THE INAUGURAL

TODOS

VOLUME 1 • RESEARCH MONOGRAPH OF TODOS: MATHEMATICS FOR ALL

PROMOTING HIGH PARTICIPATION AND SUCCESS IN MATHEMATICS BY HISPANIC STUDENTS: EXAMINING OPPORTUNITIES AND PROBING PROMISING PRACTICES

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TODOS
Mathematics for ALL

NATIONAL EDUCATION ASSOCIATION

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TODOS Mission

The mission of *TODOS: Mathematics for ALL* is to advocate for an equitable and high quality mathematics education for all students, in particular Hispanic/Latino students, by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

Goals

- To improve educators' knowledge and ability to implement an equitable, rigorous, and coherent mathematics program that incorporates the role that language and culture play in learning mathematics.
- To develop and support leaders who continue to carry out the mission of TODOS.
- To promote the generation and dissemination of knowledge about equitable and high quality mathematics education.
- To inform the public and influence educational policies in ways that enable students to become mathematically proficient.

Published April 2008

Preface

Miriam A. Leiva • University of North Carolina Charlotte

On behalf of the Board of Directors, I am proud to present you with the first *TODOS: Research Monograph*, a significant accomplishment that merits a celebration. Our founders envisioned scholarly activities that would bring together research and classroom practice into a seamless relationship to benefit K-12 students while enhancing the knowledge of teachers and researchers. This symbiotic partnership was asserted in our By-Laws, 2003, and affirmed by one of our four organizational goals:

> *To promote the generation and dissemination of knowledge about equitable and high quality mathematics education.*

We viewed the partnering of research and practice as a two-way street: best practices informing research and vice-versa to answer the question: *"what works?"* It led to the development and publication of this research monograph, a multi-year project under the leadership of Richard Kitchen, chair of the Research and Publications Committee. Professor Kitchen collaborated with Ed Silver as monograph co-editors, and we are very grateful for the outstanding outcome of their efforts.

The TODOS Research Monograph project was made financially viable by a generous grant in 2005 from the National Education Association, under the direction of Dr. Andrea Prejean, NEA Senior Policy Analyst for Education Policy and Practice. Her encouragement and support were crucial to every aspect of the monograph development and publication.

We dedicate this inaugural issue to the TODOS founding members listed below. Finally, to you the reader, I am pleased to say *read, learn, implement, and disseminate to benefit our students.*

Miriam A. Leiva, President
TODOS: Mathematics for All

Founding Members:

Cindy Chapman	Gil Cuevas
Jose Franco	Miriam Leiva
Eleanor Linn	Bob McDonald
Larry Orihuela	Yuria Orihuela
Nora Ramirez	Jeanne Ramos

Foreword

Richard S. Kitchen, Co-Editor • University of New Mexico
Edward Silver, Co-Editor • University of Michigan

It is a pleasure for us to introduce the inaugural monograph of *TODOS: Mathematics for All*. In the fall of 2006, the Research and Publications Committee, a standing committee of *TODOS* decided to initiate an annual research-based monograph that focuses on issues related to diversity and equity in mathematics education. The focus of this monograph is enhancing the achievement and learning of Hispanic-Latino/a students in mathematics. Development of this inaugural *TODOS* monograph was undertaken by *TODOS: Mathematics for All*, and the dissemination was supported in part by the National Educational Association (NEA).

The mathematics achievement of Hispanic-Latino/a students in U.S. schools is a matter that deserves considerable focused attention from both the educational research and educational practice communities. Many agree that the time has come to move beyond the conventional treatment that has focused on rigid categories of race, class, and language and which has framed the issue in terms of underachievement related to those categories. What appears to be needed now are novel theoretical, conceptual, and historical analyses of key issues associated with the mathematics achievement and learning of Hispanic-Latino/a students, and instructional examples to illustrate what might be possible, including demonstrating the elasticity and permeability of boundary conditions that many have treated as rigid and fixed. This inaugural monograph takes a step in this needed direction, offering four papers that we believe can inform the education community and policy makers about issues that affect the mathematics achievement and learning of Hispanic-Latino/a students, with particular attention to some instructional approaches and intervention strategies being used in schools to foster high levels of mathematics participation, greater achievement and advancement by Hispanic-Latino/a students in mathematics.

The first paper entitled, "The Opportunity Gap" by Alfinio Flores problematizes the notion of an achievement gap and how it stereotypes students. Flores explores the underlying causes of what he identifies as an "opportunity gap" that students of color experience when compared to white and Asian-American students. In his paper, Flores challenges unquestioned assumptions that undergird the construct of an achievement gap such as beliefs held about the abilities of students of color to learn demanding mathematics. As the initial paper in the monograph, Flores' paper establishes a tone for the remaining three papers which focus more explicitly on teaching strategies and professional development interventions that have proven to be successful to promote the learning and achievement of Hispanic-Latino/a students.

The second paper entitled, "Cultural and Linguistic Resources to Promote Problem Solving and Mathematical Discourse among Latino/a Kindergarten Students" by Erin Turner, Sylvia Celedón-Pattichis, and Mary Marshall documents teaching practices that support the mathematical learning of Latinos/as. In their paper, Turner et al. chronicle the work of a teacher and her students in a bilingual kindergarten class that regularly engaged in mathematical problem solving. From a practical standpoint, the robust student achievement that resulted serves as an existence proof of what Hispanic-Latino/a students are capable of doing when they have a committed kindergarten teacher who provided her students' access to challenging mathematics. This paper adds another example to the literature on the rich possibilities for mathematics teaching and learning for young children. Moreover, the focus here on Latinos/as raises a number of interesting issues that should be explored in research studies to probe their applicability on a broader scale.

In the third paper entitled, "Understanding English through Mathematics: A Research Based ELL Approach to Teaching All Students," Joyce Fischer working in collaboration with Robert Perez discover and confirm best practices for teaching mathematics to bilingual learners. In this paper, Fischer documents how Perez, himself a bilingual learner, developed strategies over-time that made a significant difference for the mathematical learning and achievement of his high school Hispanic-Latino/a students. A key aspect of Perez' approach to teaching as described in the paper is how he also teaches students English through the study of mathematics. The duality of the teaching/learning of mathematics and language is a critically important theme for those who are interested in the achievement of Latino/a students in the U.S. and is a fertile area for further research.

Lastly, in "The Travieso Activity," Carl Lager introduces and documents the impact that an activity has had on sensitizing secondary-level mathematics teachers to the unique challenges and needs of students who are learning English as a second language. A goal of the Travieso Activity is to change participating teachers' views of the mathematical abilities of their students who are English Language Learners (ELLs). Lager describes how he has accomplished this by having teachers concretely experience what it is like to be a second language learner during a mathematics activity.

Overall, the four papers in this monograph challenge characterizations of an achievement gap derived from deficit model perspectives of bilingual learners in mathematics. Flores' paper gives a nice global overview of achievement data that highlights the differences in opportunities that students have to learn mathematics that are based on race, ethnicity, and class. Turner, Celedón-Pattichis, and Marshall's paper provides a concrete example of how Latino/a students' learning in mathematics is dramatically improved when these students have opportunities to regularly engage in mathematical problem solving. Fischer and Perez' paper also demonstrates how Latino/a students' achievement can be dramatically increased when a teacher implements well-known, research-based teaching strategies for English Language Learners. Lager's paper provides a practical activity that can be used to challenge secondary-level mathematics teachers' negative perspectives on the capabilities of their ELL students by putting them in the disquieting position of a second language learner.

We would like to thank the many people who contributed to this inaugural *TODOS* Monograph. First, we want to thank the scholars who submitted an article for review for the inaugural monograph of *TODOS: Mathematics for All.* We had many outstanding submissions and we believe that the four papers published in this inaugural monograph best represented the goals of the monograph. Second, we would like to thank the members of the Research and Publication Committee which included Marta Civil, Gil Cuevas, Rochelle Gutiérrez, and Lena Licón Khisty. It was an honor to work with such an esteemed group of scholars on this project. Each member of the committee gave selflessly of their time, reviewing multiple papers, and providing valuable feedback to the scholars who submitted a paper.

Finally, we want to thank Miriam Leiva, *TODOS* President and Andrea Prejean, Senior Policy Analyst for the National Educational Association (NEA). Miriam devoted significant time to this project to ensure its success and has made *TODOS: Mathematics for All* a national leader in advancing the mathematics education for Hispanic-Latino/a students in the United States. Andrea and the NEA provided the financial resources that were needed to make this project a success. We wholly appreciate her support and trust in TODOS and our effort to make the initial *TODOS* Monograph a high quality, research-based publication. Our hope is that practitioners and scholars will find the content of this inaugural *TODOS* Monograph to be of great benefit in their work to advocate for and advance the mathematics education for not just Hispanic-Latino/a students, but for all students.

The Opportunity Gap

Alfinio Flores • University of Delaware

There are considerable differences of performance in national and state tests among different ethnic groups, between students from low-income families and high-income families, and among students whose parents have different levels of schooling. Often the unequal performance of Hispanic and Black students compared to European American students is described as an achievement gap. It is not uncommon to use statements like the following to describe the situation: "Students of color continue to lag behind White students and some Asian students, and the so-called academic achievement gap still exists." (A state superintendent of public instruction, as quoted by Heffter, 2006.) "Across the U.S., a gap in academic achievement persists between minority and disadvantaged students and their White counterparts." (National Governors' Association, 2005.) What kind of images do we form about the students who lag behind after reading such statements? What kind of assumptions, conscious or subconscious, do we make about their capacity of learning? Do we even ask why their performance is lower? What unquestioned assumptions do we make about the educational system?

Blanket statements about the low performance of certain groups of students in our schools that do not mention the underlying causes may reinforce prejudices and stereotypical images some people have about such groups. Unfortunately, such prejudices are not uncommon. Some authors even claim that Hispanic and Black students are "less teachable." For example, Greene and Foster (2004) state that being a minority is a disadvantage students bring to school. They claim that as the percentage of White (non-Hispanic) students decreases in a school, the "teachability index" decreases too.

Finding a proper way to frame a problem often can give us not only a better understanding of it, but can also show us better ways to address the problem and solve it. In education it is important not just to address the symptoms, but to understand, make explicit, and address the underlying causes as well. Looking at school and district practices and policies that cause to a great extent the disparity of performance in mathematics among different groups of students in schools can help us understand how to provide equal learning opportunities for all students.

In this chapter, I will first present some data that show striking and persistent differences of performance in tests among different groups of students in schools. I will then present evidence that shows that students from some groups are not as likely to have the same opportunities to learn mathematics in our schools as students from other groups.

The achievement gaps

In this section, I will present data that show gaps in mathematics learning that are important in size. These gaps have been quite persistent over the last three decades. The gap in test performance in mathematics is a wide one. At eighth grade, 91 percent of Black, 87 percent of Hispanic, and 85 percent of Native American students are not proficient in math, as measured by the National Assessment of Educational Progress (NAEP). This stands in stark contrast to the performance of European American and Asian American students (see Figure 1). Also, as Figure 2 shows, twelfth-grade Hispanic and Black students perform on

NAEP's math assessment at the same level as eighth-grade White students (Wilkins & Education Trust staff, 2006). The disparity of performance is greater on short constructed response items than for multiple choice items, and for extended constructed response tasks the difference is even bigger. For example, in the 2000 mathematics NAEP test, the percentage of correct answers given by Black eighth-grade students in multiple choice questions was 72 percent compared to that of correct answers given by European American students. For short constructed items, it was 63 percent, and for extended constructed response tasks it was only 32 percent (Strutchens, Lubienski, McGraw, & Westbrook, 2004, p. 279). That is, the more complex the kind of answer required, the bigger the difference in achievement.

