discussion

Following the research presentations, the practitioner panel, the reactor, and the participants met in small groups of six to eight for discussions. The groups included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. The task given to the working groups was to address the following questions:

- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The connections question became embedded in the discussions of the other questions. This summary represents common themes identified within and across the working groups.

What do we know?

Combining the presented research studies from the Transforming Mathematical Identities Through After-School Settings Strand with presented poster sessions, and with our own background knowledge and experiences, we are able to state what we believe to be true about the transformation of mathematical identities through after-school settings with English Language Learners (ELLs) and particularly with Latinos/as.

After-school settings can offer a unique situated approach for learning often not present or afforded in the classroom. From the three after-school projects discussed, we identified common characteristics that supported the success of Latinos/as and other minority cultures in learning mathematics and that changed their mathematical identities and other identities (bilingual, Latina/o, etc.). Although these characteristics may not all be part of regular classrooms due to current restraints from standards, assessment, etc., there are lessons to be learned and applied to the teaching of mathematics in the classroom. How mathematics knowledge is evaluated and our current classroom restraints need to be examined in light of the information coming from these after-school settings.

Typically students, and particularly Latino/a students, have negative identities in terms of their mathematics ability. These identities are reified by deficit model approaches (either in terms of language, race or ability) often used with Latinos/as and other minority groups (Lopez-Leiva, 2010). Additionally, Latina/o students often face further challenges from peer perceptions of their identity resulting in a situation of peer pressure that marginalizes Latina/o students, silencing their voices and resulting in dropping out of school. Society's privileging English over Spanish contributes to Latina/o students' tensional relationship with their bilingual identity (Lopez-Leiva, 2010).

However, students involved in these after-school settings demonstrated that their mathematics identities and other identities could be transformed. These after-school settings considered "the sociocultural resources of ethnic minority communities as foundational" (Vasquez & Marcello, 2010, p.1). The participatory pedagogy implemented included:

- informal adult-child interactions, curricular flexibility
- technology and manipulative resources
- co-constructed knowledge
- increased levels of choice
- time to explore
- cultural resources of the community
- multiplicity and diversity
- multigenerational resources
- multilingual resources,
- critical mathematics
- distribution of authority

Without the constraints of pacing calendars or standards, students engaged in problem solving activities that often focused on critical mathematics. By using mathematics to address significant problems relevant to their lives, students' perspectives on mathematics can change. Students in school typically view being "good" at mathematics as being fast when working with the four operations of addition, subtraction, multiplication and division. Often mathematics is seen as boring and of limited use in real life. Other common mathematics identities include girls not being good at math, and minorities not belonging in the upper level mathematics classes. By working on critical mathematics problems students gain a new perspective on using mathematics to address important problems. Careful scaffolding of students' mathematics learning by researchers and undergraduates resulted in students never failing (Vasquez, 2010). This practice supports students developing positive mathematics identities, and dispositions of perseverance, confidence to take risks, and agency (Lopez-Leiva, 2010; Varley Gutiérrez, 2010).

The meaningful contexts chosen and negotiated in the after-school programs afforded community and students' funds of knowledge to be valued and used as resources for solving problems. Within these programs there becomes a shared space for teaching and learning with community members, family, university, students and school staff. This becomes a true "community of learners." Limited school mathematics content generally offers very narrow entry points for students. Students' culture and language are often viewed as deficits. The problem solving contexts in these after-school programs recognized and valued students' and communities' funds of knowledge as resources, and therefore, offered much broader gateways into solving problems. Students gained access to problem solving and reasoning.

Not only do Latina/o students have negative mathematical identities, oftentimes teachers and undergraduates do, as well. Preservice teachers who participated as facilitators in the after-school programs experienced positive mathematics identity shifts.

Through the use of manipulatives, a playful approach, and other resources, mathematics became more accessible to them, also. They were later able to bring this learning to the regular classroom in their student teaching experiences. Their cultural and ethnic perceptions also changed from typical stereotypes.

What are the implications for practice and research?

The research on these after-school settings offers implications that can be applied to the regular mathematics classrooms and also to after-school settings. As demonstrated from these after-school settings, Latino/a students' mathematical identities can benefit from changes in the traditional notions of classroom structures. These participatory environments shifted the focus to understanding and making meaning of mathematics, and using mathematics for relevant problem solving purposes. Teachers will need support in learning how to do this. Students benefit from learning environments that afford choices and flexibility for learning, including whom students work with and what resources they may access, including technology. Giving students more freedom will require teaching students new expectations and how to operate in a new setting. Students will need to be taught a new culture of the collaborative classroom with negotiated norms. This involves identity shifts for teachers and students and understanding their new roles. Teachers need to be able to build community within their classroom and access students' funds of knowledge to not only motivate students, but to connect their experiences to classroom content.

The school model needs to change to one that affords more adult/student non-hierarchal relationships, multigenerational settings. Students need access to more adults within the classroom. Students also benefit from multiage groupings within the classroom. When students in these after-school settings "taught" others, their mathematics identities changed. These same tenets need to be in place for all after-school programs.

Although formal assessment should never be a part of an after-school setting, other forms of evaluation could be used to assess the program's success. By observing students participating in the after-school, data may be collected to document students' mathematical understandings and changes in identity. When problem solving is the focus of the mathematics, students develop reasoning skills that are applicable to standardized assessments requiring reasoning. Teachers and after-school providers will need professional development on becoming ethnographers to hone their observation skills in order to collect data for assessment and program planning. Narratives should become part of a student's portfolio of learning that can be used for assessment instead of relying completely on test scores for program and student evaluation. Portfolios offer a more holistic view of students' learning.

Tensions observed in bilingual students' language identity need to be addressed. Students need to value their first language and understand that it can be an important asset. Interacting with highly successful adult experts who are fluent speakers in the students' first language may help support this bilingual identity. Having discussions explicitly addressing the importance of being bilingual may also support changes in students' language identities.

Aside from the value of after-school settings to students, after-school settings can offer space for preservice and classroom teachers to learn different aspects of the teaching and learning of mathematics. They can have the opportunity to adopt new practices. The learning from the after-school setting can be brought into the student teaching experience, impacting classroom teachers not directly involved with the after-school. The after-school setting offers a platform to challenge prevailing beliefs about culture, and mathematics and language teaching and learning. Preservice teachers need to have a course that focuses on diversity, language, and culture with a component that involves an informal setting, like an after-school program, early in their education experience. They need to have reform-oriented experiences as mathematics learners in teacher preparation classes in order to change their own mathematical identities and beliefs about teaching and learning.

After-school settings like the ones discussed can provide a counter-narrative for teachers' views of learning and teaching. Teachers observing their students performing in a more relaxed after-school setting may identify students' strengths and build on them. A structure that supports teachers being able to do these observations without it being an "add-on" requirement to their list of responsibilities needs to be developed. They will require professional development to make the goals of these situated settings clear. The use of videos from after-school settings depicting student engagement can also be useful in changing teachers' perceptions. Teachers must be given support in implementing changes in the classroom power structure, and relating to students in a different way.

Administrators must also receive professional development to understand the goals in these after-school settings. The research and videos of these situated after-school settings need to be shared so they have "buy in" for establishing similar after-school programs in their schools and for supporting classroom pedagogical changes.

By educating parents about the possibilities of after-school programs, they can also be a force behind the implementation of the programs. When parents are involved in the after-school program, it impacts the home with parents more involved in working on mathematics with their children. The current typical paradigm of parent participation where parents are used for clerical work needs to change. Parents must have more participatory roles, using their funds of knowledge as a resource. This changes the current power structures that exist between parents and schools. Parents take more ownership in the school when involved and want their children to stay in the particular school with the program, resulting in a more stable student population.

Knowing that these situated after-school settings contribute to Latina/o students' success in mathematics, there must be a plan to sustain and also scale up such programs to include more communities. Community resources need to be identified and local, state and national education funds need to be targeted to support such programs. Strategies need to be developed to generalize these existing models to district levels. Training must be provided for those participating in the programs. As a long-range plan, consideration should be given to establishing a "National Center on Afterschool Learning." We need to think globally, act locally, and plan long term.

What questions do we need to research further?

We identified the following questions as needing further research and investigation:

- How does the site location (eg. school, church, community center) affect the mathematics learning, attendance and attrition rate, and identities of the Latino/a students?

- What performance-based assessments need to be developed for children in after-school programs to see how their performance compares to their scores on standardized tests?

- What are the characteristics that constitute "exemplary" after-school programs?

- How do we change the research paradigm to accommodate informal learning?

- How do we connect after-school based research to in-school research?

- How are the leadership skills of students who participate in after-school settings impacted?

- How can connections be made for students between the mathematics in after-school settings and the classrooms?

- How does the after-school setting impact the mathematics learning in the home, especially when parents are involved?

- What are the longitudinal effects on students participating in after-school programs?

- What must be in place in order to generalize these programs to other communities?

- What are "natural" ways of discussing mathematics?

- In what ways do after-school programs challenge teachers' beliefs and conceptions of learning?

- What is the impact or how might the dynamics change in the learning community when an expert (i.e., retired professional) is involved?

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Practitioners and Researchers Learning Together: A National Conference on the Mathematics Teaching and Learning of Latinos/as

Visions from the Classroom: Focus on Students

Section 9 of 9

Chair: Judit Moschkovich,
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Using Math Pen-Pal Letters to Promote Mathematical Communication

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EXPLORING THE MATHEMATICAL THINKING OF BILINGUAL PRIMARY-GRADE STUDENTS: CGI PROBLEM SOLVING FROM KINDERGARTEN THROUGH 2ND GRADE

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This paper explores young, bilingual students' longitudinal mathematical development around CGI problem solving (Carpenter, Fennema, Franke, Levi & Empson, *1999). A full description of the study is found in Marshall (2009). Analysis of students' problem solving strategies and explanations of their thinking demonstrate that problem solving is a complex process where students do not necessarily make meaning of the numbers in the problems in the same way, resulting in fundamentally different approaches to find solutions. In their creativity, flexibility and insight into problem solving, these young students highlight the richness of resources found within their Mexican immigrant community.*

INTRODUCTION

This qualitative study explores the developing mathematical thinking and conceptual development around Cognitively Guided Instruction [CGI] problem solving (Carpenter et al., 1999) of four native Spanish-speaking students as they progress from kindergarten through 2nd grade. These students come from bilingual classrooms where the majority of their mathematics instruction is in Spanish. Of particular importance in this study is that the CGI problems used for individual student interviews arose from the context of students' native language and culture (Secada & De La Cruz, 1996), a culture based in a Mexican immigrant community where the majority of residents have a lower socioeconomic status. The research and analysis presented in this dissertation are based on the understanding from sociocultural theory that there is a direct connection among concept development, social interaction and language (Mahn, 2009; Vygotsky, 1987), and from CGI theory and mathematics education research that students' mathematical concept development is reflected in the approaches they take to solve complex problems and the ways they explain their mathematical thinking (Carpenter, Fennema & Franke, 1994; Hiebert and Carpenter, 1992; Lerman, 2001). Within the CGI framework, the specific strategies students use to solve a variety of problem types help teachers target areas of mathematical concept development and understand how students are making meaning of the number operations and relationships they encounter in the problems (Carpenter et al., 1999).

While some research has shown that indeed Spanish-speaking Latina/o students and students from diverse communities can successfully engage in complex mathematical processes and that their problem-solving skills can improve through conceptually challenging activities (Kamii, Rummelsburg & Kari, 2005; Secada, 1991; Secada & De La Cruz, 1996; Turner, Celedón-Pattichis & Marshall, 2008; Villaseñor & Kepner, 1993), very little research has documented young Spanish-speaking students' mathematical problem-solving and discourse development over time within Spanish learning environments and the way these students explain their thinking in their native language (see Turner et al., 2008). This study provides a unique opportunity to explore more closely how young children make meaning of the mathematics in contextually rich word problems.

THEORETICAL FRAMEWORK

This study builds on two important theories in cognitive development that have not been previously unified. The foundational perspective comes from sociocultural theory. This perspective describes cognitive development as a refinement and expansion of the internal systems of meaning individuals construct in their constant interactions with the external systems of meaning they encounter in their social and cultural environments (Vygotsky, 1987). At the heart of sociocultural theory's notion of concept formation is the idea that language, including gesture is the first mediator of meaning in a child's life and that early spontaneous concepts are structured and represented internally by the meaning attached to words. Since making meaning is also a thought process, meaning results from the unification of thinking and speaking. Language is a central symbol system and sign operation for conscious thought, and therefore mediates the interplay between the internal and external systems of meaning (Mahn, 2009; Vygotsky, 1987). The meanings symbolized by words are generalizations and each of these generalizations is a concept, so words and meanings are the conceptual medium of thinking and speaking. In this way words and their associated meanings are the building blocks of conscious thought that lead to greater degrees of conceptual understanding. How children explain their thinking during CGI problem solving in this study gives a window into how they are making meaning of the mathematics in the problems.

The second framework used in this study is Cognitively Guided Instruction (CGI), a framework for understanding the mathematical thinking of young children during problem solving around number stories (Carpenter et al., 1999). CGI theory explains that children think about problems in specific ways, depending on the actions and relationships contained in the stories. A key idea of CGI is that children do not need to be shown specific strategies for solving these problems, but rather come to school with an intuitive ability to model the actions and relationships they encounter in a word problem when they can comprehend the situation of the story (Carpenter, Fennema & Franke, 1996). Children do not need a formal understanding of number operations before they can begin solving word problems. With only a beginning understanding of counting and one-to-one correspondence, children can be successful problem solvers using what they already know about the world to make sense of the numeric actions and relationships in the problems.

Early CGI research observed generalizations in the way children as young as five years old approach different kinds of problems, the strategies they use, and the way these strategies change and develop over time as the children incorporate a more formal understanding of numbers and operations (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993). The CGI framework contains a clear classification of problem types and associated strategies. This framework acts as a guide for teachers to understand a young student's mathematical thinking based on the strategies the child uses to solve a problem.

The Cognitively Guided Instruction framework can be understood through a sociocultural lens. In particular, the informal knowledge children bring to the classroom can be conceptualized as an important aspect of the internal system of meaning they have about number relationships. Because many of the children studied in early CGI research approached specific types of problems in similar ways and developed advanced strategies following much the same continuum (Carpenter et al., 1993, 1994, 1996), the CGI framework allows for generalizations to be made about children's early systems of meaning as they enter formal schooling. Once in school, CGI problem solving can be used to explore how children's interactions with formal mathematics influence their concept development. This process of concept development is precisely the interplay noted above when an individual's internal system of meaning incorporates new meaning and structure from the external environment. For this reason, CGI theory as seen through a sociocultural lens, helps teachers and researchers understand how children are building conceptual knowledge by targeting specific mathematical concepts.

The importance of teaching and learning mathematics in the native language complements sociocultural and CGI theory. Research described by Secada and De La Cruz (1996) in a 2nd grade bilingual classroom in Texas around CGI-type problem solving and student explanations emphasized the benefits of students having access to Spanish to make sense of mathematics problems (Secada & De La Cruz, 1996). In their chapter on teaching mathematics to bilingual students, Secada and De La Cruz argue that it is imperative to teach mathematics with understanding (Hiebert & Carpenter, 1992). In order to do so, students must be able to make sense of the problems they encounter. Ambiguity in understanding is removed when students have access to their native language to comprehend problem contexts. When students have access to their native language to explain their thinking about problem solutions, they are free to be clear and precise (Secada & De La Cruz, 1996). This communicative precision helps students organize and consolidate their thinking and (NCTM, 2000) and leads to greater mathematical development. In general, the most successful learning environments for bilingual students have strong components of conceptual development in the native language (Cummins, 2001; Thomas & Collier, 2002).

METHODOLOGY

This qualitative, three-year longitudinal study presents a substantive theory about the different ways students make meaning during mathematical problem solving (Creswell, 1998; Glaser & Strauss, 1967). Extensive problem-solving interview data was collected on eight students while solving twelve different CGI problem types as outlined in Carpenter et al., (1999). Analysis was based on the problem solving strategies described in the CGI framework, included the tools students chose to aid in their problem solving, and was expanded to include students' explanations of how they made sense of the mathematics in the problems (Mahn, 2009; Vygotsky, 1987).

The research site for this study, La Joya Elementary, is located in a low-income neighborhood in a large urban area in the Southwestern United States. The surrounding neighborhood has a large Mexican immigrant population and 86% of the nearly 700 students speak Spanish as their first language. The school services 100% free or reduced breakfast and lunch to all students. La Joya Elementary promotes a maintenance bilingual program in grades K-5 with the goal of bilingualism and biliteracy for all Spanish-speaking students. The maintenance bilingual program begins with 90% Spanish and 10% English in kindergarten and ends with 50% Spanish and 50% English in 5th grade. The primary grade teachers who participated in this study introduced all mathematical concepts in Spanish and reinforced much of the material in English. All of the student participants had a portion of each day devoted to learning English as a second language (ESL).

Four final research participants were chosen for analysis, including two girls, Yolanda and Gina, and two boys, Omar and Gerardo. These children were chosen to balance the research between boys and girls and because each of these four students demonstrated confidence in problem solving. All were able to think independently and move away from prescribed classroom methods for problem solving.

Language also played a role in my selection of the four students as I attempted to balance their linguistic strengths. While Yolanda is Spanish dominant, Omar is English dominant, and Gerardo and Gina appear to use both languages more or less comfortably. Gerardo prefers to use English and has had this preference since 1st grade. Gina made a transition to conversational English usage in 2nd grade, but used more Spanish for problem solving. Gerardo and Gina are also quite verbal and like to explain their thinking. Omar and Yolanda are less verbal, although Yolanda appears to have greater access to her own thinking than Omar. These final four were chosen before the intensive coding process began to prevent a bias in the emerging themes.

The analysis of students' mathematical thinking over three years of the study focuses on the CGI strategies they use to solve problems, the tools and aids they choose to help them find their solutions, and the way they explain their thinking and problem solving. Each student's problem solving strategy for each CGI problem was classified using the CGI framework. Final problems for analysis were chose to give a balance of problem types that reflected both actions and relationships. I chose the Compare, Multiplication, Part-Part-Whole, and Join Start Unknown problems. Compare and Part-Part-Whole problems involve relationships and Multiplication and Join problems involve actions. As I refined my coding and themes began to develop, I decided to include Partitive Division problems, which represent actions, because they reflected the most striking differences in student approaches.

To begin the analysis, I transcribed student explanations of the final five problem types over three years. From here I was able to summarize the tendencies I saw in each of the four children's problem solving over time and do a cross student comparison based on problem type. I paid particular attention to the distinction between direct modeling and advanced strategies, as I explain below.

DISCUSSION AND FINDINGS

Before I present the findings, I want to discuss mathematical thinking and compare this thinking to the verbal thinking I outline in my theoretical framework. As a psychologist, Vygotsky (1987) believed that consciousness is composed of a broad range of thinking processes and that verbal thinking is only one of these, along with mathematical thinking, musical thinking kinesthetic thinking to name three others (Gardner, 1993; John-Steiner & Mahn, 1996). Since the research presented in this dissertation is built upon CGI mathematics word problems that were presented to children orally, and the children were asked to explain their thinking in association with their solutions, the children had to think both verbally and mathematically during problem solving.

Making meaning is critical to both verbal and mathematical thinking. Just as words have been internalized as meanings in connection with verbal thinking, so too are the representations children encounter in the mathematics classroom such as the individual numerals "1", "4", or "9", the sequential number line, and the chart of numbers from 1 to 100 arranged in rows of 10. Meanings associated with these symbols develop in social interaction, similar to meanings associated with words, and can be complex and must be sorted out by children (Fuson, 1988). For example, the simple number 5 can either represent a cardinal quantity, an ordinal position, or a measurement amount. Five can be the number of discrete objects in a group, it can be the fifth item in a sequence and represent a relative position, or it can be the length in units of an object.

To add to the complexity, the meanings related to specific symbols can shift during an activity, as when children are learning to count. They need to understand that the last word they use in the sequence of numbers assigned to objects represents the whole set of objects not just the last item, even though in the sense of one-to-one correspondence between object and number, it is the discrete 5^{th} object that is associated with the word "five." In her work with young children, Fuson (1988) notes that, "An important development throughout the age range…(age 2 to 8) is the increasing ability to shift among meanings and, finally, to integrate several of these meanings. Adults shift so easily and have such integrated meanings that it is difficult for adults even to comprehend how separate these meanings are for young children" (p. 5).

To understand how children make meanings during problem solving, we have to look at both their words and actions and attempt to undercover the symbol systems they are using for their thinking. The basis for classifying their words and actions in this study was students' CGI problem solving strategies as developed by Carpenter et al. (1993, 1994, 1996, 1999). Two important classes of strategies in CGI theory are direct modeling and counting. Direct modeling reflects meaning making that focuses on discrete objects and counting reflects meaning that focuses on number sequences. Carpenter et al. (1999) explain,

> Direct Modeling is distinguished by the child's explicit physical representation of each quantity in a problem and the action or relationship involving those quantities before counting the resulting set. In using a Counting strategy, a child

> essentially recognizes that it is not necessary to actually construct and count sets. The answer can be figured out by focusing on the counting sequence itself. (p. 22)

With a counting strategy, the child makes sense of the problem through an abstraction of the concrete objects to a number sequence. Counting strategies are considered advanced by CGI researchers and are thought to follow direct modeling as children develop their sense of number. Carpenter et al. (1999) note,

> Counting strategies are abstractions of the corresponding Direct Modeling strategies they [children] used previously…Gradually over a period of time children replace concrete Direct Modeling strategies with more efficient Counting strategies, and the use of Counting strategies is *an important marker* in the development of number concepts. (p. 28)

I have added the italics in the above quote to make a point. If the difference between a direct model and a counting approach represents a marker from concrete to abstract mathematical thinking, then this distinction provides important theoretical information and a window into the system of meaning students are using to solve these CGI problems. I asked during analysis, do students think about each object in the story and manipulate them to find their answers, or do they abstract the objects to numbers in a sequence and operate on that sequence to find their answers?

Using the distinction between direct modeling and counting strategies, I began to analyze the strategies used by the four students in this study by categorizing their approaches into one of two groups. I labeled their strategy a direct model if they began by represented each object in the problem even if they used a counting strategy within their model to find the answer. I labeled their strategy advanced if they did not represent each object, but abstracted the problem to counting, a recalled fact, a derived fact, or a number operation like addition or subtraction.

Findings, Strategies

The table below shows a breakdown of student strategies for the five problems examined in this study over three years. See Marshall (2009) for an extensive explanation of the problems given to each child.

Table 4: Breakdown of All Students' Strategies, Kindergarten – 2nd Grade

Student	Number of Problems	Number Correct	Direct Model and Combinations of DM and Counting	Advanced Strategies: Counting, Recalled Facts, Add., Sub., Carrying or Borrowing
Omar	37	33 (89%)	5 (14%)	***32 (86%)***
Yolanda	39	33 (85%)	10 (26%)	***29 (74%)***
Gerardo	35	31 (94%)	***24 (69%)***	11 (31%)
Gina	36	35 (97%)	***30 (83%)***	6 (17%)

As seen above, Omar and Yolanda preferred to use advanced strategies 86% and 74% of the time, respectively. They did not reach as many correct solutions as the other two students, but were still quite successful problem solvers. Gerardo and Gina used direct modeling 69% and 83% of the time, respectively. They were more successful in finding correct solutions to the problem. From the perspective of correct answers, it could be argued that the CGI problems lend themselves to direct modeling as they are based in contextually rich story problems about objects (Carpenter et al., 1993, 1999), which gives an advantage to the direct modelers. One could also argue that Omar and Yolanda pushed themselves toward more advanced approaches and therefore took more risks. Correct solutions aside, the table above presents a clear distinction between the strategies Omar and Yolanda used to solve CGI problems over time versus the strategies Gerardo and Gina used.