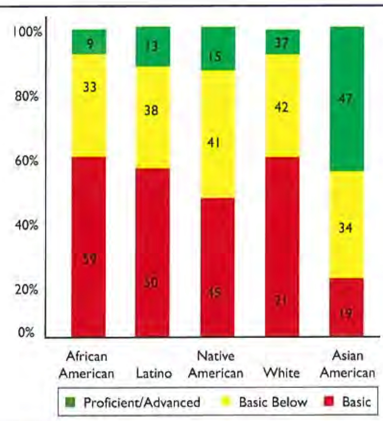

Figure 1. 2005 NAEP Grade 8 by ethnicity. Source: National Center for Education Statistics, NAEP Data Explorer as cited by Haycock, 2006, Slide 17

The gap has also been very slow to close. There was some narrowing of the gap in the 1970s and 1980s, but since 1988 the gap widened somewhat or remains about the same (see Figures 3 and 4). There is also a considerable gap in test performance between students who come from poor families and those who come from non-poor families (Figure 5). There are also clear and persistent differences in the performance of students when grouped according to the education level of their parents (Figure 6). Differences

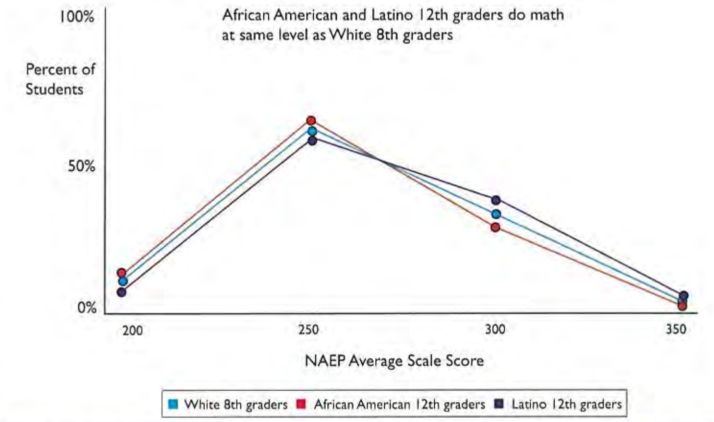

Figure 2. NAEP performance for different groups. Source: Wilkins et al., 2006. ©The Education Trust. Used with permission.

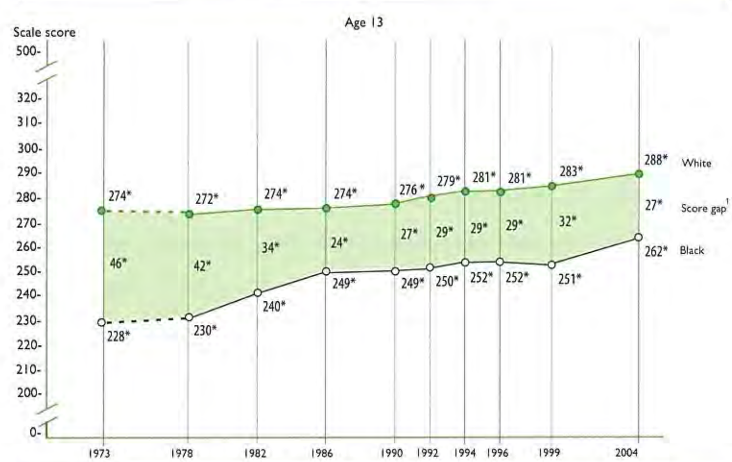

Figure 3. Trends in average mathematics scale scores and score gaps for White students and Black students age 13, 1973–2004. Source: Perie, Moran, & Lutkus, 2005, p. 42

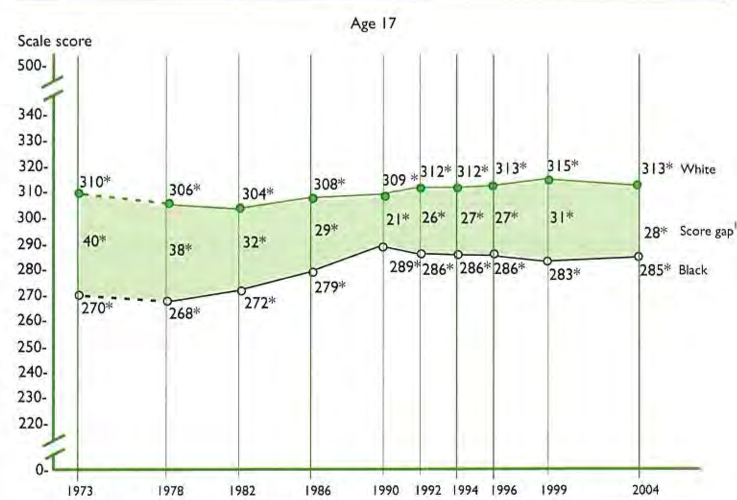

Figure 4. Trends in average mathematics scale scores and score gaps for White students and Black students age 17, 1973–2004. Source: Perie, Moran, & Lutkus, 2005, p. 42

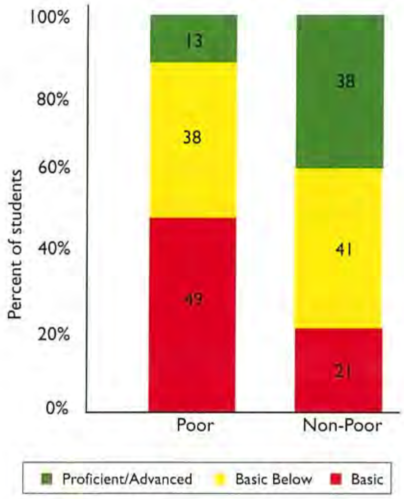

Figure 5. 2005 NAEP grade 8 math by family income. Source: Wiener, 2006, p. 3.

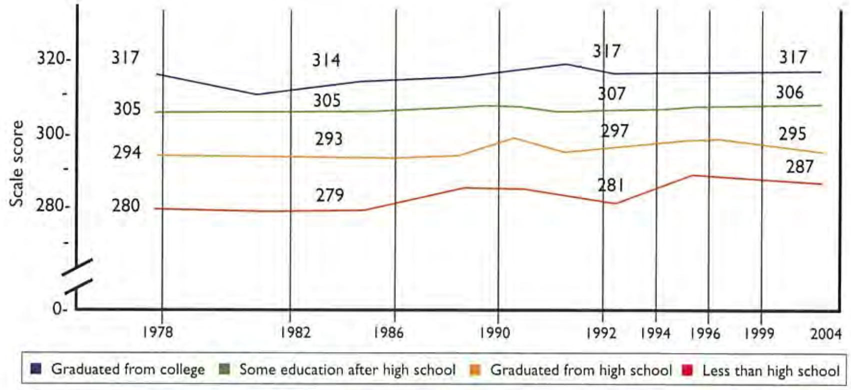

Figure 6. Trends in average mathematics scale scores for students age17, by student reported parents' highest level of education: 1978–2004. Source: Perie, Moran, & Lutkus, 2005, p. 45.

in parents' school achievement tend to be reproduced in the schools, rather than erased.

It is thus clear that students from some groups are not learning as much in our schools as students from other groups. The important questions are: *Why do such disparities exist? What are the causes?*

The opportunity gaps

A very different way to describe the distinct experiences among students in schools is to understand how Hispanic and Black students are less likely than White students to have qualified teachers, high quality mathematics instruction, and appropriate use of resources. For example, Black and Hispanic students are less likely than White students to —

- Have teachers who emphasize reasoning and nonroutine problem solving
- Have access to computers
- Have teachers who use computers for simulations and applications (Strutchens & Silver, 2000).

Black and Hispanic students often experience a lesser form of education, in mathematics and in general (Diversity in Mathematics Education Center for Learning and Teaching, 2007). In contrast, White students usually have adequate mathematics course offerings, qualified mathematics instructors, quality mathematics curricula, and teachers who respect their culture and hold high expectations of them.

The National Council of Teachers of Mathematics (2005) answers the question of "How can we close the achievement gap in mathematics" by stating that all students "should have equitable and optimal opportunities to learn mathematics free from bias," and that "all students need the opportunity to learn challenging mathematics from a well qualified teacher who will make connections to the background, needs, and cultures of all learners." The solution to the achievement gap is thus framed as opportunity to learn.

In the following pages, I will describe ways in which many Hispanic and Black students are less likely to have equal opportunities to learn. I will illustrate how many low-income and minority students are systematically shortchanged by the educational system by having less access to experienced and qualified teachers, by facing low expectations, and by receiving less funding per student. These three dimensions of opportunity are, of course, not the only important factors. Classroom practices, for example, have been empirically demonstrated to be linked to student performance (Hiebert and Grouws, 2007). Other important factors are briefly mentioned in the final comments section. For each of the three factors discussed in this chapter, I provide a brief rationale of why that factor is important at the beginning of the corresponding section.

Gaps in opportunities to have qualified and experienced teachers

Qualified teachers who are committed to the learning of their students are the single most important factor for students' success (Darling-Hammond, 2001). As can be documented by multiple research studies, a good teacher can make a big difference (Darling-Hammond, 2000). Having several experienced and highly qualified teachers in a row can improve dramatically the performance of students (Sanders & Rivers, 1996). By pointing out inequities in the distribution of teacher quality among schools, my intention is not to denigrate the many outstanding, talented and dedicated teachers who are teaching our most disadvantaged children, often under very hard conditions. Quite the contrary, the intention is to underline the importance for all children to have access to their fair share of such exemplary teachers. By stressing the importance of experienced and qualified teachers, the intention is not to put the blame on inexperienced teachers, but to focus on structural inequities in opportunities for students from different groups at the school, district, and state level. In this section, I will describe how Black and Hispanic students and low-income students are shortchanged in their opportunities to have access to teachers who have experience, to teachers who are well qualified in mathematics, and teachers who are generally well prepared.

Access to experienced teachers

Black and Hispanic students are less likely to have experienced teachers than their European American counterparts (Mayer, Mullens, & Moore, 2000). Classes at schools that serve mostly Black and Hispanic students are twice as likely to be taught by teachers with three years of experience or less than classes at schools where there is a majority of White students (Figure 7). Similarly, classes at high poverty schools are more likely to have inexperienced teachers than at low poverty schools (Figure 8) (Mayer, Mullens, & Moore, 2000). In schools that are hard to staff and that have a high turnover of teachers, students do not always have a permanent teacher. For example, at the Brooks Academy in Phoenix, where more than 99 percent of the students are Hispanic, Native American, or Black, and where 99.5 per-

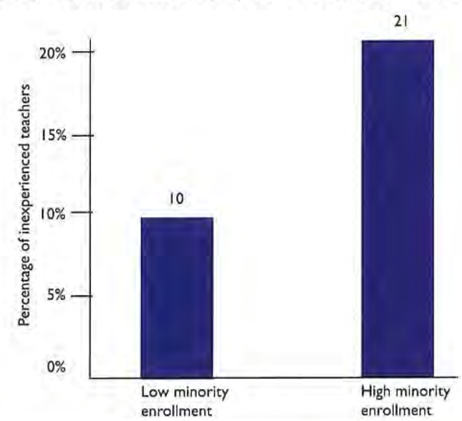

Figure 7. Percentage of inexperienced teachers by minority enrollment. Source: Mayer, Mullens, & Moore, 2000, p. 13.

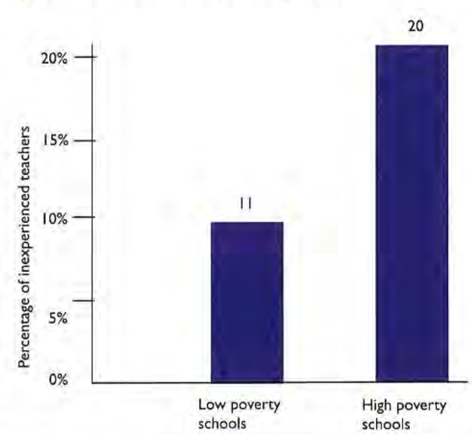

Figure 8. Poor students get more inexperienced teachers.Source: Mayer, Mullens, & Moore, 2000, p. 13.

cent of the students qualify for free or reduced price lunch, one fourth-grade class had 17 substitute teachers before a full time teacher was hired in January (Kossan, 2006).