The above data suggest that the children had two distinct ways of making meaning. Two of the children tended to think about the numbers in the problems ordinally and two of the children tended to think about the numbers cardinally (Fuson, 1988). Careful examination of individual student strategies showed that Yolanda and Omar approached the numbers as linear sequences as early as kindergarten. Both Omar and Yolanda generally used counting strategies to solve CGI problems (Carpenter et al., 1999), made mental calculations frequently, quickly reached problem solutions, and looked for ways to apply formal operations and algorithms. When they compared two discrete sets of objects, they abstracted the numbers to a sequence and then counted either up or down to find their answers (Carpenter et al., 1999). Neither student chose direct modeling as his or her first approach and they only used this method when they did not have an idea about a counting or advanced strategy. It is important to note that Omar and Yolanda consistently had difficulty falling back to a direct modeling strategy and when they did, many times their answers were incorrect. See Marshall (2009) for a more detailed description of this phenomenon.

Gina and Gerardo, on the other hand, tended to approach problems from a discrete perspective. They preferred to draw direct models of the problems or construct them out of cubes. Frequently they found their solutions by partitioning the whole quantity into parts. For most of the compare problems, Gina and Gerardo built complete comparison sets even if they used a counting strategy to find their solutions (Carpenter et al., 1999). Gina continued to use connecting cubes when they were available to build her direct models, even though as she got older she was able to find the answer to the problems in her head. In the same way, Gerardo frequently drew sets of numbers, but then showed he could find the answer mentally by counting or a recalled fact.

Findings, Tools

The tools students chose to help them solve the problems played a major role in revealing how these students were thinking about the problems. Omar and Yolanda had strong preferences for the number line and 100 chart. These tools are based on a sequential ordering of numbers, and show these two students were attaching an ordinal meaning to the numbers in the problem. Gina and Gerardo preferred connecting cubes and base ten blocks, which showed they were assigning meaning to the numbers from a cardinal perspective as sets of objects. Interestingly, analysis showed that when students were thinking about the numbers in the problems from one perspective, but trying to use tools that facilitated the other numeric meaning, for example trying to manipulate discrete sets using a sequential 100s chart, then the tool worked against their sense-making efforts.

Findings, Language

The language students used to describe their thinking was a reflection of their verbal thinking and gave an additional clue into how they were making sense of the problems. According to Vygotsky (1987), social communication through language gives us one of the best avenues to study conscious thought. The problem situations used in this study drew on students' native language and cultural contexts, thus giving the bilingual students in this study the best opportunities to comprehend the problems (Secada & De La Cruz, 1996), employ both verbal and mathematical thinking (Mahn, 2008, Vygotsky, 1987), and then discuss their solutions in the language of their thinking (Sfard, 2001).

An example of Omar's verbal explanations highlight how he abstracted the objects in the problems to a sequence of numbers and then counted to find his answers (Carpenter et al., 1999; Fuson, 1988). In 2nd grade Omar solved a Join Start Unknown (7, 22) problem about candies. The question was how many candies did he have to start with if he was given seven more candies to have a total of 22? While finding his solution he said, "I'm counting up and counting down," as he wrote the numbers from 1 to 7 on a piece of paper and 21 to 15 on a piece of paper, "…til I get seven right here…I counted up till seven, and then I counted down until I got fifteen." Omar used number sequences in his explanation and also wrote the sequences on a piece of paper as seen in the figure below.

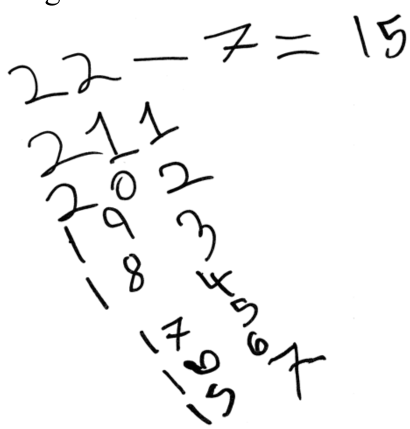

Figure 1. Omar, Join Start Unknown (7, 22), 2nd grade.

I propose that he used number line representations in his mind to mediate his thinking and that he had incorporated sequential representations of numbers into his system of meaning to make sense of join problems (Mahn, 2009).

Yolanda's explanations also highlight her sequential thinking to solve the CGI problems. At the end of 1st grade she used a recalled fact and base ten thinking to solve Part-Part-Whole (30, 20), where 20 of 30 balloons are blue and the rest are red. How many are red? This problem lends itself to discrete thinking, but Yolanda approached it ordinally. She answered 10 and said, "Porque veinte más diez es igual a treinta. *(Because 20 plus 10 is equal to 30.)*" When I gave her a harder pair of numbers, (75, 50) for the same problem, she initially began by counting up from 50 to 75 by twos using a Counting On To strategy. Figure 2 shows how Yolanda wrote the sequence of numbers, adding one to the end to go from 74 to 75, but she did not immediately arrive at 25 as an answer.

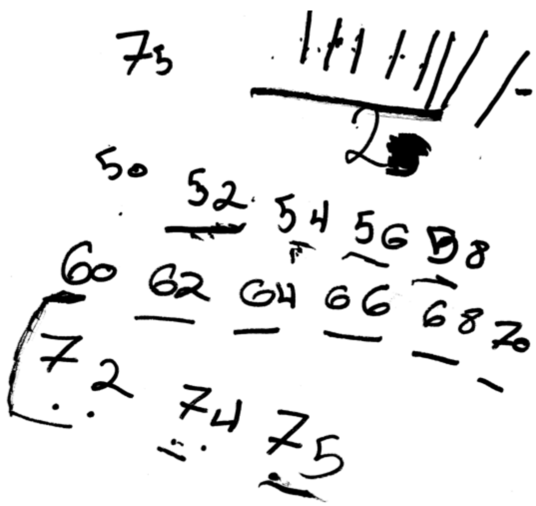

Figure 2. Yolanda, Part-Part-Whole (75, 50), 1st grade.

After writing the numbers, she thought of a better way so she switched to counting by fives to get the correct answer of 25. She was able to self-correct her flawed counting strategy with another way of counting.

Unlike Omar and Yolanda, Gerardo preferred to think about the number discretely. He gave an interesting explanation for his thinking about the Join Start Unknown problem in the final interview from 2nd grade. I initially misspoke in Spanish, saying "cien doce" instead of "ciento doce" for the number 112, and he understood the problem to be JSU (40, 100) instead of JSU (40, 112). He mentally solved the situation where he has an unknown number of candies to start, is given 40 more, and then winds up with 100 candies. To discover how many candies he had to begin, he manipulated the groups of tens in the problem, and then explained his thinking. He said, "Because, if my, if I get a hundred at last, and I just take the forty away and give it back to my friend, I'll have sixty candies. But if I keep having those, I already know that I have sixty and forty together." Here he is flexibly adding and removing a set of 40 candies to 60 candies to get 100 candies. I believe he thought about two discrete sets of candies while at the same time used base ten thinking to manipulate the groups of ten (Fuson, Smith & Lo Cicero, 1997).

As a consistent discrete thinker, Gina's language demonstrates how she made sense of a problem with discrete sets and thought in terms of parts and wholes. In a problem from kindergarten she approached the Part-Part-Whole (10, 6) problem about balloons by initially drawing 10 lines. The problem states that she had ten balloons, six were blue and the rest red. How many were red? Explaining her solution, she says, "Como *(Because,)*," "ya sabía…ya sé cuantos son seis, y luego puse la rayita, *(I already knew, I already know how many are six, and then I put a little line,)*" "y luego me quedaron cuatro. *(and then I was left with four.)*" She explains that she knew there were six balloons in one group, and then she removed this set of lines from her total to find the remaining set. She thought about the problem discretely as dividing the total into two distinct groups.

In the middle of 1st grade, Gina modeled the Join Start Unknown (5, 13) problem again by creating the whole amount and removing a set to leave the rest as her answer. She said, "Pues, um…hay…le puse 13 y luego conté 5, y le puse…ay…una cajita [around the five]…y luego conté los demás. *(Well, um…there are…I put 13 and then counted five, and it put…ay…a little box [around the 5]…and then I counted the rest.)*" In her solution she operated on the discrete sets of objects rather than the counting sequence, and therefore made sense of the problem based in the cardinal meaning of the numbers (Fuson, 1988). As part of the figure below we see Gina's solution to Join Start Unknown with the smaller numbers (3, 5) in the same interview. She solved the problem quickly by directly modeling the two parts of the problem to see that two plus the know quantity three results in the known quantity five.

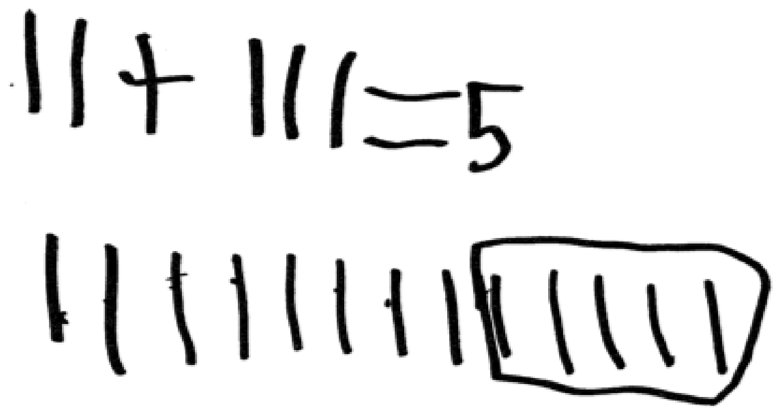

Figure 3. Gina, Join Start Unknown (3, 5) and (5, 13), 1st grade.

Sequential versus Discrete Thinking

CGI literature promotes children abstracting the numbers in the problems away from the objects and toward counting sequences, and considers counting strategies more advanced than direct modeling (Carpenter et al., 1993, 1994, 1996 1999). This would imply that sequential thinkers are more advanced mathematically than discrete thinkers, and it is true that the children in this study spent a great deal of time in their classes developing counting by ones, twos, fives and tens. Sequential thinking is critical if students are to develop a sense of the number patterns and relationships necessary for advanced mathematics including algebra (Fuson, 1988; Fuson et al., 1996; Jordan, Kaplan, Oláh & Locuniak, 2006; Kamii et al., 2005). Omar and Yolanda, the sequential thinkers, were confident and successful problem solvers and by CGI standards would be considered more advanced in their mathematical development than Gina and Gerardo.

However, discrete thinking is a critical component of formal mathematics (Van Wagenen, Flora & Walker, 1976) and this research found that Gina and Gerardo, the discrete thinkers, were equally powerful and successful problem solvers. All four children brought strengths to the CGI problem solving interviews and each child encountered challenges when the structures of the problems did not match the way they were making sense of the numbers. In the CGI framework, some problems tend to be easier to solve with sequential counting strategies and some lend themselves to discrete thinking. Multiplication, for example, can be solved easily by skip counting because the size of groups is known. Partitive Division, on the other hand, is an example of a structure that is not easily solved by a counting because the size of each group is not known (Carpenter et al., 1999). Analysis showed that Omar and Yolanda excelled at Multiplication, but had difficulty with Partitive Division problems (Marshall, 2009). Gina and Gerardo rarely had difficulty with Partitive Division because they could approach the problems in terms of discrete sets.