Access to qualified teachers in mathematics

The great majority of the least prepared teacher recruits are very frequently placed in under-resourced, hard-to-staff schools serving high proportions of low-income and minority students in central cities and poor rural areas. Students who most need skilled teachers are least likely to have well prepared teachers, thus magnifying inequalities (Darling-Hammond, 2001). For example, in California, the percentage of underprepared teachers in mathematics rises as the percentage of students of minority groups increases (see Figure 9) (Esch, Chang-Ross, Guha, Humphrey, Shields, Tiffany-Morales, Wechsler, & Woodworth, 2005).

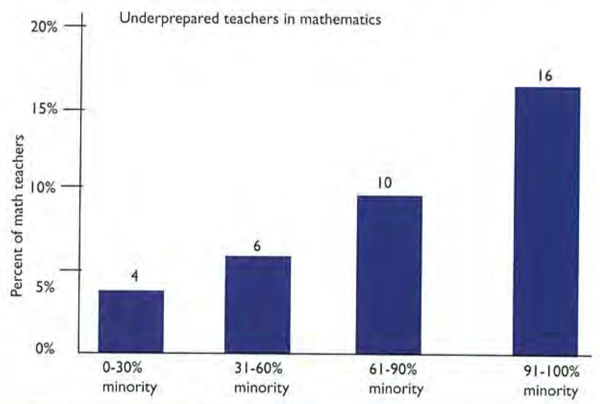

Figure 9. Distribution of underprepared teachers with a math assignment by school level percentage of minority students. Source: Esch et al., 2005, p. 75

More classes in high poverty schools and high minority schools are taught by out-of-field teachers, that is, by teachers who do not have at least a minor in the subject area they teach (Figure 10). Classes in high schools and middle schools with high percentages of Hispanic and Black students are more likely to be taught by teachers who lack even a minor in the subject area than classes in schools with low percentages (Wilkins et al., 2006). This is a problem that has existed for quite some years. There was no progress in reducing the percentage of classes taught by of out of field teachers between 1994 and 2000 (Jerald, 2002). On the contrary, there was a slight increase, but this increase does not affect all students equally. While the percentages in low-poverty and low-minority schools remained essentially unchanged, the percentages of out of field classrooms in high-poverty and high-minority schools increased significantly (Figure 11).

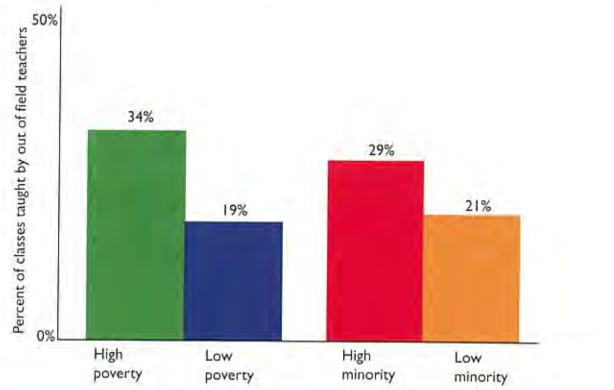

Figure 10. More classes in high poverty, high minority schools are taught by out-of-field teachers. Source: Richard M. Ingersoll, University of Pennsylvania, as cited by Haycock, 2006, slide 61

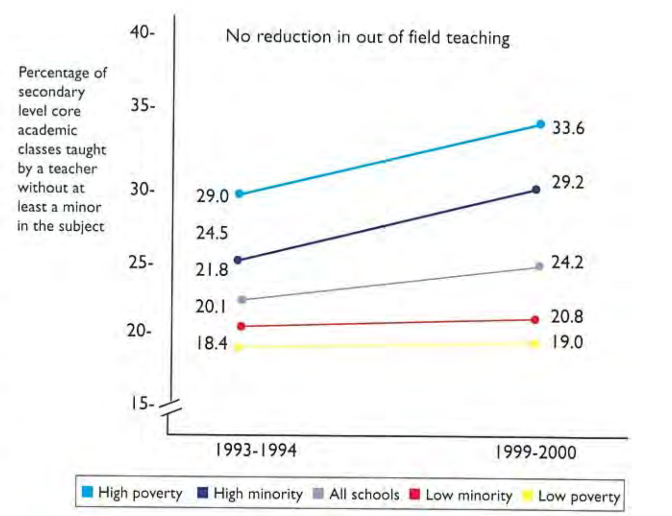

Figure 11. Changes in the percentages of classes taught out of field. Source: Richard M. Ingersoll, University of Pennsylvania, as cited by Jerald, 2002, p. 5

To solve the problem of out of field teaching, action is needed at the school and district levels. Part of the problem is due in some places to a shortage of teachers in science and mathematics. However, a considerable part of the problem, about half (Jerald, 2002) could be solved with the present cadre of teachers by assigning teachers to teach in their field of expertise. At present, teachers in disadvantaged schools "are far more likely to be misassigned than are those in advantaged schools" (Ingersoll, 2002, p. 17).

Access to well prepared teachers in general

Preparation in the content area and experience are not the only ways to measure teacher quality. The Illinois Education Research Council used five measures of teacher quality to define an overall "index" for schools, called the Teacher Quality Index (TQI). The five attributes they included are the percentage of teachers with degrees from more competitive colleges, the percentage of teachers with provisional or emergency credentials, the percentage of teachers who did not pass the Basic Skills tests their first time, the percentage of teachers with less than four years of experience, and the average ACT (American College Test) composite score of teachers in the school (Peske & Haycock, 2006). The researchers assigned to each school a Teacher Quality Index rating. Then they ranked all schools according to this rating and divided them into quartiles. Thus, schools in the top quartile had more teachers who were stronger on the measures included in the index than schools in the lower quartile. Then they analyzed patterns of distribution of qualified teachers according to the demographics of the student population. The data revealed that students in the schools with the highest concentrations of poverty and minority populations were assigned teachers who were qualitatively different from teachers in schools with low concentrations of students from minority groups and poverty (Figures 12 and 13). Large numbers of schools with the most percentages of students of minority groups had very low teacher quality indices. Of the schools with high percentages of minority students, 61 percent of them had TQIs in the lowest 10 percent of the state. Eighty-eight

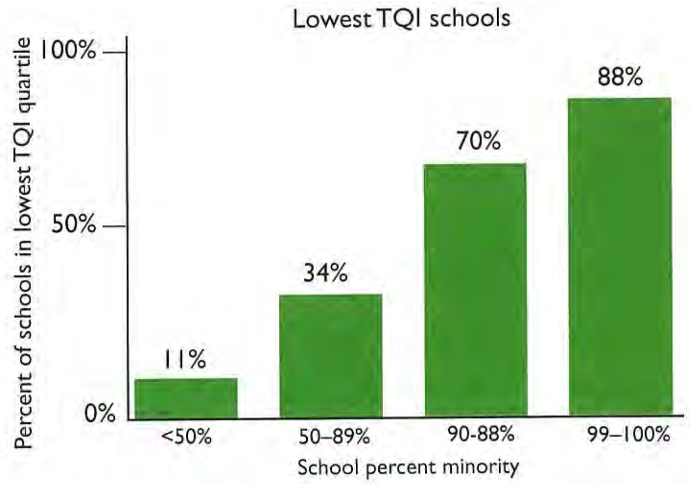

Figure 12. Low teacher quality index and percent minority Source: Peske & Haycock, 2006, p. 7

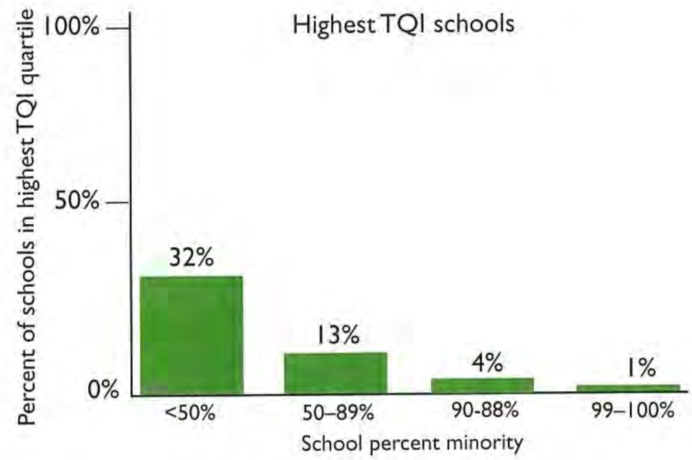

Figure 13. High teacher quality index and percent minority. Source: Peske & Haycock, 2006, p. 7

percent of those schools had TQIs in the bottom quartile of the state. On the other hand, only 11 percent of the schools that had the fewest minority students were in the lowest quartile, and only 1 percent of these schools were in the lowest 10 percent (Peske and Haycock, 2006).

The distribution of schools with high and low indices was similar when grouping schools by income. Eighty-four percent of the schools with the largest concentration of low-income students had indices in the bottom quartile in teacher quality, and 56 percent of those schools fell in the bottom 10 percent of the state. Only 1 percent of schools in the state (i.e., three schools total) serving high concentrations of poverty students had indices in the highest quartile of the state. In contrast, 46 percent of the schools with the lowest percentages of low-income students had a teacher quality index in

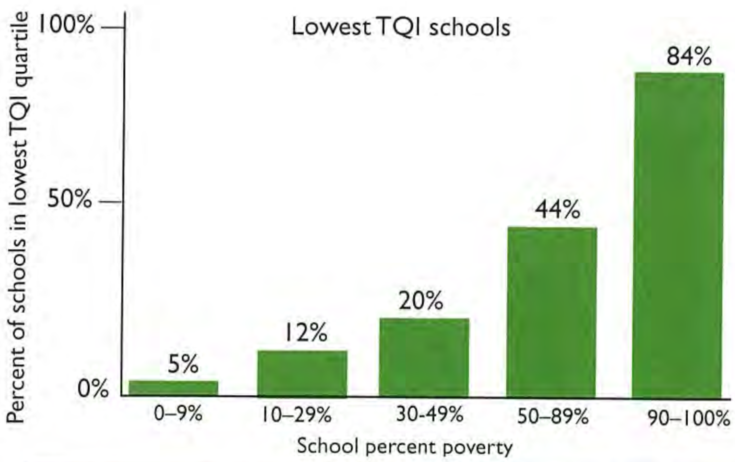

Figure 14.. Low teacher quality index and percent poverty. Source: Peske & Haycock, 2006, p. 7

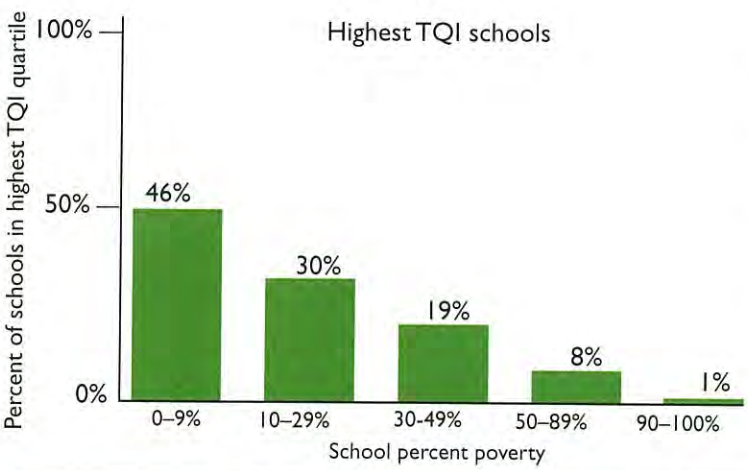

Figure 15. High teacher quality index and percent poverty. Source: Peske & Haycock, 2006, p. 7

the top quartile, and only 5 percent had an index in the lowest quartile (Figures 14 and 15) (Peske & Haycock, 2006, p. 7-8).

The case of Illinois is by no means unique. In many other states, across and within school districts the distribution of well prepared and experienced teachers is not equitable when schools are grouped according to their concentrations of low-income and minority students. For example, Kain and Singleton (1996) found that in Greater Dallas the percentage of teachers with advanced degrees declines from 37.7 percent for schools that are 6.9 percent Black to 26.4 percent for schools that are 92.2 percent Black. Such data provide evidence of systematic and important differences in the fraction of teachers with advanced degrees within low-income, minority and high-income, majority schools in the same district. Their results for teachers with 20 or more years of experience provide evidence that fewer senior teachers are found in schools with large proportions of low-income, Black students.

There are also stark contrasts between teachers in schools in New York City and the rest of the state. Thirty-one percent of teachers in the public schools of the city failed the main certification exam at least once, whereas for the rest of the state the figure was only 5 percent. About 47 percent of teachers in the city who took the math state certification test failed at least once, compared with about 21 percent of teachers elsewhere. Twenty-seven percent of city teachers who took the state test for elementary teaching skills failed, compared with 3 percent elsewhere (Goodnough, 1999).

In California, the lowest performing schools have the highest percentages of underprepared and novice teachers. In 2005–06, in schools in the lowest achievement quartile of the state's Academic Performance Index, 21 percent of teachers were underprepared, novice, or both, whereas in the highest achievement schools it was 12 percent of teachers (Guha, Campbell, Humphrey, Shields, Tiffany-Morales, & Wechsler, 2006). The distribution of underprepared and novice teachers follows a similar pattern when using performance on the California High School Exit Exam. Thirty-one percent of teachers in schools with the lowest passing rates in the mathematics section were underprepared or novice in 2006, whereas in schools with the highest passing rates it was 17 percent (Guha et al., 2006). In 2005–2006 in high minority middle and high schools, 16 percent of mathematics teachers were underprepared, compared with 4 percent in low minority schools. The distribution using the school's Academic Performance Index follows a similar pattern. In low performing middle and high schools, 18 percent of mathematics teachers were underprepared, compared with 5 percent in high performing schools (Guha et al., 2006).

There are also differences in distribution of qualified teachers within schools. Students in low tracks are more likely than students in high tracks to have a teacher without a minor or major in mathematics (Figure 16) (Ingersoll, 1999).

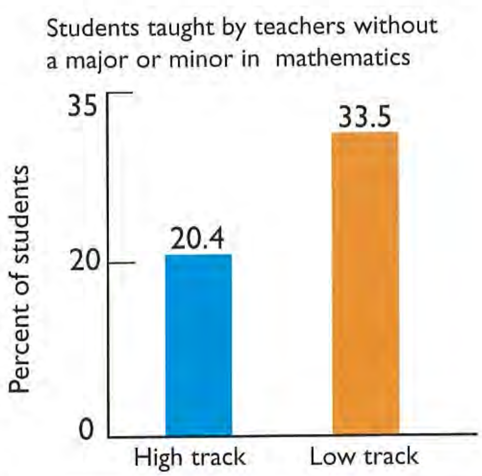

Figure 16. Students in lower track more likely to get less qualified teachers. Source: Ingersoll, 1999, p. 30

As Haycock (2000) points out, "No matter how you measure teacher qualifications—licensed vs. unlicensed, in- vs. out-of-field, performance on teacher licensure exams, or even actual effectiveness in producing learning gains—low-income and minority youngsters come up on the short end" (p. 1). To guarantee that the students with the most needs also get their fair share of strong teachers requires action at the school, district, state, and federal levels.

Gaps in opportunity due to low expectations

Low expectations can manifest themselves and be detrimental to students in several different ways. Low expectations often result in self-fulfilling prophecies. Once placed in the low tracks, it is very difficult for students to move to higher tracks. Academic talent does not flourish and develop unless it is nurtured. Students who are not given the opportunity to learn challenging and interesting mathematics and instead are subject to rote learning of remedial mathematics are often turned off from mathematics and do not take mathematics beyond the minimum required, thus loosing options to mathematically oriented careers. Students who do not have access to advanced courses in mathematics in school are placed at a disadvantage when competing for a place at the best institutions of higher education.

Often students from a cultural background different from that of the teacher are put in situations where the teacher assumes deficits in the students, rather than locating and teaching to their strengths. Teachers may assume that the failure of a student to thrive intellectually is due to a deficit in the student rather than a deficit in teaching. As a consequence, teachers may be teaching less when they should be teaching more (Delpit, 1992). Many mathematics teachers fail to use the culture of Black students in instruction. This results in the school culture being alienating to many Black students and being inconsistent with their cultural experiences, and also their dreams, hopes and struggles (Malloy, 1997, p. 27). The same could be said for many Hispanic students (Getz, 1997).

Lower expectations, ineffective teaching, and reinforced stereotypes often result from a sense of helplessness of the teacher based on the teacher's beliefs of what students are capable of doing and the kind of support they receive at home (Irvine and York, 1993). Low expectations lead, in turn, to fewer opportunities for students to learn more challenging and advanced mathematics.

Different expectations for different students are often reflected in the ways teachers teach and test. Teachers who had at least 60 percent Black or Hispanic students in their classrooms were far more likely to spend classroom time using multiple-choice testing and other means of testing low-level cognitive objectives than teachers who had a majority of White students in their classrooms (Madaus, West, Harmon, Lomax, & Viator, 1992).

Giving high grades for work that in other schools would earn lower grades (see Figure 17) or would be more appropriate for younger students is another form in which low expectations are manifested. This can have a devastating effect later. A student may earn all A's in her school and not realize how much farther behind she is with respect to her peers in other schools. It is not until such student has to compete with students from other schools to enter a school in high demand that she becomes fully aware of how inadequate her preparation was.

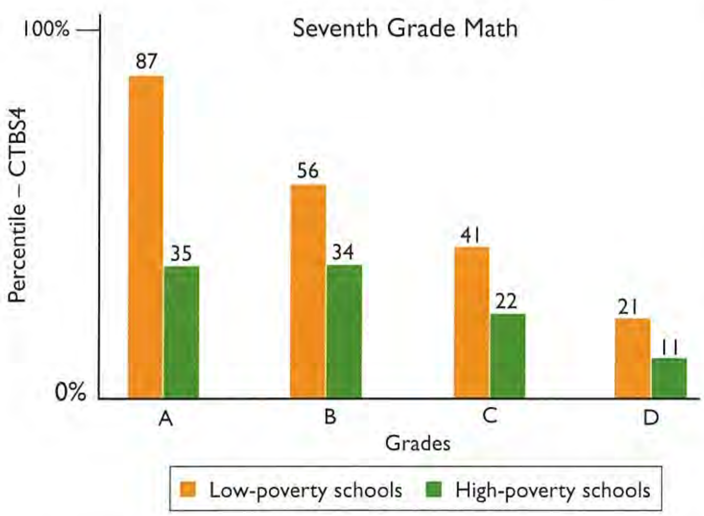

Figure 17. Students in poor schools receive A's for work that would earn C's in affluent schools. Source: Prospects (Abt Associates, 1993), as cited by Haycock, 2006, slide 56.

Blacks and Hispanics are more likely than Whites to be placed in low-ability and remedial classes or in special education programs; hence, they are more likely to learn fewer topics and skills (Oakes, 1990). Black and Hispanic students are also less likely to be identified as capable learners and placed in enriched or accelerated programs. For example, in 2000, 32 percent of White eighth graders were in what teachers considered high ability classes, but only 16 percent of Hispanic and 16 percent of Black eighth graders were in such classes (Strutchens, Lubienski, McGraw, & Westbrook, 2004). Consequently, Black and Hispanic students are likely to have fewer opportunities to learn science and mathematics than their White peers (Oakes, 1990, p. 160).

Enrollment of Black and Hispanic students is significantly lower than that of European American students in eighth-grade courses that to a great extent determine whether students will have the opportunity to take advanced mathematics (pre-calculus and calculus) before they graduate from high school. Only 49 percent of Hispanic and 47 percent of Black students have taken pre-algebra or algebra in eighth grade compared to 68 percent of European American students (Strutchens et al., 2004).

Unfortunately, it is not unusual to see Black and Hispanic students placed in low tracks even in cases where they scored as well as or better than their White or Asian American peers on standardized tests (Education Trust, 1996, as cited by Love, 2002, p. 258). Often Black and Hispanic students are tracked out of advanced mathematics courses based on false assumptions. For example, Love (2002, p. 3) quotes an urban high school mathematics teacher:

> *We thought we were tracking students in or out of higher level mathematics courses by their ability. Then we looked at the data on student achievement on standardized tests. We learned that African American and Latino students who scored as high as White students were getting tracked out of college level courses.*

Black and Hispanic students typically take fewer high school science and mathematics courses than do Whites

(Oakes, 1990). Only 22 percent of Hispanic and 25 percent of Black high school graduates were enrolled in the college track courses at their high school (Wilkins et al., 2006). In part, this is because students in nonacademic tracks often lack the prerequisites to enroll in academic courses. It is also due in part to the fact that they are very often required to take fewer science and mathematics courses than are college-bound students (Oakes, 1990). However, as Oakes (1990) points out, tracking is not the only factor in differences in course-taking. In many schools with large numbers of low-income students, even those students in college preparatory programs typically take fewer academic classes.

Students need the opportunity to take more advanced level courses in mathematics. Not surprisingly, students who take more advanced mathematics courses do better on tests (see Figure 18). However, participation in more advanced mathematics courses is uneven among groups of different ethnic backgrounds (Figure 19). We need to encourage and expose students from all backgrounds to opportunities to take more advanced courses. Oakes (1990) reports that girls and students from minority groups usually receive less encouragement and have fewer science- and math-related opportunities both in school and out than do White males. However, when girls and students from minority groups do receive encouragement and are exposed to opportunities, they show interest and participate.

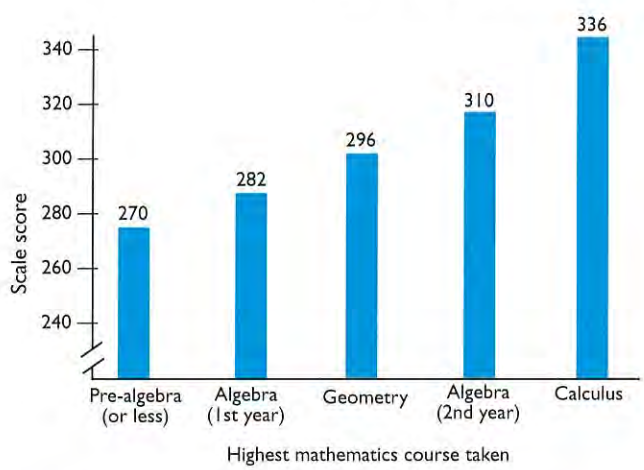

Figure 18. Mathematics scale scores, students age 17, by highest mathematics course taken: 2004. Source: Perie, Moran, & Lutkus, 2005, p. 57.

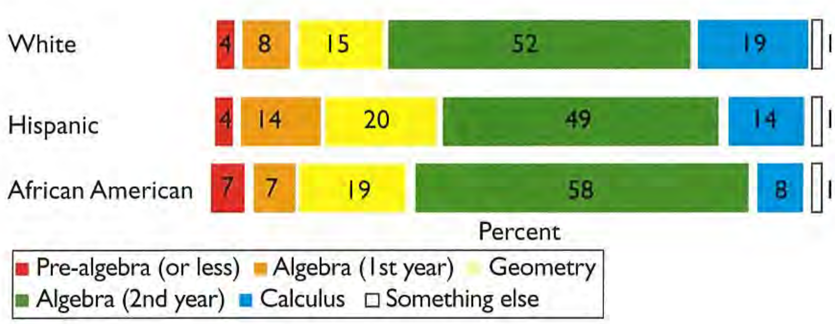

Figure 19. Percentage of students, by race/ethnicity and highest mathematics course taken: 2004. Source: Perie, Moran, & Lutkus, 2005, p. 58.