Implications

The development of students' mathematical thinking is complex. The findings from this study show that students can have fundamentally different ways of making sense of CGI problems as early as kindergarten and the meanings they attached to numbers continue to influence their mathematical problem solving throughout the primary grades (Fuson, 1988; Mahn, 2009). CGI theory generalizes the idea that students move from direct modeling to counting strategies, that students' counting strategies are abstractions of their direct models, and that students tend to leave behind direct modeling as they develop the ability to focus on the number sequences as objects of manipulation instead of the discrete objects in a problem (Carpenter et al., 1993, 1994, 1999). The analysis presented in this research refines the theoretical perspective of CGI that all students develop mathematical problem solving along this type of trajectory. The students' mathematical problem solving in this study was influenced by more than one meaning attached to the numbers in problems (Fuson, 1988) and students' dominant way of thinking about the numbers continued to influence their mathematical development.

All four students in this study brought unique talents to CGI problem solving and all were impressive problem solvers. Because they tended to make sense of problems either sequentially or discretely, some types of CGI problems were easier for them and others presented challenges, as noted above. The most powerful mathematical development for children comes when they can think about problems from both sequential and discrete perspectives (Fuson, 1988). When they can shift between meanings and take multiple perspectives, they have learned mathematics with understanding (Hiebert & Carpenter, 1992; Secada & De La Cruz). To gain this flexibility, children need problem types that promote their sequential thinking and also problems that draw on their ability to think discretely and use decomposition strategies to break numbers down into parts of a whole. Lessons that focus on counting skills and sequential thinking are part of the standard elementary curriculum. I suggest that an increased emphasis be placed on Partitive Division and Part-Part-Whole type CGI problems to help students develop the flexibility they need in discrete thinking for advanced mathematics.

In conjunction with a focus on developing both sequential and discrete thinking, mathematics educators need to recognize the important role that tools play in facilitating children's thinking. Not all tools help children solve all types of problems, as seen in this research. If the problem type and the meaning children attach to the numbers in the problem match the tool they are using, then the tool helps them find a solution. On the other hand, if the tool does not match the meaning children are attaching to the numbers and/or does not help students make sense of the problem using a more appropriate meaning, then the tool interferes with the sense making process. When teachers recognize that certain tools are better for certain types of problems, they are more able to direct children toward appropriate tools that help the children develop the flexibility in thinking important for mathematical development.

Language plays a vital role in mathematical development. In this study, students' explanations reflected how they were thinking verbally during problem solving (Mahn, 2009; Vygotsky, 1987). Their verbal thinking was in turn connected to how students made meaning of the problems and further illuminated the system of meanings students were constructing about mathematics. This window would have been obscured if the students were forced to make meaning in a language other than the language of their thinking (Cummins, 2001; Mahn, 2009; Thomas & Collier, 2002). The words that mediated their thoughts became part of the external expression of their thinking (Mahn, 2009; Vygotsky, 1987). There was no conflict between the language they used for learning mathematics and the language they used for thinking. The four students in this study had the power of two languages to make sense of the mathematics in the CGI problems (Cummins, 2000; Khisty, 1997; Moschkovich, 2002).

Finally, I want to close with a word about equity in mathematics education for Latina/Latino students. When given the opportunity to solve challenging mathematics problems and share with us how they made sense of the problem situations, these four students showed a richness of thinking and sophistication of language that moved us closer to understanding children's mathematical development. This research shows that great potential lies within diverse communities and all students must be given opportunities to learn mathematics with understanding (Hiebert & Carpenter, 1992; Secada, 1989a, 1989b, 1991, 1992, 1995).

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INSIGHTS FROM TWO STUDIES OF SMALL-GROUP MATHEMATICAL DISCUSSIONS AMONG STUDENTS IN BILINGUAL CLASSROOMS

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This paper describes results from two studies of bilingual students' mathematical discussions with peers. Grounded in a sociocultural approach and using methods from sociolinguistics as well as the analysis of mathematical discourse practices, we document 1) how two groups of students constructed the role of authority in their small group discussions, and 2) how the students' use of two languages functioned within their discussions. We show how authority was constantly negotiated and reconstructed in students' talk-in-interaction during mathematical discussions. We also show how using two languages functioned to manage social and mathematical elements of these small group discussions.

When confronting the challenge of teaching mathematics in linguistically diverse classrooms, both researchers (Esmonde, 2009; R. Gutiérrez, 2002; Moschkovich, 1999) and practitioners (Hoffert, 2009) have recommended using small group work as one possible classroom activity structure that can meet the demands of a heterogeneous student population. More than twenty years of research has shown that under the right conditions, working in small groups with peers can have a positive impact on students' social relations and domain-specific learning (Cohen, 1994). Following up on these encouraging findings, Cohen (1994) called for more research that opens the "black box" of group processes and investigates how and under what conditions small group work is successful.[1] While a number of mathematics education researchers have responded to this call (Barron, 2000, 2003; Cobb, 1995; Forman & Larreamendy-Joerns, 1995; Jurow, 2005; Sfard & Kieran, 2001), more research is needed to investigate how group work promotes mathematics learning for bilingual students and English Learners[2] (Esmonde, 2009).

In this paper I summarize findings from my research (done under the guidance of Judit Moschkovich and in collaboration with the Mathematical Discourse Research Group at UC Santa Cruz) investigating how bilingual students learn mathematics in the setting of group work (Zahner & Moschkovich, in press-a, under review). At the end of this paper I use recently collected data from my dissertation research to refine findings from my earlier studies. The two foci of this paper are (a) how students construct the role of authority in their peer discussions and (b) how bilingual students use multiple languages and code switching to engage in mathematical reasoning.

Conceptual Framework

Underlying my work is a theory of mathematics learning rooted in sociocultural approaches to the study of human interaction. Specifically, I treat learning mathematics as a process of appropriating socially shared tools for thinking (Newman, Griffin, & Cole, 1989; Rogoff, 1990). These tools for thinking include semiotic systems such as the use of computational algorithms (Wertsch, 1998), and the use of particular types of talk, such as engaging in mathematical discourse practices (Moschkovich, 2007a). Learning may be documented by observing changing patterns of participation in cultural practices (Rogoff, 2003). In mathematics classrooms where talk is a central feature of daily activity, one observable indicator of that learning may be the transformation of students' mathematical reasoning and participation in mathematical discourse practices (Forman, 1996; Lampert, 1990; Moschkovich, 2002; Sfard, 2008).

My analysis of students' talk during group work is rooted in functional approaches to language (Austin, 1962; Eggins & Slade, 1997) as well as ethnomethodological approaches to

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[1] Defining "success" for small group work is a challenging issue because researchers from different fields have different criteria for a successful educational intervention. For this paper I will consider small group work successful if it promotes student learning and equity.

[2] I use the term "English Learner" (EL) to describe students who are designated by schools as English Learners. While there is considerable and justifiable controversy about the classification of students and the use of appropriate labels (*cite), I am following the guidelines of the California Department of Education (http://data1.cde.ca.gov/dataquest/gls_learners.asp) since this agency sets the institutional definitions that impact students and teachers in my studies.

studying (classroom) interaction (Garfinkel, 1967; Heritage, 1984; Mehan, 1979). Adopting a functional approach implies that the meaning of an utterance is not inherent in the words spoken by the speaker, but rather, meaning is constructed through the combination of an utterance, what has come before it, what follows, and it's effect ("illocutionary force" in Austin's terms). Adopting an ethnomethodological approach reflects my commitment to understanding how members of a social situation make sense of their everyday interactions (Garfinkel, 1967). That is, from a student's perspective, how she successfully navigate participating in small group mathematical discussion with peers?

My overall theoretical framework also builds on activity theory (Engeström, 1987; K. D. Gutiérrez, Baquedano-López, & Tejeda, 1999) and the analysis of group work as a functional system. In my analysis of how students learn specific subsets of school mathematics content I focus on the microgenetic level of development and examine transformations in students' use mathematical discourse practices (Moschkovich, 2002, 2007a; Sfard, 2008). To understand the mediation of social interaction in students' discussion, I look at regularities in students' talk. In this area I draw on methods from sociolingusitics, specifically conversation analysis (Sacks, Schegloff, & Jefferson, 1974; Schiffrin, 1994) as well as more general forms discourse analysis (Gee & Green, 1998).

Sociocultural studies of learning attempt to specify how learning and development is related to participation in cultural activities (Rogoff, 2003). A constant tension in such studies is that investigators must balance attending to individual variation and documenting of regularities shared by participants in cultural communities. To avoid the danger of reducing my participating students to stereotypes, I strive to follow the recommendations of Gutiérrez and Rogoff (2003), a) recognizing that the same person may participate in constellations of cultural practices and multiple cultural communities, b) avoiding equating culture and ethnicity/race, and c) using the past tense when describing empirical observations to emphasize that observations represent discrete events rather than timeless truths.

Throughout my research I focus on mathematical discussions in bilingual settings. I adopt a broad, social definition of bilingualism (Sánchez, 1994) and bilingual classrooms, focusing on the language practices of the students who are in the class, the social characteristics of the community beyond the walls of the classroom, and the type of instructional program in a school. This holistic approach means that "bilingual" is not limited to an individual trait or a specific type of instructional program. Below I will describe the research site of my two studies in more detail, though here I note that each of these research sites was a mathematics classroom where bilingualism was the norm.

Data and Methods

The findings reported here synthesize results from two distinct, but related, studies. For the sake of simplicity I will describe the two research sites using the pseudonyms Classroom A and School B. My research in Classroom A took place in one middle school mathematics class, and my study at School B includes two algebra classes at a comprehensive high school. The naming of the sites reflects the chronological order in which the data were collected. My work in classroom A is complete, and my work in school B is ongoing. The three mathematics classes I've studied in sites A and B were bilingual—either by formal program designation or due to the students' and teachers' language practices. The data for this paper are from one of the two classes in School B. Below I describe each classroom and the data and methods for each study.

Classroom A

Classroom A was a sixth-grade mathematics class in a dual-immersion Spanish-English bilingual charter school located in California. Over 90% of the students in this school identified as Latino/a[3] and all of the sixth-graders were bilingual in Spanish and English. The sixth-grade students in this school took two math classes, one taught in Spanish and the other class taught in English, so the sixth graders studied half of the sixth grade mathematics curriculum in English and the other half of the curriculum in Spanish. The observations for this study took place in the English-language mathematics class. Although she lectured and conducted whole-class discussions in English, the teacher, Ms A, was bilingual and interacted with individual students and small groups in both Spanish and English.

My research in classroom A focused on the mathematical discussions of one group of students in the class over the course of one week. Ms A. selected students to sit at the focal group (in the range of my video camera and microphone) by choosing a mix of "typical" students who she thought would be likely to engage in lively discussions. I observed the class for approximately a month and I videotaped all interactions of the focal group during their mathematics class for one week. Adopting a naturalistic (Moschkovich & Brenner, 2000) and ethnomethodological (Garfinkel, 1967; Mehan, 1979) approach, I intentionally chose to record the students' "everyday" peer mathematics discussions in the class without designing activities or starting an intervention. Ms A. structured the groups and selected which mathematical activities the class would do, but she made an effort to use group work each day of my data collection. Between four and seven students sat at the table where the focal group worked.[4] All seven students were bilingual, and four of them reported speaking primarily Spanish outside of school while two reported speaking primarily English outside of school (the seventh student did not complete the language preference survey).

I video recorded a total of five hours of the focal students' in-class interactions in Classroom A. I selected excerpts from the videos when the students engaged in sustained mathematical discussions for detailed analysis. To guide the selection of relevant discussions, I used Pirie and Schwartzenberger's (1988) definition of a mathematical discussion. Based on that selection process, I transcribed a total of 56 minutes of the students' interactions using a modified version of Jefferson's transcription conventions (APPENDIX A). Besides video and transcripts, other data sources include copies of students' written work, field notes, and a short survey on students' language practices and impressions of group work.

School B

The second research site, School B, includes two algebra classes in a comprehensive high school located in an agricultural town in central California. The student population at this school is 94% Latino and 83% of the students at the school were classified as either English Learners (35%) or "Fully English Proficient" (48%-FEP means a student is fluent in English but speaks a language other than English at home). 77% of the students at this school are eligible for a free or reduced price lunch.