Low expectations are also reflected in the courses schools offer. Not all schools offer the same number of options for advanced courses in mathematics. Many schools do not offer courses beyond Algebra 2; many do not offer advanced placement (AP) courses in mathematics. In California, "regardless of high school size, the availability of AP courses decreases as the percentage of African Americans and Latinos in the school population increases" (Oakes, Joseph, & Muir, 2004, p. 75). In 1999, one student filed a statewide class-action suit against her school district and the state of California to achieve equitable access to advanced placement courses. Her school, Inglewood High School, which serves primarily Black and Hispanic students (99 percent), offered only three advanced placement courses, none in science or mathematics. Other public high schools, which serve large numbers of White, Asian American and affluent students such as Beverly Hills High School and Arcadia High School (each with 9 percent of Black and Hispanic students), offered more than 14 advanced placement courses, including calculus, computer programming, and physics (Oakes, Joseph, & Muir, 2004).

Gaps in opportunities to receive equal funding

Inadequate funding for schools often results in conditions that are detrimental for students and teachers such as school buildings in disrepair, large numbers of students in each classroom, general overcrowding in the school, and insufficient or inadequate instructional materials (Oakes, Rogers, Silver, Horng, & Goode, 2004). Limited funding also puts some districts at a disadvantage when competing for experienced and highly qualified teachers. "Some educators manage—by ingenuity, resourcefulness, and sheer force of will—to get very high achievement for students in high-poverty and high-minority schools despite egregious funding gaps. But it is undeniable that in the aggregate poor children have fewer opportunities in public schools in most states because they have fewer resources available to them" (Education Trust, 2005, p. 2).

Gaps across districts

In many places in the U.S., school funding is based mainly on local property taxes and revenues (Howell and Miller, 1997). School districts with a large number of well-to-do people have more funds per student than school districts with a large number of people in poverty. Thus students of poverty tend to go to schools with fewer funds. In many places also, a large proportion of Black and Hispanic students live in districts with less funding available. According to NAEP data from 2000, only 3 percent of White eighth graders are in schools where more than 75 percent of students qualify for free or reduced-price lunch, whereas 34 percent of Black and 34 percent of Hispanic eighth graders are in such schools. The majority of White eighth graders attend schools with less poverty. While 64 percent of White eighth graders attend schools with less than one quarter of the students being eligible for free or reduced price lunch, only 15 percent of Black and 25 percent of Hispanic eighth graders do so (Strutchens et al., 2004 p. 281).

There can be no real equal opportunity to learn as long as the "savage inequalities" in our schools continue to exist (Kozol, 1991). In many places, schools are highly segregated. For example, in the Bay Area in California, although 28 percent of the students are White, 65 percent of White students go to White-majority schools (Oakes, Rogers, Silver, Horng, & Goode, 2004). Schools are not only segregated, they are also unequal. School districts that educate the greatest number of Black and Hispanic students receive less local and state money to educate them than the districts serving the fewest number of minority students (Wilkins et al., 2006). Teachers in schools with the highest concentrations of low-income students are less likely to obtain the resources they need. Barton, Coley,

Table 1. Per student spending in several metropolitan areas 2002-2003. Adapted from Kozol, 2005

Metropolitan Area	School District	Spending per student	% Hispanic + Black	% Low income
Chicago area	Highland Park and Deerfield (HS)	$17,291	10	8
	Chicago	$8,482	87	85
Philadelphia area	Lower Merion	$17,261	9	4
	Philadelphia	$9,299	79	71
Detroit area	Bloomfield Hills	$12,825	8	2
	Detroit	$9,576	95	59
Milwaukee area	Maple Dale - Indian Hill (K-8)	$13,955	20	7
	Milwaukee	$10,874	77	76
Boston area	Lincoln (K-8)	$12,775	19	11
	Lawrence	$7,904	86	69
New York City area	Manhasset	$22,311	9	5
	New York City	$11,627	72	83

and Goertz (1991) report that while more than 80 percent of the teachers in schools with middle- to upper-SES students received all or most of the materials they requested for instructional purposes, only 41 percent of teachers in schools with mainly low-SES students received all or most of the instructional materials they requested.

In California, schools where 90 percent or more of the students are Black, American Indian, Hispanic, Filipino, and/or Asian students are much more likely than White-majority schools to have serious problems such as instructional materials that are inadequate, facilities in disrepair, and overcrowding. Forty-two percent of schools with high concentrations of Black, American Indian, Hispanic, Filipino, and/or Asian students have such serious problems compared to only 7 percent of White-majority schools. In Los Angeles County, 63 percent of schools with large proportions of Black, American Indian, Hispanic, Filipino, and/or Asian students suffer such serious problems (Oakes, Rogers, Silver, Horng, & Goode, 2004).

In some cases, legal actions are necessary to redress disparities in funding. For example, the difference of per student expenditure in New York City and other parts of the state was so big that the state was sued to allocate funds for students in more equitable ways. Recently, the state's highest court ruled that the city should be allocated at least $1.93 billion more per year. Although this is far less than the $4.7 billion set by a lower court (Herzenhorn, 2006), it is a clear indication that the funds allocated for students in the city were not sufficient.

The funding inequities of New York City are not unique by any means. In many urban areas there are huge disparities in per student spending between districts serving large numbers of Hispanic and Black and low-income students and districts in the suburbs with low concentrations of Black and Hispanic students and low numbers of students who qualify for free or reduced price lunch. Table 1 shows per student spending for two districts in each of several metropolitan areas (Kozol, 2005).

It also gives the percentage of students who are Black or Hispanic, and the percentage of low-income students for each district. The pattern is unmistakable. In each metropolitan area, the higher the percentage of Hispanic and Black students, the lower the per student spending. The differences are also impressive. In some cases, the per student spending in a low-minority district is twice as much as in the neighboring district with large numbers of Black or Hispanic students.

Districts with fewer resources often are not able to compete in teachers' salaries with wealthier districts. At the time the state of New York was sued for inequitable funding, the plaintiffs pointed out that the starting salary for New York City teachers was about 25 percent less than starting salaries in wealthy suburban counties (Goodnough, 1999). Who would blame a teacher who needs to tend for her or his family for moving to another school district where the pay is better?

Gaps in opportunity to receive equal funding within districts

Inequities in funding across districts are easy to see just by visiting school buildings in wealthier districts and in poorer districts. However, less known is the fact that the problem of unequal funding exists also *within* districts. Schools with a larger proportion of minority or low-income students within the same district often have a larger proportion of inexperienced teachers. Wiener (2006) describes two schools in the same district in San Diego. One school has 55 percent White students, 32 percent students on free or reduced price lunch, and an Academic Performance Index of 808 (the statewide performance target is 800). The other school has 79 percent Hispanic and Black students, 75 percent of the students receive free or reduced price lunch, and the Academic Performance Index is 648. The average pay for teachers at the first school was $6,800 higher than at the second. This difference in salaries happens because as more experienced teachers migrate from one school to another, they take their higher salaries with them. Teachers with more experience tend to migrate to schools with a larger proportion of European American students, to schools with less poverty, and schools that perform better overall in state mandated tests (Wiener, 2006).

The San Diego example is not an isolated incident. In many urban districts there are huge differences in average salaries for teachers from one school to another. This inequity is not transparent due to the fact that urban districts calculate school budgets using average teacher costs. Thus, a school with a staff consisting of mainly senior staff with higher salaries does not appear in the official budget as receiving more money than another school that is staffed mainly by beginning teachers with lower salaries. The hidden differences can be staggering. For example, in Baltimore City Schools the real costs were very different from the official budgets. While the district average was $47,178, in one school the average salary of the teachers was only $37,618, and at another school the average was more than $57,000 (Roza & Hill, 2004). Thus, the teacher expenditure per student is very different from school to school. In Austin, the distribution of non-categorical spending per student is not equitable across schools. Teachers who teach at schools with the greatest enrollment of low-income students receive just 85 percent of the district salary average, while those teachers at schools with the smallest need receive 108 percent of the district average (Roza, Miller, & Hill, 2005).

Unfortunately, the schools that are thus shortchanged are schools with large numbers of low-income children. The schools that benefit from this budgeting system are schools with larger proportions of upper income students. By using average costs for the school budgets, districts mask the fact that they are taking away from the poor to benefit the rich. Federal programs that allocate funds that are meant to supplement and not supplant allow this practice. The money received from the federal government is meant to be used to give additional resources to students of poverty, not to replace money that had been allocated to serve other purposes. For many years, Title I legislation allowed districts to use average salary figures when comparing expenditures among schools (Roza & Hill, 2004, p. 212). "Districts were henceforth allowed to maintain major inequities in school funding, as long as these were driven by teacher allocation" (Roza & Hill, 2004, p. 216). The biggest part of school budgets comes from teachers salaries, typically more than 80 percent of the school allotment (Roza & Hill, 2004), so inequities in teacher salaries across schools amount to large inequities in per student expenditure.

We clearly have huge opportunity gaps. By focusing on the gaps of opportunities, it becomes clear that the achievement gap is just one of the symptoms. To solve the problem, we need to address the causes not just the symptoms. As Lee Bollinger, President of Columbia University states:

> *The achievement gap that exists in American education is not a gap in ability, but a gap in resources and a gap in expectations. We know that students from all backgrounds can succeed at the highest levels of education, when they are given the support they need to succeed—the support that is regularly given to the students from the top income brackets. (Quoted by Scatton, Coley, & McBride, 2006, p. 2)*

Final Comments

As we have seen, some important groups of students have considerably less opportunities to learn in our schools than other groups. We need to be aware of the structural inequities that exist across school districts, as well as within districts, schools, and classrooms. Often, inequities are assumed to reflect a hierarchy of competence. Students who had more opportunities are assumed to be more capable or have more aptitude to learn than other students who did not have as many opportunities.

Talking about achievement gaps without mentioning the opportunity gaps that cause them is an invitation to look at the students who lag behind and perform poorly through deficit models to try to "explain" low performance in terms of factors such as cultural differences, poverty, low educational level of the parents, or experiences these students lack. As Khisty (1995) points out, we can look at the problem in a fundamentally different way and identify the instructional and organizational decisions that harm some of our students. By describing the problem in terms of opportunity gaps, we focus our attention on some of the important things that some of our students are not receiving at school. This focus makes it more clear what the actions are we need to take to guarantee that all students do indeed have the same opportunities to receive a high quality education in school.

As mentioned before, the opportunity gaps described in this chapter—less access to experienced and well qualified teachers, facing low expectations, and inequities in per student funding—are not the only challenges that many Hispanic students face in our schools. Other issues such as language (Cuevas, 1990), cultural differences, and bias in testing are also important. Hispanics constitute a widely diverse cluster of groups and some among them, for example, recent immigrants, have special needs. They face not only the challenge of learning mathematics in a new language, but also important differences in the way they or their parents learned mathematics in their countries of origin (Perkins & Flores, 2002). It is important that schools do more to accommodate the special needs of these groups of students, and this will require additional expertise, resources, and funding. However, a great deal would be gained if the opportunities to learn for Hispanics and other groups that have been systematically underserved by the educational system were the same as for other students. A great deal would probably be advanced toward having the same achievement across groups by providing real equity in opportunities to learn within schools and across schools.

As many examples across the nation show, given the opportunity, students from any cultural or ethnic background and any socioeconomic level can excel. Kitchen, DePree, Celedón-Pattichis, & Binkerhoff (2007) found three salient characteristics in all the schools they studied that were highly successful educating students of poverty, especially in mathematics. These characteristics are "(a) high expectations and sustained support for academic excellence, (b) challenging mathematical content and high-level mathematics instruction that focused on problem solving and sense making (as opposed to rote instruction), and (c) the importance of building relationships" (p. xiv). The opportunity to have access to schools and teachers with such characteristics should be the rule rather than the exception for students from all groups. Concerted actions are needed at the classroom, school, district, state, and federal levels to guarantee that all students receive the same opportunities to learn mathematics.

Of course, achievement and participation are not the only aspects that are important for equity in mathematics education. The practice of mathematics "to analyze, reason about, and especially critique knowledge and events in the world" (Gutiérrez, 2002, p. 158) is also crucial to achieve an equity agenda. Developing rich mathematical activities based on the wealth of knowledge of the families and communities of the students is another avenue for equity (Civil, 2004). Much more than access to a traditional curriculum is needed to prepare students to develop a sense of agency and "write the world" with mathematics (Gutstein, 2006).

Equity of opportunities in schools is but one of the aspects needed to achieve equity of opportunities in society. As Ladson-Billings (2006) points out, injustices of the past still affect us today, a situation she has described as the "education debt." Today's children whose parents were shortchanged in school in the past will probably face greater challenges in school and in life than children whose parents had all the opportunities and advantages. The least we can do is to make sure that this education debt does not keep increasing for present and future generations.

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Cultural and Linguistic Resources to Promote Problem Solving and Mathematical Discourse Among Hispanic Kindergarten Students

Erin E. Turner • University of Arizona
Sylvia Celedón-Pattichis[1] • University of New Mexico
Mary Marshall • University of New Mexico

INTRODUCTION

A central tenet of the Principles and Standards for School Mathematics is that all students, even young children, should participate in solving problems and communicating their mathematical thinking (NCTM, 2000). Problem solving creates opportunities for children to construct understanding of important mathematical ideas (Hiebert, Carpenter, Fennema et al., 1996), to engage in mathematical practices such as presenting explanations and comparing solutions (Cobb, Wood & Yackel, 1993; Yackel & Cobb, 1996), and to develop positive dispositions towards the subject matter (Carpenter, Fennema, Peterson et al., 1989; Cobb, Wood, Yackel et al., 1991; Franke & Carey, 1997).

A substantial body of research has investigated the problem solving capacity of young children. As early as first grade (Carpenter, Fennema, Peterson et al., 1988, 1989; Carpenter, Hiebert & Moser, 1981; Fennema, Carpenter, Franke et al., 1996) and kindergarten (Carpenter, Ansell, Franke et al., 1993; Outred & Sardelich, 2005; Warfield, 2001), children can solve a range of word problems, often by modeling the quantities and relationships involved. While prior research has documented that a) young children can solve problems, and b) certain teacher practices seem to support students' learning, with notable exceptions (i.e., Carey, Fennema, Carpenter & Franke, 1995; Villaseñor & Kepner, 1993), the majority of the research has been conducted in predominantly White, English speaking, middle class schools (e.g., Carpenter et al., 1993; Fennema et al., 1996; Warfield, 2001). Much less is known about how young Hispanic students, nearly half (45 percent) of whom are English language learners (Kohler & Lazarín, 2007), learn to solve and discuss mathematical problems, and equally important, about the knowledge, strategies, and practices that their teachers draw upon to support their understanding.

Given that Hispanic students are the fastest growing group in our nation's public schools (Kohler & Lazarín, 2007), and the persistent gap in achievement between Hispanic students and their White and Asian counterparts (NAEP, 2005), this is a critical void in the literature.

[1] The second and third authors contributed equally to this paper.

A recent national assessment of 22,000 young children, specifically kindergarteners, reported that Black and Hispanic students entered kindergarten with more limited mathematical knowledge and skills than their White peers, and made fewer gains in achievement over the course of the year (NCES, 2000). In particular, while low-income Black and Hispanic students made some progress in basic mathematical skills (i.e., counting), at the end of the year they lagged even further behind White and Asian students on measures of more advanced mathematical knowledge such as solving simple addition and subtraction word problems (NCES, 2000). In other words, over the course of one year of formal schooling in kindergarten, the gap had widened (see also Jordan, Kaplan, Oláh & Locuniak, 2006, for similar findings).

While a widening "achievement gap" is certainly cause for concern, documenting the gap does little to inform our understanding of what actually happens in these kindergarten classrooms. We need to know more about the kinds of *opportunities students have to learn* to solve problems and discuss their thinking (Tate, 2005), and we need to know the availability of these kinds of opportunities to young Hispanic students (e.g., Oakes, 1990). We contend, as many others have argued, that the underachievement of lower income, minority students may be largely attributed to differential school and classroom-based experiences (e.g., Fryer & Levitt, 2004; Khisty, 1995; Oakes, 1990). For this reason, there is an urgent need to document teaching practices that support low income and minority students' success in problem-solving oriented classrooms (Boaler, 2002), including practices that address the specific learning needs of Hispanic students, many of whom are in the process of learning English.

In this article, we report on what happens when Hispanic kindergarten students in three classrooms have repeated opportunities to participate in solving and discussing basic word problems. We focus particular attention on the classroom practices and cultural and linguistic resources that teachers draw upon to support students' learning. We begin by presenting a set of theoretical tools that inform our work and analysis, and then briefly review prior work related to young children's problem solving.

Theoretical Tools

Socio-cultural Perspective on Learning

Consistent with situated and sociocultural perspectives on learning (Moschkovich, 2002; Nasir & Hand, 2006), we see mathematics classrooms as communities of practice (Lave & Wenger, 1991; Wenger, 1998) where students learn to solve problems, explain their reasoning, and engage in discussions about mathematical ideas. Learning mathematics is inherently a social and cultural endeavor. As Nasir and Hand (2006) argue, "understanding learning requires a focus on how individuals participate in particular activities, and how they draw on artifacts, tools, and social others to solve local problems" (p. 450).

Language, Discourse and Learning Mathematics. As a primary tool for knowing and interacting with the world, language plays a key role in learning. It mediates thought and new understandings via socially situated interactions (Vygotsky, 1978). In mathematics classrooms structured around solving and discussing problems, attention to language is particularly important (Moschkovich, 2002). Children not only construct mathematical ideas through language-rich interactions (Yackel & Cobb, 1996), they learn to "speak mathematically" (Pimm, 1987), which includes communicating their thinking, explaining solution strategies, and appropriating specialized mathematical vocabulary. Teachers play a critical role in providing students access to mathematical words and ways of talking (Khisty & Chval, 2002), by modeling mathematical discourse (Hufferd-Ackles, Fuson & Sherin, 2004) and providing students with repeated opportunities to use the language in context.

Given that language mediates learning, it is important to emphasize the role of students' native language in supporting their mathematical understanding (Khisty, 1995, 1999). If young children are to solve problems and communicate their thinking about those problems, it is reasonable to assume that they must first have opportunities to make sense of the problem, and that such sense making is facilitated by opportunities to work in their native language, or to draw on their native language for support.

Participation, Identity and Learning Mathematics. Sociocultural perspectives on learning also draw attention to the relationships between participation and identity, specifically, how the ways that students participate can impact how they view themselves as learners, and vice versa (Nasir & Hand, 2006). When students have repeated opportunities to participate in ways that honor their experiences, ideas and particular ways of knowing, the learning can be transformative (Wenger, 1998). In mathematics classrooms, when teachers position students as knowledgeable and competent problem solvers who have something important to contribute to the classroom community, they not only support students' understanding, but place them on a trajectory toward greater competence and participation (e.g., Empson, 2003). In such communities of practice, it is possible for students to create productive relationships with the discipline of mathematics (Boaler, 2002b; Cobb & Hodge, 2002) and identities that imagine greater competence in the future (Wenger, 1998).

Cultural Knowledge, Practices, and Learning Mathematics. Sociocultural perspectives view culture as a dynamic and socially constructed way of knowing and interacting with the world. Culture is not a set of traits, customs, and celebrations, but the practices that people engage in, what "people actually do and what they say about what they do" (González, Andrade, Civil, & Moll, 2001, p. 118). Research has argued that incorporating the cultural practices, knowledge and skills of students' households and communities into classroom activity can enhance students' learning (Civil, 2006; González, et al., 2001; González, Moll, & Amanti, 2005; Moll, 1992) by allowing students to tap into familiar, everyday experiences and practices—ways of talking, solving problems, understanding, and making sense of the world. Specific to mathematics, teachers and researchers have worked collaboratively to uncover mathematical Funds of Knowledge (González et al., 2005) in the households of low-income and minority students, and to design specific classroom activities or units of study that integrate these knowledge bases (for examples see Civil, 2002, 2006; Civil & Kahn, 2002; Kahn & Civil, 2001).

In summary, sociocultural perspectives emphasize the importance of language, social interaction, cultural knowledge and experiences, and participation in communities of practice as key elements of understanding and supporting children's learning. From a sociocultural perspective, understanding how young Hispanic students learn to solve problems and explain their thinking requires attending to these aspects of instruction. However, existing research on young children's problem solving rarely gives these aspects of instruction the attention we feel is necessary to address the needs of Hispanic students. Below we comment on this research.

Prior Research on Young Children's Problem Solving

An extensive body of research has documented that young children can solve a broad range of simple word problems by directly modeling the actions and relationships involved (Carpenter, 1985; Carpenter, Moser, & Bebout, 1988; Fuson, 1992; Outred & Sardelich, 2005). Carpenter, et al. (1993), found that kindergartners who had repeated opportunities to solve a variety of basic word problems demonstrated remarkable success on an end-of-the-year-assessment. Almost half of the 70 students interviewed used valid strategies on all of the problems, which included multiplication, division, and multistep problems, and the majority of students were successful on the most basic problem types (e.g., addition and subtraction). It is important to note that all teachers participated in a professional development program, *Cognitively Guided Instruction*, focused on the development of children's thinking about basic operations. Although teachers were not provided with specific guidelines for instruction, they were encouraged to use information about children's thinking to plan and adapt problem-solving tasks. Teachers generally did not show students how to solve problems, but instead provided students with concrete materials, such as counters, that they could use to model the problem.

It is relevant to note, given the focus of our research, that students in the Carpenter, et al. (1993), study were predominantly White (72 percent in one school and 77 percent in another) and from middle, or upper-middle class communities. That being said, research with *first-grade* students in more ethnically diverse urban schools (57 percent to 99 percent students of color) has documented similar findings (Villaseñor & Kepner, 1993). When students have repeated opportunities to solve and discuss word problems, they perform significantly better on assessments of problem solving and number facts than peers in more traditional classrooms (see Carey, Fennema, Carpenter & Franke [1995] for a study in schools with predominantly Black student populations).

In summary, extensive research has described how teachers draw upon knowledge of children's mathematical thinking as they organize problem solving based instruction, as well as the impact of their instruction on young children's learning (Carey et al., 1995; Carpenter et al., 1993, 1989; Fennema et al. 1996; Villaseñor & Kepner, 1993). It is not our intent to reiterate the value of such instruction here. Rather, there remains a need to investigate how young Hispanic students, including those who are ELLs, learn to solve problems and communicate their mathematical thinking, and more specifically, how teachers draw upon *other* knowledge bases, such as students' language, culture, or home experiences, as resources to support students' understanding. Our study helps to address this critical void in the literature. In the sections that follow, we describe the setting and participants of our study, as well as our methods for generating and analyzing data.

METHODS

Participants

We focused on three kindergarten classrooms in schools with predominantly Hispanic student populations (87 percent, 75 percent, and 72 percent, respectively), where almost all students (over 90 percent) qualified for free or reduced lunch. In Ms. Arenas'[1] classroom, all students were native Spanish speakers with varying degrees of English language proficiency. Ms. Arenas, also a native Spanish speaker, followed a bilingual model of instruction and all of her mathematics lessons were in Spanish. Ms. Field, who was trained in ESL (English as a Second Language) strategies, taught mathematics in English. Close to half of Ms. Field's students were English language learners and the rest were native English speakers. Ms. Perales, who was trained in bilingual education, taught mathematics in both Spanish and English, following a dual language model of instruction. Approximately half of Ms. Perales' students were native Spanish speakers and half spoke English as

1 All names are pseudonyms.

their first language. All three teachers taught in all-day kindergarten programs, and had approximately 18 to 20 students in their class.