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[3] Demographic data about the schools are from http://www.ed-data.k12.ca.us/ (Accessed January 2, 2010).

[4] The size of the group varied due to student absences. When seven students were sitting at the table, the group's discussion frequently broke into two or more parallel discussions.

The study at School B focuses on how algebra students reason about rates, linear functions, and the accumulation of change in the context of small group work during an algebra unit on linear functions. This focus is slightly different from the study in Classroom A, which was not tied to specific area of mathematical content. The participating students at School B were drawn from two remedial algebra classes—one class, B1, had a relatively high proportion of English Learners, and the teacher used both Spanish and English during her daily teaching. The other class, B2, had a relatively low proportion of English Learners and the teacher in B2 used English during class.

The data collection in School B occurred while the algebra classes were doing a unit on rates and interpreting graphs of linear functions. The data sources include six weeks of in-class videos focusing on one group of students per class, video and written work from three out-of-class group discussions with each focal group (see APPENDIX B for one task used as a prompt for these discussions), a survey of students' language practices, field notes from in-class observations, and informal and formal interviews with the participating teachers. This study is ongoing, and though the bulk of the data collection is completed, I continue to analyze the data and work with the teachers and students in School B.

FINDINGS FROM PREVIOUS STUDIES

The two research projects in sites A and B had slightly different foci, goals, and research questions. Despite differences in the design, both studies focused on how students use talk, semiotic meditational means, and mathematical discourse practices to reason in the setting of small group work. Below I detail two findings based on research in Classroom A. In each section I give a short review of the most relevant literature, detail one research question and one finding, and illustrate the finding with an example. Next, I use newly collected data from School B to extend and test findings. Finally, I describe implications for researchers and practitioners.

Construction of Authority during Group Discussions

Investigating the role of authority in students' group discussions of mathematics is important because of the prominent role authority plays in mathematics teaching and learning. While reasoning by an appeal to authority is considered unsound in academic mathematics (Harel & Sowder, 1998), Lampert (1990) argues that one of the primary features of a traditional mathematics classroom is that the teacher and the text are treated as arbiters of mathematical authority. Beyond Lampert's historical analysis, the pervasive role of authority in mathematics classrooms can be observed in classroom talk. For example, teacher and student talk in mathematics classrooms frequently includes phrases that create positions of authority (e.g. "what you must do is"), and this phenomenon is more common in mathematics classrooms than in other types of classrooms (Herbel-Eisenmann, Wagner, & Cortes, 2008). While using group work is a common suggestion for transforming relations of power and authority in mathematics classrooms, group work is also not immune from issues of authority (Cobb, 1995; Esmonde, 2009). For example, in one detailed analysis of four pairs of second graders solving problems in a reform mathematics class, Cobb (1995) noted that one student in each pair was often treated as an authority (or acted as one), and that this role had deleterious effects on both students' learning new strategies for doing arithmetic problems.

For both teachers and researchers, a critical question is how is authority constructed during group interactions and how can relations of authority be transformed? One research question in the study of classroom A was, how did authority mediate students' interactions during small group work? Moschkovich and I (Zahner & Moschkovich, 2008, under review) documented how authority was constructed moment-to-moment through students' interactions. This was a reflexive process—authority was constructed through students' discourse, and simultaneously, being constructed as the authority appeared to lead to the use of particular conversational moves. While there were regularities in who was constructed as the authority for the focal group, we observed that authority shifted rapidly. To see the reflexive construction of and conversational impact of authority, consider the following excerpt from a discussion between two of the students in the focal group, Amber and Francisco.

Three students, Amber, Francisco, and Lorenzo were sitting at a table made of six desks. They were working on a series of problems from their text and their assignment was to put the solution to one problem on a poster for presentation to the class. Lorenzo was writing the students' names on the poster while Amber and Francisco were choosing which problem to solve. Immediately before the start of the discussion in Excerpt 1, Francisco read question number eight from the textbook: "At 6:00 AM the temperature was -4°F. By 6:00 PM, the temperature had risen 17°. What was the temperature at 6:00 PM?"

Excerpt 1

1. Francisco:	But who ge- do you get number eight at all? ((referring to question number eight from the text))
3. Amber:	Seventeen minus six (.) Du:h
4. Francisco:	Oye ((“listen” or an interjection)), I found it already
6. Francisco:	Look. Subtract ss seventeen minus negative four [xxx
7. Amber:	[Thats that I sai::d
8. Francisco:	You said subtract thirtee:n
9. Amber:	No, I said seventeen=
11. Amber:	=(minus) negative four
12. Francisco:	Now I get [it
13. Lorenzo A.:	[Ahhh ((teacher approaches))
14. Amber:	I got number eight ((looking up at teacher))
15. Francisco:	I got number eight too, but (kinda) she's helped me

This excerpt highlights the ways in which authority was created, negotiated, and reified through students' spoken interactions. First, Francisco's question to Amber in line 1 positioned her as a "knower" of mathematics (this point was bolstered by our analysis of all of the students' questions during small group discussions that week). Second, Amber's response ended with the tag "duh," which indicated her negative assessment of the quality of Francisco's question. This tag also helped reify her position as an authority. In lines 4-6, Francisco challenged Amber's authority by proposing an alternative solution to the problem. Amber's response to this challenge was to interrupt Francisco and claim that she had already said his solution (even though she had not). She also used another elongated syllable for emphasis ("that's what I sai::d"), which may be another, less obvious judgment than the word "duh." Amber and Francisco's disagreement appeared to continue through line 12 and until line 14 where Amber told the teacher that she knew the answer to the question. Perhaps the most interesting part of this excerpt is that Francisco ultimately credited Amber for helping him in line 15, even though his proposed solution in line 6 was the closest suggestion to a correct answer in this discussion!

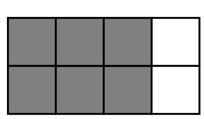
Excerpt 1 highlights how authority is created and negotiated through students' interactions during peer discussions. It also shows that this construction of authority is a reciprocal process: Amber did not just "take" authority in this discussion, there was a process of mutual construction, negotiation, and struggle for authority. This observation stands in stark contrast with the "traditional" presentation of authority in school mathematics where Lampert (1990) claims the teacher and text act as authorities and there is little question about who or what is the source of correct answers. In terms of sociolinguistic patterns, we observed three ways that authority was constructed in these students' peer discussions: 1) interruption and claiming the conversational floor, 2) the voicing of private speech while doing computations (which we call "computational private speech" Zahner & Moschkovich, in press-b), and 3) the use of direct contradiction. These findings are elaborated further in Zahner and Moschkovich (under review).

Use of Two Languages

Bilingual students often use both of their languages to reason mathematically and this linguistic diversity in groups of bilingual students and English learners can be seen as a valuable asset (Moschkovich, 2000). One common type of move includes a single speaker using more than one language. Linguists distinguish among a number of types of language switches (e.g. using loan words, switching at prepositions within a single move, or switching at the end of a turn). While the distinctions are interesting, they are more detail than we require. In this paper we refer to the use of two or more languages during the same discussion as "code switching." One purpose of code switching in the setting of a small group mathematical discussion is to manage the conversational floor and preserve "face" (Lakoff, 1973). In several studies Moschkovich has shown that some students use two languages during mathematical discussions in repeated bids for the floor or to modify the potential negative impact of contradicting another conversational participant (Moschkovich, 2007b; Zahner & Moschkovich, in press-a).

In Excerpt 2, we see how the students in Classroom A skillfully used Spanish and English to manage the conversational floor. The students were working on a problem that involved finding the percent of a figure that was shaded (see below). After Claudia convinced her group mates that the answer was 75% (by appealing to the teacher's authority), Francisco asked for clarification. Excerpt 2 starts with Franscico's question and continues through Claudia's explanation of solving the problem via long division.

Prompt "Find the percent that is shaded in the figure"



Our reconstruction of Claudia's Computation

$$\begin{array}{r} 0.75 \\ 8\overline{)\,^{2}\cancel{6}.^{1}00} \\ \underline{56\downarrow} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Figure 1. The problem the group was discussing in Excerpt 2 and our reconstruction of Claudia's computation

Excerpt 2

1. Francisco:	ah- why six over eight seventy five percent?
2. Claudia:	((bobbing head slowly)) Divide it.
3. Amber:	OK watch. Six goes inside divide by eight [xxx=
4. Claudia:	[Por eso 'ira ((“that's why, look”))
5. Amber:	=and [that
6. Claudia:	[goes in the casita ((“little house”)), you take out eight you put a decimal porque no se puede ((“because you cannot”)) [put a decim- you put a zero=
7. Amber:	[Uh-huh
8. Claudia:	=ocho por siete ((“eight times seven”)) put a seven here it's fifty-six
9. ...	
10. Dennis:	four=
11. Francisco:	=(and that's) four
12. Dennis:	[[four
13. Claudia:	[[five it's ten is four
14. Francisco:	Oh yeah, seventy five percent
15. Claudia:	y (luego) ((“and then”)) [xxx zero
16. Dennis:	[Zero then five
17. Claudia:	y luego es ((“and then it is”)) seventy five percent

In lines 2-7 of Excerpt 2 Claudia and Amber appeared to dispute who would take the floor and answer Francisco's question. After Claudia's brief answer in line 2, Amber elaborated on how to set up the long division. As Claudia re-took the floor from Amber she used a common bid for the floor in Spanish, “por eso 'ira” (“because of that, look”) while interrupting her group mate. As the interaction progressed, Claudia frequently mixed Spanish into her talk, especially when stating prepositions and doing computations. While we cannot know exactly why Claudia did this, we do know at least three things about Claudia and her use of Spanish and English. First, Claudia could express the computations in both Spanish and English, so she did not use Spanish due to gaps in her English vocabulary. Second, Claudia's use of Spanish in this excerpt and throughout the data collected at School A was not systematically related to using “everyday” or non-academic terms (for example, in Excerpt 2 Claudia does a multiplication in Spanish and uses an informal metaphor for the division sign that she had learned in English). Finally, Claudia's (and her group mates') use of two languages both here and in other data frequently aligned with efforts to control the conversational floor, repeat a point, or mitigate the effects of a direct contradiction.

We do not view bilingual students' use of two languages during mathematical discussions as a deficit. Quite the contrary, in detailed analysis of the discussions of the students in Classroom A, and in analysis of mathematical discussions among other bilingual students (Moschkovich, 2007b; Zahner & Moschkovich, in press-a), we have shown how the use of two languages allows bilingual students to navigate the difficult, face-threatening terrain of a mathematical discussion. This observation that code switching facilitates the skillful management of “face” may similar to Rowland's observation that hedges in mathematical statements lesson the face threatening stance of making mathematical assertions. Rowland found that these hedges are used by elementary students and expert mathematicians alike (Rowland, 2000). Our data indicate that code switching in bilingual students' mathematical discussions functions in a similar way.

Our relatively neutral assessment of code switching should be interpreted in light of the fact that Classroom A was in a bilingual school that actively promoted students to use both Spanish and English. In school settings were bilingualism not institutionally supported, there is much more evidence of how switching between languages may also be used to maintain status relations or to include/exclude participants in the discussion (c.f. Lopez Leiva's work in CEMELA). For this reason we reaffirm the notion that claims about sociocultural phenomena, such as code switching, must be limited in scope to the time and place of the observation (K. D. Gutiérrez & Rogoff, 2003).

COMPARISONS WITH OBSERVATIONS IN SCHOOL B

In my dissertation study, I am testing, refining, and extending my previous findings. This most recent research focuses on how students learn and reason about rates and linear functions in the setting of group discussions in two remedial algebra classes. The research site for my dissertation is School B, a comprehensive high school located in an agricultural region of California described above. In this study my primary interest is how the students learn and reason about rates and accumulation (e.g. the slope of a linear function showing displacement or the accumulated area under a linear function of velocity) in algebra classes where small group discussions are a common form of classroom interaction. To narrow the scope of the research I focus on discussions among students in two small groups in two different algebra classes in School B. One group is in a class, B1, taught in both Spanish and English. This class has a relatively high proportion of recent immigrants and students classified as English Learners. The teacher in class B1 is highly experienced, highly regarded by her peers, and has a long history of using groups in her instruction. The other group is from an algebra class, B2, is taught exclusively in English. The student population of this class consists primarily of students who are bilingual but not classified as English Learners. The teacher in B2 is new to teaching and she models her teaching and curriculum after the teacher in B1.

The overarching research questions for this study are:

1. What interactional and sociomathematical norms emerge during students' discussions about rate and accumulation? How do these emergent norms relate to student reasoning about rate and accumulation?
2. How do students use mathematical discourse practices and semiotic resources such as tables, equations, and diagrams to reason about rates and accumulation during small group discussions?
3. How do students' use of mathematical discourse practices and semiotic resources change over the course of a unit on linear functions and analyzing rates of change?
4. Are there systematic differences in how English Learners and bilingual students use mathematical discourse practices to reason about rates and accumulation?

Throughout my study, I focus on the students' domain-specific learning in through group mathematical discussions. I use videos of the focal groups recoded in-class as well as video and written work from out-of-class task-based group discussions to document the development of the students' reasoning about rate and accumulation. This analysis is ongoing, so for the purpose of this paper, I will focus on a brief excerpt of one group's interactions during an out-of-class task-based group discussion to show how the two findings I have outlined above relate to the discussion of one group of high school algebra students.

In the following excerpt, a group of bilingual students (who were in the class B1) were working on the "hexagon desks" problem in APPENDIX B.[5] The students in this group were bilingual, but the majority of their in-class interactions were in English. This excerpt is taken from the third discussion this group had on this problem (the three discussions were spaced across eight weeks), and they were discussing question 1 of the task, filling in the table for the number of students who could sit at a row of hexagon desks pushed together. In their final written responses to this task, the group had previously alternated on their solution for the last row of the table. During their first discussion of the problem the group wrote 30 in the last line of the table and during their second discussion of the same problem they settled on 31. In Excerpt 4, taken from the group's third discussion of the problem,[6] all four students were actively negotiating whether the final line of the table should be 30 or 31 and whether to "add five" at some point while completing the pattern.

Before reading this excerpt, it may be useful to try the first part of the problem in APPENDIX B. As you read the excerpt, focus on the following.

1. Mathematically, how did Krystal and Susana solve the problem, and how did Mateo and Jaime solve the problem?
2. How did authority get constructed in this discussion? Whose proposals were take up, whose proposals were ignored?
3. What are the possible functions of code switching in this discussion?

Excerpt 3

PV C1 Discussion 3 (15:15-19:14). Krystal, Mateo, Jaime, and Sandra were sitting at a table with Krystal and Mateo on one side, and Jaime and Sandra on the other. The students were working on question 1 of the task in APPENDIX B.

1. Krystal	wait what? OK, I agree with eighteen and twenty two ((referring to values in lines 4 and 5 of the table))
2. Mateo	and twenty six is [inc-
3. Jaime	[Cause the last one you add on xxxx
4. Mateo	yeah
5. Krystal	eighteen [nine:teen-
6. Mateo	[you just add four to all of them
7. Krystal	then you add four and then you add like two
8. Mateo	the last one is five
9. Krystal	what

[5] This problem is similar to a released eighth grade NAEP problem and also a MARS/Balance Assessment task.

[6] I used the same problem multiple times to document changes in the students' reasoning over time. While these students initially tried to recall their previous answers, the quickly fell into a thorough discussion of the problem, as seen in Excerpt 3.

10. Mateo	see six plus four is ten plus four is fourteen and the last one you just add five that's all these
10. Susana	[[why add five
11. Krystal	[[(really)? yeah
12. Mateo	cause it that's the last one [and somebody can sit on this ((appears to point at the end of the chain of hexagons))
13. Krystal	[one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six twenty seven [[twenty eight twenty nine thirty
14. Mateo	[[(so what if) they do that I don't know ((Mateo was looking at Jaime while speaking. This may be a continuation of a previous discussion unrelated to this problem))
15. Krystal	I got thirty watch. 'ira ((“look”)) one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six twenty seven twenty eight twenty nine thirty
16. Mateo	((looking on while Krystal counts, then gestures his finger along the figure, pointing at vertical segments that are the intersections of polygons))
17. Jaime	wait
18. Krystal	I'm not counting these ((looking at Mateo))
19. Jaime	On the fourth one it's not eighteen it's nineteen
20. Mateo	huh
21. Krystal	yeah (.) wait what
22. Jaime	it's nineteen
23. Krystal	On the four no::
24. Jaime	yeah
25. Krystal	no
26. Susana	xxx
27. Jaime	cause ’ira ((“look”)) four these are already three cuantos son ((“how many are they”))
28. Krystal	ok ’ira ((“look”)) four one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen
29. Jaime	xxx
30. Krystal	OK you guys [draw another one right here
31. Jaime	[but on three (.) con ((“with”)) three son ((“there are”)) (.) fourteen. It's like counting this one just block this side
32. Mateo	I know
33. Jaime	fifteen sixteen [seventeen eighteen nineteen
34. Susana	[exactly and it's eighteen
35. Krystal	((takes paper from J)) and then you go like this and like this and like this and like this OK ’ira ahorita ((“look now”)) one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen ((hands paper back to J))
36. Jaime	it's like the same thing [you just block this one right

37. Susana	[no it isn't
38. Krystal	And then you block the [[other one
39. Jaime	[[pues 'ira (.) why this one
40. Krystal	cause xxx
41. Jaime	Oh yeah cause xxx I thought it was the last
42. Krystal	is that right? OK we've got another one ((starts singing)) OK so one two three four five six seven for seven xx OK so its one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six twenty seven twenty eight twenty nine thirty
43. Jaime	you forgot to do the six
44. Krystal	what
45. Jaime	you forgot to do the six
46. Krystal	oh yeah huh. one OK one two three four five six seven eight. one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two twenty three twenty four twenty five twenty six. Twenty six ((looking at M)) Did you get twenty six for number six too
47. Mateo	yeah
48. Krystal	yeah but I get thirty right here
49. Susana	it is thirty

Mathematical Reasoning

Before jumping into an analysis of authority and code switching in this discussion, I first offer a few remarks about the students' mathematical reasoning in Excerpt 3. Recall that the students were filling in the table in question 1 of the problem in APPENDIX B. In lines 6 and 8 Mateo said "you add four to all of them … the last one is five." One plausible interpretation of Mateo's statements is that he meant that number of students in each line of the table is four more than the number of students in the previous line, except in the last line where the increase is five. Recall that this same issue, especially focused on the final line of the table, came up all three times this group of students worked through this problem. For this group, finding the correct value for the final line of the table was a persistent challenge. While Mateo's suggestion to add five is not correct, it does have an interesting relationship to how the chain was drawn on the students' papers. In both his talk and gestures (lines 12 and 16), Mateo highlighted the contribution of the final (rightmost) vertical segment in the chain of desks when he argued that the number of students in the final row of the table should be five more than the previous row. Note in the figure below, taken from the students' written response, how the five new segments on rightmost hexagon are drawn darker than the segments in the hexagon immediately to the left. The fact that drawing the last hexagon means drawing five new segments is one possible source of Mateo's thinking on this problem.

10. Mateo	see six plus four is ten plus four is fourteen and the last one you just add five that's all these
10. Susana	[[why add five
11. Krystal	[[(really)? (.) yeah
12. Mateo	cause it that's the last one and somebody can sit on this

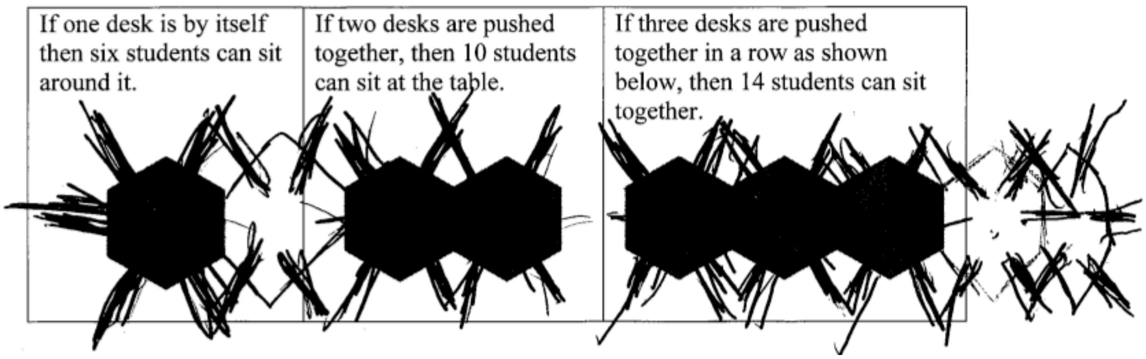


Figure 2: Part of the students' written work in Excerpt 3.

From a mathematical perspective, Mateo's "last one just add five" technique does not generalize. The contradictions inherent in this perspective are revealed in Jaime's contention (line 19) that for four desks the answer should be nineteen since the fourth iteration should be five more than the total with three desks. Jaime's argument may have been an attempt to generalize Mateo's "add five at the end" rule, and when he drew a chain of four desks, the fourth desk was the last one.

Susana questioned Mateo's "add five" strategy in line 10 and in lines 28 and 34 she disagreed with Jaime (and, implicitly, Mateo) when she argued that eighteen students could sit at a row of four desks. In line 28 Krystal demonstrated Susana's point by counting the eighteen sides of a row of four desks. Beyond this contribution, however, Susana's participation in this discussion was limited so it is difficult to infer much more about how Susana was reasoning through the problem.

Krystal used two strategies for solving these problems. Her statement "you add four and then you add like two" in line 7 may have been related to finding a formula for the number of students who could sit around n hexagon desks. Later in this session the group derived the formula 4n + 2 in response to question three. Krystal's suggestion in line 7 may indicate that she, like Mateo, was already trying to find the pattern for the nth term while filling in the table. This also matches a regular method these students had for solving these problems. In class the students often made a pattern from a table.

At the same time that Krystal may have been looking for the pattern (indicated by line 7), Krystal also repeatedly checked the answer for each row of the table by drawing a figure with the given number of hexagons and counting all of the sides. In this four-minute excerpt, Krystal counted the sides of a hexagon chain five times (lines 13, 15, 28, 35, and 42). Three of those times (lines 15, 28, 35) she explicitly invited a group mate to watch as she counted ("I got thirty watch" in line 15 and "ok 'ira" ["watch"] in 28 and 35). Two of Krystal's episodes of counting aloud may be instances of computational private speech (Zahner & Moschkovich, in press-b). In line 11 Krystal expressed doubt in Mateo's "add five" strategy and in line 15 she explicitly contradicted Mateo's claim that the last line should be 31. By the end of the excerpt, Mateo had abandoned his argument that the final line should be 31, and the group moved on to solving the next question.

Ultimately the group wrote 30 in their final answer to the last line of the table in question 1. They also successfully found the number of students who could sit at a row of 100 desks (402) and they derived the formula 4n + 2 for the number of students who could sit at a row of n hexagon-shaped desks pushed together in a row.

Authority

The students' discussion in Excerpt 3 did not have the same pattern of questions, judgments, and outright competition for the conversational floor evident in Excerpts 1 and 2. There are several possible explanations for this difference: 1) these students were in a mathematics class where they had received extensive training on the use of group discussions, and 2) the task that these students solved was more open-ended than the routine exercises discussed by the students in Excepts 1 and 2, and 3) These students were older than the students in Excerpts 1 and 2, which may explain some differences in how they handled mathematical disputes. These explanations are not exhaustive, and there were likely other issues at work, especially in terms of interpersonal dynamics. However the current data do not allow me to make inferences about those dynamics.