We selected these three classrooms because teachers had participated in professional development focused on young children's mathematical thinking (i.e., Cognitively Guided Instruction, see Carpenter, Fennema, Franke et al., 1999), and they were interested in conducting problem-solving lessons with their students. For two of the teachers, Ms. Arenas and Ms. Field, this was their first year using what they had learned about children's thinking to plan and implement problem-solving tasks. The other teacher, Ms. Perales, had previous experience.

Data Collection and Analysis

Pre- and Post-Assessments. As one method of documenting students' learning, we conducted pre- and post-task-based clinical interview assessments (Ginsburg, Kossan, Schwartz et al., 1983). At the beginning of the year (October), we selected 21 students (seven from each classroom) who represented a range of achievement levels to participate in a pre-assessment, which included both counting and problem-solving tasks (e.g., join, separate, multiplication, and division word problems). Table 1 displays selected pre-assessment items. All problems were presented orally in students' dominant language, and students had access to multiple tools (counters, cubes, paper and pencil). Interviewers reread each

Table 1. Selected Pre-Assessment Items (English Version)

Problem Structure	Problem
Join Result Unknown	a. Maya has six candies. Her brother gives her three more candies. How many candies does Maya have now?
Separate Result Unknown	b. Jason has 10 pennies. He loses four of them. How many pennies does Jason have now?
Multiplication	c. Javier has three pockets. He puts two pennies in each pocket. How many pennies does Javier have now?
Partitive Division	d. There are eight marbles. Two friends want to share the marbles so that they each get the same amount. How many marbles can each friend have?
Compare, Difference Unknown	e. Sarita has nine toy cars. Her brother Jorge has six toy cars. How many more toy cars does Sarita have than Jorge?

problem as many times as needed, and clarified specific information if the child asked. After each problem, children were asked to explain their solution strategy. ..

At the end of the year (May), we administered a post-assessment that included a broader range of problem types, modeled after the problems included in the Carpenter, et al. (1993), study. (Table 2 displays selected post-assessment items). A total of 45 students participated in the post-assessment, approximately 15 from each classroom. All students who received parental consent participated in these interviews, conducted in their dominant language. As in the pre-assessment, students had access to multiple tools and after each problem were asked to explain their reasoning. All interviews were conducted by trained members of the research team, and videotaped for later analysis.

We coded students' responses to pre- and post-assessment items using the coding scheme reported in Carpenter, et al. (1993). Each response was coded in terms of a) the strategy that the child used (i.e., direct modeling, counting, recalled fact), b) whether the strategy was valid or invalid, and c) whether the answer was correct. Valid strategies were those that absent a miscount would have resulted in a correct answer.

Table 2. Selected Post-Assessment Items (English Version)

Problem Structure	Problem
Join Result Unknown	a. Julio has six candies. His sister gives him six more candies. How many candies does Julio have now?
Separate Result Unknown	b. Karla had 13 cookies. She ate five of them. How many cookies does Karla have left?
Join Change Unknown	c. Francisco wants to buy a toy plane that costs $11. Right now, he only has $7. How many more dollars does Francisco need so that he can buy the toy plane?
Multiplication	d. Sara has three bags of marbles. There are six marbles in each bag. How many marbles does Sara have altogether?
Partitive Division	e. Estevan had 15 marbles. He shared the marbles with three friends so that each friend got the same number of marbles. How many marbles did each friend get? (Estevan did not keep any marbles for himself.)
Measurement Division	f. Alan had 10 cookies, and some little bags. He wants to put two cookies in each bag to give to his friends. How many bags can he make?
Compare	g. Fernando has 12 toy cars. His sister Anabel has nine toy cars. How many more toy cars does Fernando have than Anabel?
Multistep	h. Javier has two bags of candy. There are four candies in each bag. Then, Javier gets hungry and eats three of the candies. How many candies are left?
Measurement Division With Remainder	i. In art class, 15 children are going to paint. To paint, they need to sit at tables. Only four children can sit at each table. How many tables do they need so that all of the 15 children can paint?

Classroom observations. The second source of data involved ongoing observations of problem-solving lessons in the three classrooms. We visited Ms. Arenas' and Ms. Field's classrooms on a biweekly basis to observe and/or videotape instruction. Because of distance and resources, Ms. Perales' classroom was observed once a month. For each teacher, a typical lesson lasted between 30 and 45 minutes. Over the course of the year, we videotaped and fully transcribed at least six lessons in each classroom. Transcription attended not only to what was said, but to other aspects of interactions, such as gestures and use of tools and representations.

Teacher interviews. Each teacher was interviewed once in the fall and once in the spring. The interviews asked teachers to reflect on the strategies they used to support students' learning, the knowledge that they drew upon to design and implement tasks, and their thoughts about the understanding of particular students. All interviews were audiotaped and transcribed.

Analysis of observation and interview data. Classroom and interview transcripts were coded according to the principles of grounded theory (Strauss & Corbin, 1990). This involved chunking the data into meaningful units, and then coding selected statements or interactions using words or phrases that specifically addressed the research questions (Erlandson, Harris, Skipper et al., 1993). For example, classroom observations were coded with particular attention to specific teacher and student actions related to solving and discussing mathematical problems. We used a computer-based qualitative research tool, TAMS© Analyzer, to code all data. To establish reliability (Miles and Huberman, 1994), transcripts were coded by at least two members of the research team. Differences in interpretation were discussed until agreement was reached. The TAMS© Analyzer tool allowed us to search across transcripts to establish recurring patterns, or themes, related to particular instructional practices. Each theme was then triangulated across data from all participants, and across data of various forms (i.e., classroom observations, field notes, teacher interviews) (Erlandson et al., 1993).

In the sections that follow, we begin with a brief overview of the pre-assessment results, and then describe a typical problem-solving lesson in each of the kindergarten classrooms. We continue with a detailed analysis of specific instructional practices that teachers used to support students in learning how to solve problems and explain their reasoning. We conclude with a discussion of students' performance on the post-assessment measures.

FINDINGS

Analysis of Pre-assessment Measures

Typical of many kindergarten classrooms, children began the year with a range of mathematical experiences and proficiencies. On a *Kindergarten Developmental Progress Record [KDPR]* that teachers administered during the first month of school, most students (80 percent) were able to count a small set of objects (up to eight items) and to recognize some numerals from one to 10. However, less than half of the students could count past 10 or 15, and there were several students in each classroom who did not count past two or three or demonstrate one-to-one correspondence at the beginning of the year. As Ms. Field noted, "When I look over those [KDPR] screenings, and I remember testing some of these kiddos, and they didn't know more than 1, 2, 3 … there were quite a few kids that couldn't even count to 10 which, with my first time [teaching] in kindergarten, really amazed me."

In terms of problem solving, almost half of the 21 students (nine out of 21, or 43 percent) who participated in the pre-assessment successfully solved a basic addition problem, and slightly more than half (11 of 21 students, or 52 percent) solved a basic subtraction problem (table 1, problems a and b). Given that many students began the year with somewhat limited counting skills, and that national assessments have found that only 4 percent of kindergartners are able to solve basic addition and subtraction problems at the beginning of the year (NCES, 2000), we find students' success on these easier problem types to be remarkable, and further evidence of the typically underestimated problem-solving capacity of young children. However, other items on the pre-assessment, such as multiplication (14 percent solved), partitive and measurement division (28 percent and 23 percent solved, respectively) and compare problems (0 percent solved) were significantly more difficult.

Strategy Use

Of all problems solved correctly, the vast majority (82 percent) was solved using direct modeling strategies (i.e., students modeled the quantities and relationships involved using fingers, objects, or drawings). Only four of the 21 students used more advanced counting strategies, and these strategies were all used on easier problem types (i.e., Join Result Unknown). It is important to note that students' performance on the pre-assessment was relatively consistent across the three classrooms. On average, students solved two of the seven items correctly, ranging from an average of 1.67 items correct in Ms. Field's class to 2.4 items correct in Ms. Perales' class.

In summary, students' mathematical performance on pre-assessment measures reflected a range of understandings that is typical of kindergartner students. If anything, some students began the year with less developed number concepts and skills than kindergarten teachers might expect.

Portrait of Instruction

Ms. Arenas, Ms. Field and Ms. Perales drew on a variety of instructional formats in their problem-solving lessons. Common to all lessons was that the teacher orally presented a word problem, and then encouraged students to solve the problems in ways that made sense to them. Students often used concrete materials, such as counters and cubes, or drew pictures on small white boards to support their reasoning. After most students had solved a problem, the teachers facilitated a group discussion in which multiple students shared their strategies.

In some instances, teachers worked on problem solving with one small group of students at a time. Typically, teachers grouped students heterogeneously, although there were instances when specific students were grouped together because the teacher felt they would benefit from working on a certain type of problem. On other occasions, teachers presented the entire class with a problem, and students worked individually or with a partner to generate a solution. In addition to problem solving, each of the teachers used center activities that involved building and counting sets to support students' number sense and counting skills. It is important to note that teachers did not wait until students had mastered a set of "basic skills" to begin problem-solving lessons. They introduced problem solving at the beginning of the year, and used contextual problems to strengthen students' number-related concepts and skills. As Ms. Arenas noted:

> *[At the beginning of the year] my students, most of them didn't recognize any of the numbers, you know, higher than three they probably didn't know. Some of them counted 1, 2, 100 - and they didn't know how to count so I had to develop that little by little at the same time that I was doing the CGI [problem solving].*

Given this general portrait of instruction, we now shift our focus to a detailed analysis of the *specific practices* that teachers used to help students solve problems and communicate their thinking. While documenting every practice is beyond the scope of this paper, we focus here on the *cultural and linguistic resources* that teachers drew upon to support students' learning. We conclude our findings section with an overview of students' performance on the post-assessment.

Analysis of Classroom Practices

1. Generating mathematical problems through authentic, storytelling conversations

One practice common to the teachers in our study was the use of authentic, "story-like" conversations to generate mathematical problems. Teachers often presented stories in an informal, conversational manner, including rich contextual information, and inviting students to respond with questions or comments. By framing problem solving around telling and investigating stories, teachers drew upon ways of talking and negotiating meaning that were familiar to children. That is, all children have experience listening to stories and using stories to communicate meaning, which is particularly prevalent in some Hispanic families (Delgado-Gaitan, 1987; Villenas & Moreno, 2001). The dialogic nature of the stories that teachers told invited students to enter the situation and imagine themselves as active participants and contributors.

For example, Ms. Arenas typically began her lessons by telling students to listen carefully ("*Fíjense, amorcitos*"), because she was about to share a story ("*Les voy a contar otra historia*"). In one conversation, she built a story around a current classroom activity: decorating and filling Easter baskets. The structure of the problem she generated (9÷3) reflected an actual classroom event in which the student teachers needed to equally distribute nine eggs among three people.