Nonetheless there were differences of opinion among the four students in Excerpt 3. The contrast in how this group handled mathematical conflicts and how Amber and Francisco handled their conflict in Excerpt 1 is worthy of further reflection because the issue of authority has been identified as a critical component of mathematical discussions. In Excerpt 3 the group's discussion showed more of a relationship of balanced authority, similar to the relationship between the students in one of the dyads described by Cobb (1995).

One indication that this group exhibited more balance in the distribution of authority was that they used evidence from their diagrams and logical reasoning to support their claims. For example, during each of Krystal's repeated counts of the perimeter, she was looking and pointing at the diagram on her paper (or Jaime's paper in line 35). When Mateo contradicted Krystal in lines 12 and 16 he pointed at her diagram and referenced the vertical segments. This use of evidence and a shared diagram contrasts with Amber and Francisco's discussion in Excerpt 1. In that discussion the two students apparently were disagreeing (based on what they said and Amber's judgmental statements), but there is little evidence that they had achieved intersubjectivity, i.e. the actual source of their disagreement is ambiguous. The lack of shared written referents may have been part of the problem. This practice of making claims from evidence is a valued (school) mathematical discourse practice (Moschkovich, 2007a), and using evidence to support claims (and establish some semblance of intersubjectivity) may be one way to promote equitable small-group interactions (Esmonde, 2009).

A second, related, indication that the authority was more distributed among the group from classroom B1 is that the students appeared to take up and investigate each other's claims. For example, in lines 7 and 8 Mateo and Krystal offered competing strategies for finding the sum.

5. Krystal	eighteen [nine:teen-
6. Mateo	[you just add four to all of them
7. Krystal	then you add four and then you add like two
8. Mateo	the last one is five
9. Krystal	what
10. Mateo	see six plus four is ten plus four is fourteen and the last one you just add five that's all these
10. Susana	[[why add five
11. Krystal	[[(really)? yeah

Susana and Krystal's questions in lines 10 and 11 gave Mateo a brief opportunity to explain his reasoning. This is not an example of a successful proposal, and Mateo's opportunity to explain himself was short-lived. Krystal interrupted Mateo and turned back to the evidence, counting sides of a hexagon chain made of seven hexagons–twice. The second time Krystal counted, she explicitly told Mateo to watch ("I got thirty watch. 'ira") while she counted.

The group's investigation of another disputed claim was repeated in lines 17-28. This time Jaime made another claim—that the fourth row of the table should be nineteen. Krystal and Susana both took up Jaime's idea and responded to him by drawing the figure and counting sides. By line 41, Jaime's "Oh yeah cause xxx I thought it was the last" indicates that he accepted Krystal and Susana's answer (but was still endorsing Mateo's idea of "last one add five"). While Krystal and Susana disagreed with Jaime and Mateo, they reasoned about the problem by counting the diagram, rather than arguing their point from a position of personal authority.

Even though this group of high school algebra students handled disagreements differently than the group of middle school students from Classroom A, there clearly was a disputed answer, and by extension, disputed authority. The parallels between these groups help us understand more about how authority is constructed, even during more equitable group interactions. One parallel between the discussion among the algebra students in Excerpt 3 and the middle school students in Excerpts 1 and 2 is that the students who were contradicting their peers and implicitly assuming a position of authority (Krystal in Excerpt 3 and Claudia in Excerpt 2) both held the floor for the majority of the discussion and gave direct commands to their group mates. For example in Excerpt 3 Krystal repeatedly used inclusive imperative verbs ("watch/mira" and "draw another one right here" [30]), while her group mates used the imperative sentences to a much lesser extent.[7] In Excerpt 2, all of Claudia's turns included direct commands while in Excerpt 1 Fancisco's imperative, "Look subtract seventeen minus negative four" might be interpreted as a challenge to Amber's authority, and she responded forcefully to this suggestion ("that's what I sai::d"). A second parallel between the constructions of authority within two groups is that disputed answers (or disputed "rights" to answer) often corresponded with overlapping talk and a contested conversational floor. Krystal's interruption in line 12 is one example of this, and Caludia's taking of the floor in Excerpt 2 is another example. In a separate paper (Zahner & Moschkovich, under review) I show how this move by Claudia followed a dispute in her group.

CODE SWITCHING

The students' use of Spanish and English in Excerpt 3 generally aligns with our earlier finding that language switching may be a tool for "face management" during mathematical discussions among bilingual students. Rowland (2000) has detailed how mathematical talk can be particularly face-threatening. Here we see some evidence of how code switching may have functioned to mitigate this phenomenon. Recall that the group in Excerpt 3 came from a bilingual mathematics class. While the use of languages was not free of ideology, for this group (at least in

[7] Morgan (1996) and Rotman's (1988) analyses of mathematics texts highlight how the use of verbs in the imperative mood is part of the genre of written mathematics. Morgan notes how the use of this mood is related to constructing a relation of authority between the author and reader of mathematics texts. Krystal's use of the imperative is inclusive; she invites her group mates to watch what she does (rather then telling them to do something for her).

Excerpt 3), the alternation between using Spanish and English does not necessarily carry the same ideological baggage that it might in other contexts. The students did skillfully use both languages when making potentially face-threatening claims.

The pattern repeated in Excerpt 3 and across most of this group's discussions is that the students used common Spanish expressions and switched languages while disagreeing, when making a bid for the floor, and in mid-turn transitions. For example, in lines 27, 31, and 39 Jaime used Spanish when disagreeing with his group mates. In 15, 27, 28, 35, and 39 Jaime and Krystal gave direct commands in Spanish, lessening the impact of such a face-threatening move. Also in 35 Krystal's "OK 'ira ahorita" served as a pivot in her turn, marking when she had finished drawing the new figure and when she was going to start counting the sides.

It is notable that neither Mateo nor Susana used Spanish in Excerpt 3. In terms of their daily language practices, both students reported speaking Spanish outside of school. Also, during one part of this group's discussion there was one instance where Mateo clarified the meaning of the term "sack race" for Susana by switching to Spanish. One interesting follow-up on this phenomenon will be to investigate how she used Spanish and English during this group's in-class discussions.

This analysis of the students' use of Spanish and English is not to say these students weren't aware of the power of languages and larger social realities of language ideology. In the following interaction, Krystal appeared to express wonder at the fact that the second richest man in the world, a telecommunications magnate from Mexico, only speaks Spanish (N.B., the truthfulness of this claim in immaterial; Krystal' apparent pride and amazement in the person's knowledge of English is most important here). The following interaction occurred during an informal discussion that was recorded while the researcher was setting up equipment for the group's discussion in Excerpt 3.

Excerpt 4

The students were sitting around a table while the researcher set up cameras and microphones. The topic turned to the world's richest people.

1. Mateo	wait Bill Gates is the guy he made Microsoft?
2. Susana	yeah
3. Mateo	oh yeah
4. Susana	he's hella rich
5. Mateo	that guy's the chief
6. Krystal	you know who's like the second richest guy in the world
7. Susana	who
8. Krystal	the cell phone guy from Mexico
9. Susana	haha the cell phone guy
10. Krystal	yeah seriously
11. Researcher	I heard that too. Cause like every call you make actually it goes like through his his phone system
12. Krystal	yeah (.) second richest guy in the world
13. Jaime	who
14. Krystal	from México ((pronounced in Spanish)
15. Jaime	yeah
16. Krystal	oh yeah, he like it he doesn't even know how to speak English

Further analysis of this group's interactions may be needed to disentangle their use of Spanish and English while solving mathematics problems. There are several interesting possible next directions of inquiry. I have already noted that it will be interesting to compare how the students use two languages in the out-of-class and in-class mathematical discussions. Across groups, comparing this group's use of Spanish and English with the code switching of a group from a classroom where the teacher only used English might be another interesting follow-up.

CONCLUSION

In this paper I have presented two findings from my previous study of middle school students' mathematical discussions in a bilingual classroom and I have used recently collected video of a group discussion from a bilingual algebra classroom to offer a comparison with earlier findings. The new data add depth to earlier findings about the construction of authority and students' use of Spanish and English in their group discussions. Specifically, in the discussion detailed in Excerpt 3, the students showed more balanced relations of authority and they relied on evidence from the task and their drawings to support their claims. The students' use of Spanish and English appeared to serve similar face-saving functions and functions related to managing the floor, as seen in Excerpt 2.

There may be several implications for practice in this analysis. First, from both a practical and research-oriented perspective, our analysis has shown how mathematical reasoning and social/interpersonal relations in mathematical discussions are related. An analysis of either "the math" or "the students' use of language(s)" alone would be incomplete; both parts of the analysis are important for understanding the meaning that students make through engaging in mathematical discussions. Second, the group in Excerpt 3 offers a nice contrast in how differences of opinion and contested authority might happen in a mathematical discussion. I am not necessarily using the students' discussion in Excerpt 3 as not an ideal; but there are several practices worthy of emulation. For example, reasoning from data and making claims explicit are critical for promoting equitble mathematical discussions (Esmonde, 2009) and they are part of valued mathematical discourse practices (Moschkovich, 2007a). Finally, the data in Excerpt 3, once again, show how bilingual students' use of two languages can be very productive in terms of promoting mathematical reasoning.

APPENDIX A

The following transcription conventions are a modified version of Jefferson's conventions, detailed in (Schiffrin, 1994). Line numbers in some excerpts are not consecutive because we did not include utterances from side discussions (e.g., teacher instructions to neighboring groups).

:	Elongated syllable
[	start of overlapping speech
[[	two speakers start simultaneously
(uncertain transcription)	
xxx	inaudible speech
((comment))	
((“translation of utterance in Spanish”))	
=	latched phrases
-	self-interruption or sudden stop
.	falling intonation
?	rising intonation

APPENDIX B

Hexagon Desks Task used in Excerpt 2.

Ms. West wants to know how many students can sit around a row of hexagon shaped desks.

If one desk is by itself then six students can sit around it. 	If two desks are pushed together, then 10 students can sit at the table. 	If three desks are pushed together in a row as shown below, then 14 students can sit together. 

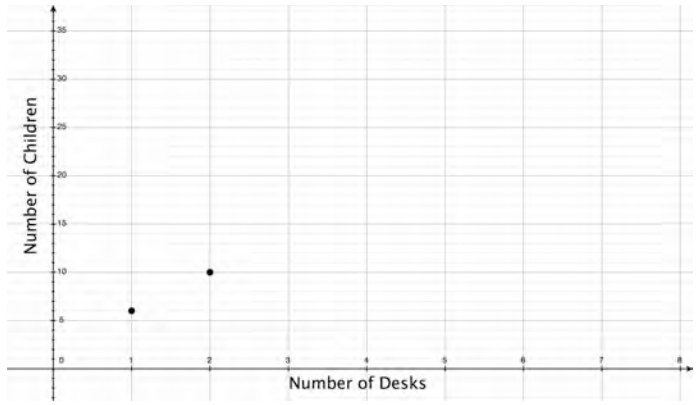
1. Fill in the following table for the number of students who can sit together for the number of desks pushed together in a row:

Number of Hexagon Desks	Number of Students
1	6
2	10
3	14
4	
5	
6	
7	

2. Imagine that 100 of the hexagon desks were pushed together in a row. How many students could sit around that row of desks? Show the work you used to find that solution.

3. If n hexagon shaped desks, were pushed together, then how many students could sit at the row of desks? Give your answer as a formula in terms of n.

4. Use the table you made in problem 1 to draw a graph showing the number of children who could sit at a row of desks



5. If you connect the dots between points in the graph to make a line, what is the slope of that line? How do you know?

6. What is the meaning of the slope of the line in terms of the problem about children sitting at desks? Explain your answer in terms of the problem and using words and ideas that you know from math class.

7. What if n <u>octagon</u>-shaped desks were pushed together? How would this problem be different? How would it be the same? Explain your answer in as much detail as possible (you may use equations, tables, graphs, words, etc.).

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Visions from the Classroom—Focus on Students Discussion

The following section presents comments from the discussion that took place after the research presentations and the practitioner panel. The participants met in small groups that included teachers, school administrators, mathematics educators, mathematicians, bilingual/ESL educators, and policy makers. We have captured multiple simultaneous discussions and have attempted to be as faithful as possible to the participants' comments.

The task given to the working groups was to address the following questions:
- What do we know?
- What are the implications for practice and research?
- What else do we need to know?
- What connections exist between this strand and the other strands at this conference?

The working groups reflected on the connections that exist between this strand and the other strands within the context of the discussions of the other questions. The following summary represents common themes identified within and across the working groups.

What do we know and what are the implications for practice and research?

Based on the research studies presented as part of the Visions from the Classroom—Focus on Students Strand, along with the poster sessions, and our own professional experiences we are able to state what we believe to be true about the teaching of mathematics for English Language Learners (ELLs) and particularly with Latinos/as. The teaching of mathematics, especially to ELL students is a complex endeavor that requires specialized training. First we will identify specific knowledge that supports the success of learning mathematics for ELL students.

Foundational is sociocultural theory that "describes cognitive development as a refinement and expansion of the internal systems of meaning individuals construct in their social and cultural environments…Language including gesture is the first mediator of meaning in a child's life and that early spontaneous concepts are structured and represented internally by the meaning attached to words" (Marshall, 2010, p. 2). Language, therefore, is a system of thinking and mediates meaning making. Internal sense making and conceptual development is dynamically linked to external linguistic and cultural experiences (Marshall, 2010). Use of one's first language is yet another form of representation that could support different models for thinking about mathematics.

We should not prevent students from drawing on their native language as it serves as a cultural and linguistic resource for students' mathematical thinking and reasoning. Restricting students from using their native language also violates a student's civil rights and encourages the loss of one's identity as they lose their native and/or family's language. Students and teachers should be allowed and encouraged (by design) to use any and all tools available to them including linguistic and cultural resources. We are losing intellectual and creative power by not encouraging students to draw on all the resources they bring into the classroom. More use of the home language as a resource in communicating mathematical ideas needs to be supported. "Classroom conversations that include the use of gestures, concrete objects, and the student's first language as legitimate resources can support students in learning to communicate mathematically. Instruction needs to support students' use of resources from the situation or the everyday register, in whichever language students choose" (Moschkovich, 2002, p. 208).

The bilingual language practice of code switching is often considered a deficiency and stigmatized. However, a large body of linguistic research finds that "code switching is a social language practice connected to community norms" and "code switching is not a reflection of low level of proficiency in a language or the inability to recall a word" (Moschkovich, 2007, p. 131). In their research, Zahner and Moschkovich (2010) documented middle and high school bilingual mathematics students used code switching during small group mathematical discussions for the purposes of:

- managing the conversational floor
- modifying the potential negative impact of contradicting another student

Code switching actually can benefit bilingual students' mathematical discussions. Work with educators, administrators, and policy makers to change perceptions of code switching and further understanding of its purposes is critical.

The complexity of the sociocultural aspects of mathematical learning must be recognized. How students construct mathematical authority is dependent on interaction with the teacher, the textbook and is also interactively constructed through group mathematical discussions (Zahner and Moschkovich, 2010). Considerations and attention must be given to student status and how it affects mathematical authority within group work. Careful selection of problems that are "group worthy" tasks, consideration of group composition and establishing norms for group work can benefit students' mathematics learning (Cohen, 1994).

The type of audience can impact students' explanations and justification of mathematics solutions to problems. When students' engage with authentic audiences and not solely with their teacher, a mix of social and mathematical talk can arise that integrates school mathematics and students' own funds of knowledge (Crespo, 2003). When students' wrote to an authentic audience (a pen-pal), Crespo found that students' explanations of the mathematics were better developed and justifications more complete than when they wrote to their teacher or other classmates. When learning becomes more personal, and mathematical and social talk are integrated, students' retention of the mathematics learning increases, their understanding is deeper, and their mathematical confidence and identity develop (Crespo, 2003).

Opportunities to engage with authentic audiences in mathematical problem solving especially using written communication benefits the ELL student since written communication is the last system of language to develop. Possible contexts can be pen-pals across grade levels within the school, pen-pals with preservice teachers, or using the internet to connect with other students with similar cultural and linguistic backgrounds. The internet can be especially useful to build cultural connections when there are small populations of ELLs within the classroom or school.

To develop deep mathematical understanding, students need to have time to engage in and explore mathematics in meaningful ways and to make sense of the mathematics (Khisty, 1999). In work with primary students using a Cognitively Guided Instruction (CGI) framework along with students' native language, the ambiguity in understanding problems is removed, allowing students to consolidate and organize their thinking. The level of student explanations is more precise than when they are required to communicate in their second language (Marshall, 2010). Conceptual understanding is dependent on making meaning. Dependent on the students' perspective, manipulatives and other mathematical tools can either hinder or support the child's sense making efforts (Marshall, 2010).

For students to develop and adapt more sophisticated strategies they need significant amounts of time (Hiebert & Grouws, 2007). However, students often receive mixed messages about what is valued in school mathematics. Speed and a sense of urgency to move on are often the message they receive and are not given the time needed for deep thinking and meaningful work. Often this urgency eliminates teachers' willingness to allow students to struggle to make sense of the mathematics. (Hiebert, J., et. al., 1997; NCTM, 2006; National Research Council, 2001). The traditional mathematics classrooms often enable student helplessness by doing too much for the students (Sousa, D. A. 2008).

Students have the capacity to learn mathematics, but it is the role of the teacher to capture and foster this capacity through an intellectually rich, problem-based curriculum that is not culturally dissonant with their experiences.

> "…the process of learning requires the use of signs, symbols, and other cultural tools (e.g. language, skills, knowledge, and beliefs) used by individuals to embody their collective experiences in external forms such as material objects (words, pictures, books, etc.,) which is the essence of learning. Teachers who have an understanding of these aspects of student learning and cognition are better equipped to teach students who come from a variety of diverse cultural and linguistic backgrounds" (Howard and Aleman, 2008, p. 162).

Teachers' content background is critical along with professional teaching knowledge to recognize the power of children's thinking and also recognize how school mathematics can be presented by students using non-conventional representations (Ball, D.L., Hill, H. and Bass, H., 2005; Hill, H. and Ball, D. L., 2009). However, often instruction is approached with a deficit view of learners. "…if the goal is to support student participation in mathematical discussions, determining the origin of an error is not as important as listening to the students and uncovering the mathematical competence in what they are saying and doing. It is only possible to uncover students' mathematical competence if we use a complex perspective of what it means to communicate mathematically" (Moschkovich, 2002, p. 208).

At the middle school level, students' also need time to connect with and engage in investigations, and to talk about the mathematics. Current instructional schedules in middle schools create tensions for the students and teachers to be able to deeply connect with the mathematics for conceptual understanding and retention. When mathematics becomes the focal point of instruction for the underserved and underachieving students they can succeed and then carry that success to other content areas. This was the model for The Algebra Project by Robert Moses (Moses, R., West, M. & Davis, F., 2009). If given substantial amounts of time, mathematics can become an empowerment tool for students.

Currently, ELLs are underrepresented in higher-level mathematics courses in high school (Taningco, M. T., Mathew, A. B., and Pachon, H.P., 2008). This issue needs to be addressed by examining and changing our current structures for teaching mathematics to Latinos/as who are ELLs. Particular attention must be given to assessments and educational policies. More advocacy with departments of education, superintendents and legislators is crucial and policy makers need to have first-hand experiences with the high school exit exams being required.

To improve the present teaching of mathematics for Latinos/as careful attention must be given to needed preservice preparation and teacher professional development. Preservice teachers need to have experiences working with diverse populations of students and more focus on the importance of language and culture in teaching and learning. Understanding what is embodied in the Equity Principle (NCTM, 2000) for teaching mathematics to Latinos/as is paramount and must be emphasized in their teacher preparation courses. Master teachers must be identified for student teaching placements. Preservice and inservice teachers must reflect on, identify and address any of their own biases that may exist towards ELLs and students from other non-dominant populations. Syllabi that focus on equity and ELL strategies in preservice teacher mathematics coursework need to be developed and accessible to teacher educators.

Professional development needs to bridge knowledge between research and practice. Teachers require access to "teacher-friendly" research and researchers, and time to discuss and reflect. Professional development must be continuous and sustained, with teachers developing relationships with researchers as "action researchers." Researchers in tandem with teachers should develop research questions and collect case studies. New teachers should be provided support and guidance from mentor programs and/or networks of experienced teachers for at least their first two years. Communities of support need to be built for teachers working with Latinos/as and ELLs, especially at the high school level.

Lists of resource materials to use in professional development that address the teaching of mathematics with Latinos/as and ELLs need to be developed, along with a collection of video cases that illustrate best practices in mathematics classrooms with English learners. Particular focus needs to concentrate on developing mathematical reasoning and communication. Regional networks need to be created to support teachers with similar situations and populations of ELLs.

What questions do we need to research further?

We identified the following questions as needing further research and investigation:

- What can we learn and use from CGI at the middle and high school level?

- What is the balance that instruction should aim for in terms of balancing procedural accuracy and flexibility? Between using advanced strategies and other strategies that arrive at correct answers?

- How can instruction include language as another form of representation?

- Are different types of models evoked when students think or talk about mathematics in Spanish than when they use only English? Do some of these models not exist or exist less frequently or differently in English?

- How do Latino/a students themselves view and understand mathematical authority in group interactions?

- How do we get students to view math as a "tool" for understanding real world phenomena and for problem-solving?

- What is involved and required to create an environment where the classroom becomes an authentic audience for student explanations and reasoning?

- What models could we create, investigate, or develop in order to understand how to foster teacher facilitation that supports students building off one another's thinking?

- How do we get teachers to be more cognizant of the importance of equity in mathematics education? What experiences will help transform and/or challenge teacher beliefs that will lead to equitable practices?

- How can we design research that evaluates and demonstrates (over time) the intrinsic value of instruction that supports students' learning at deep levels in learning communities that are not constrained by policies that place little value on teaching/learning for understanding?

- More research needs to focus on Latinos/as and ELLs mathematics learning at the high school level.

• How do mid-career teachers and those in alternative programs to receive teaching credentials compare in effectiveness to those from traditional teacher preparation programs?

• More quantitative and also longitudinal studies are needed to support the qualitative research that has been done.

• What structure can be created that will allow new teachers to build on information from researchers and veteran teachers?

• How do we bridge the knowledge between research and practice?

• How can we provide "authentic" experiences for preservice teachers in understanding the needs of ELLs?

• What needs to change in teacher preparation?

Next Steps

The participants in the Visions from the Classroom: Focus on Students compiled this list of next steps:

1. Provide a list of resource materials (connect with website TE-MAT.org)
2. Develop teacher professional development materials for mathematics education.
3. Create and make accessible video cases for mathematics classrooms with ELLs.
4. Be alert to help and have specific proposals for schools labeled underperforming.
5. Call this conference the Inaugural Conference, and have a continuing conference (stay connected with TODOS through the internet).
6. Influence policy makers by having a conference with superintendents and legislators directly and get them to do math and try the high school exit exams.
7. Create regional networks (i.e. Virginia local settings)
8. Create and have access to syllabi that incorporates ELL methods into preservice teacher coursework—make available on AMTE website or present in a session.
9. Connect to DOEs—write memorandums to contacts
10. Encourage teachers to become researchers
    -provide teachers mentoring
    -provide resources so teachers have time to do this
11. Take conversation outside of us and have more political involvement
    -go into districts, contact school superintendents, DOEs, Ed Utopia
12. Include students/youth in next gatherings (at all levels) to get student experiences

13. Get involved with Latino media
    -Latino magazine
    -UNIVISION/Gate Foundation

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