Ms. A:	*Vamos a ver otra. Fíjense, que estamos ya comprando los huevitos de pascua para sus canastas. Los huevitos de pascua. Y tenemos* – (Let's do another one. Listen, you know we're already buying the little Easter eggs for your baskets. The Easter eggs. And we have ---)
Julieta:	*Maestra, mi mamá y una prima ya compraron las canastas.* (Teacher, my mom and a cousin already bought the baskets.)
Ms. A:	*¿Ya compraron las canastas?* (They already bought the baskets?)
Julieta:	*¡Sí!* (Yes!)
Ms. A.:	*Ah, Ms. Maribel y Ms. Aracely (two classroom assistants) ya las van a empezar a decorar. ¿Verdad? Las canastas de pascua. Entonces, adentro les vamos a poner unos huevitos.* (Oh, Ms. Maribel and Ms. Aracely (classroom assistants) are going to start decorating them now. Right? The Easter baskets. Then, inside, we are going to put some little eggs.)
Alonso:	*¿Maestra? ¿Maestra?* (Teacher? Teacher?)
Ms. A.:	*¿Sí?* (Yes?)
Alonso:	*Mi mamá me va a comprar una canasta de basquetbol.* (My mom is going to buy me a basketball basket.)
Ms. A.	*¿Para la pascua?* (For Easter?)
Alonso:	[nods]
Ms. A.	*¡Ay, qué bonito!* (Oh, how nice!)
Julieta:	*Y maestra, mi mamá ya le echó dulces.* (And teacher, my mom already put the candies in.)
Ms. A.:	*¿Ya le echó huevitos? Pues ahora, Ms. Anita y Ms. Crystal (student teachers) trajeron 9 huevitos.* (She already put in little candy eggs? Well, today, Ms. Anita and Ms. Crystal (student teachers) brought nine little [candy] eggs.) ...
Ms. A.:	*Escuchen. Pero los quieren repartir, escuchen. Primero escuchen. 9 huevitos trajeron. Pero los quieren repartir entre Ms. Maribel, Ms. Aracely y Ms. Maggie. ¿Cuántos le van a quedar a cada una?* (Listen. But they want to share them, listen. First listen.They brought nine eggs. But they want to divide them between Ms. Marbiel, Ms. Aracely, and Ms. Maggie. How many is each one going to get?)

Interestingly, although this practice of generating mathematical problems through "storytelling" conversations was not explicitly discussed in the professional development that teachers received, it was typical of lessons across the three classrooms. Ms. Perales engaged students in a story about Mariela (a student in the class) and the shoes that she had at home in her closet. She discussed with students what a "pair" of shoes meant, taking off her own shoes to demonstrate, and posed questions such as, "If Mariela had 6 shoes, how many pairs would that be?" Mariela added to the story, saying she just had four shoes at home, because she was wearing one pair of shoes to school. Students then used Mariela's contribution to figure out how many pairs of shoes she had at home.

Teachers commented that they viewed sharing and constructing stories as a way of drawing on students' cultural knowledge and experiences. Ms. Arenas noted, "Yes, I always try [to relate to their cultural experiences], and that is what I am doing with the CGI. I have been trying to give my stories. Other teachers put their own stories. A lot of stories are very cultural." Using stories to connect students' cultural knowledge and experiences with their mathematical activity has been documented in prior research (Lo Cicero, Fuson, & Allexsaht-Snider, 1999; Lo Cicero, De la Cruz, & Fuson, 1999). In what follows, we extend this work by contributing a more detailed analysis of *how* this practice supports young Hispanic kindergarten students as they learn to solve and discuss mathematical problems.

a. *Stories that reflect familiar contexts invite students to draw upon lived experiences and cultural funds of knowledge to make sense of mathematical ideas.* Teachers rarely presented "generic" problems (e.g., Sara had three apples. Johnny gave her five more. How many does she have now?) or problems that were based on unfamiliar contexts. In fact, we only identified five such problems across all the lessons we observed. Instead, the stories reflected community events (e.g., going to the fair), family practices (e.g., purchasing fruit at a local market), shared classroom experiences (e.g., field trips), or common play activities (e.g., games of marbles, sharing toys with friends). Teachers seemed to be familiar with many family practices of their students and how these practices might relate to mathematics. Ms. Arenas noted:

> *[Children] bring rich experiences in going to the market with their parents. ... The market experience, the open market experience, [experience] with the money and how much do you pay for this or that. ... I know the kids know, maybe they can estimate how many strawberries are in that basket, because they have had the experience. ... They have other cultural experiences too. A lot of times they plant with their parents and even counting the seeds or transferring the seeds is math[ematics].*

When teachers employed relevant contexts to frame mathematical ideas, students were invited to draw on their own experiences and funds of knowledge as they assumed an active role in making sense of mathematical ideas.

b. *The narrative structure of stories scaffolds students' explanations.* Teachers also drew upon the *structure* of stories to scaffold students as they learned to explain their thinking. When students struggled to explain their ideas, teachers often reminded them of the events in the story, and then used the narrative as a framework to guide students as they explained the steps they took to solve the problem. For example, Ms. Field presented the following story about a game of marbles between Sarita and Kyle: "Sarita had eight marbles, and then Sarita gave some of those marbles to Kyle. She gave them away. And now she only has four left. So how many did she give to Kyle? Go ahead, try it." After several minutes, Penny volunteered to share her solution with the class.

Ms. Field: Sure, come on up Penny. All eyes up here. Tell us what you did.

Penny:	First I started with four, and then I started with four more; then I counted and it made nine, and then I counted, and then ---- (she pauses, seems uncertain, and looks up at Ms. Field).
Ms. Field:	OK Penny, wait a minute, let me tell you the problem one more time. We said that Sarita had eight marbles, and then she gave Kyle some, and she had four left. So how many did she give him?
Penny:	Four, four. (Points to her picture. She has drawn a line of eight marbles, with four marbles on one side of her white board and four marbles on the other).
Ms. Field:	So she gave him four marbles?
Penny:	Yes.
Ms. Field:	And then how many did she have?
Penny:	She gave [him] four and had four more left. Cause [it's] like four and four is eight. (Points to the two groups of four on her board).

Ms. Field then restated Penny's solution, emphasizing that Sarita started with eight marbles, she gave away four and then had four left, and that Penny figured out that she gave away four because she knew that four and four is equal to eight. In this episode, we see how Ms. Field drew Penny back into the story when she seemed unsure about how to explain her strategy. Then, Ms. Field asked focused questions about various events in the story ("So how many did she give him?") as a way of guiding Penny to use the structure of the story as a resource to explain her thinking. In the end, when Ms. Field restated Penny's ideas, she again offered Penny a model of how she might use the story to frame her explanation.

As we compared lessons across the year, we noticed that while teachers initially provided substantial guidance and modeling to help students communicate their reasoning, as the year progressed, students began to contribute clearer and more complete explanations. We suspect that repeated interactions in which teachers used the familiar, narrative structure of stories to scaffold students' explanations contributed to this growth.

c. *Stories help students learn to represent mathematical ideas and connect multiple representations.* Teachers also used stories to support students as they learned to represent mathematical ideas and connect multiple representations (e.g., drawings, symbols, objects). In the following episode from Ms. Perales' class, after students solved a problem about equally distributing 15 toy cars among three friends, Brenda volunteered to share her solution.

Brenda:	I put 15 right here, 15 lines, and I put one line in each [circle], one for Beto, one for David, and one for Juan. (she holds up her slate, and gestures how she "passed" out one tally, representing a car, to each friend.) Cinco. Cinco! {Five. Five!} (meaning that each friend got a total of five cars).
Ms. Perales:	Cinco. Juan, David y Beto, todos van a agarrar cinco, cada uno igual. Equally. Good job! {Five .... Juan, David and Beto, they're all going to get five, each one the same. Equally. Good job!)

As the discussion continued, Ms. Perales helped Brenda to re-represent her solution on the board. They first drew a row of 15 tallies to represent the cars, and then three large circles to represent each of the friends. Brenda then directed Ms. Perales to distribute one car (a tally) to each boy, and to erase three tallies from the row of 15. Ms. Perales followed Brenda's instructions, drawing lines to link one tally to each boy's circle, and then re-representing the tally inside the circle with a dot, just as Brenda had done. When Brenda came to the board to continue enacting her solution, Ms. Perales paused to clarify the meaning of her representation for other students.

Ms. Perales:	*What is she putting in here?*
Yanitza	*Cars!:*
Ms. Perales:	*Carritos. Cars. And then how come she is taking them away? How come he is putting some and then taking some away?* (points to where she has erased 3 tallies from the row of 15).

Mariela: Because three and three and three. Three for David, three for Beto, and three for Juan. (Indicating that so far, Brenda has distributed three cars to each boy).

Ms. Perales: Okay, she is giving each kid a car.

When Brenda finished representing her solution, Ms. Perales asked students how many cars each boy received and they enthusiastically responded, "Cinco! Cinco! Y Cinco!"

Ms. Perales: *¡Sí! Hay cinco carritos aquí adentro* (points to one boy's circle, and writes the number 'five' below it), *hay cinco carritos aquí adentro* (points to the next circle, and again writes 'five'), *y hay cinco aquí* (points to final circle, and writes 'five' below it). (Yes! There are five carritos here inside [the circle], there are five here inside, and there are five here.)

In this example, we see how Ms. Perales repeatedly referred to the story context to help students make sense of the various components of Brenda's representation (e.g., "What is she putting in [the circle]?...Okay, she is giving each kid a car.") She then carefully pointed back and forth between a group of five tallies and the number 'five' as she noted that each representation stood for the five cars each boy received. Teachers' use of stories to help students make sense of different representations may have been especially helpful for ELL students, because each representation created an additional opportunity for students to make sense of the mathematical ideas.

In summary, teachers frequently used authentic, "story-like" conversations to generate mathematical problems. The use of relevant stories not only created opportunities for students to draw on cultural knowledge and experiences to make sense of problems, but also scaffolded their explanations and supported their understanding of multiple mathematical representations.

Scaffolding students' ability to communicate their thinking through strategic use of positioning and questioning coupled with access to multiple resources.

Each of the teachers in our study spoke explicitly about the importance of mathematical discourse in their classrooms. They wanted students to communicate their thinking, and they were aware that learning to communicate mathematically could be challenging for young children, particularly children who were in the process of learning the language of instruction. Teachers described a variety of practices that they drew upon to address these challenges. For example, Ms. Field noted that creating a safe environment that encouraged students to take risks supported mathematical communication in her classroom:

> *What's good is that they're [English language learners] experimenting, and they might not know the exact term but they are trying to use it. And having a safe environment where nobody is allowed to laugh at anybody – I think that really helps the communication also. Even the ones that are learning the language, the risk takers like Rogelio, have made so many gains. He doesn't care if he makes a mistake, he's trying.*

Ms. Arenas added that giving students opportunities to hear peers explain their strategies was important.

> *[I] support[ed] them first by guiding them because they didn't know what to do. And then modeling from other students, because some of those students mastered that ability before and they were pretty good models of how to explain and verbalize their strategies. They became models to other students.*

Teachers also supported students' participation in mathematical discourse by encouraging the use of multiple resources to communicate ideas. For example, when teachers orally introduced a problem, they used gestures, held up fingers to represent quantities, pointed to relevant objects in the room, and in some cases translated particular words or phrases. In one sense, their actions were aimed at supporting students' understanding of the problem. At the same time, their actions modeled how students might draw upon multiple resources to explain their *own* ideas. We noticed that almost invariably students did use a variety of resources to explain their thinking, and that this enhanced students' ability to contribute their ideas and make sense of the ideas of others. Another way teachers supported students' participation in mathematical discourse was by modeling mathematical ways of talking. Teachers frequently restated students' strategies, and in doing so, used more mathematically precise language (e.g., "Oh, so you are saying that you *counted on*, you started at four and then *counted on*, five, six, etc.")

We include these brief examples to demonstrate that teachers drew on a variety of instructional practices to support Hispanic kindergarten students as they learned to communicate their thinking. Our intent is not to focus on these practices, which though productive, are not unique to the teachers in our study, and have been well documented in prior research (e.g., Echeverria, Vogt, & Short, 2004; Khisty & Chval, 2002; Moschkovich, 1999). Rather, we intend to highlight how teachers scaffolded students' mathematical communication through the intentional use of positioning and questioning strategies. To demonstrate the power of these practices, we present in-depth analyses of episodes from two classrooms.

In the first example, students in Ms. Arenas' class solved a problem about Diego, who had $6 but wanted to buy a toy plane that cost $10. Students' task was to figure how much more money he needs to buy the plane. Belén was the first student to share her strategy.

Ms. Arenas: *Vamos a empezar ya con Belén, a ver Belén, explícanos cuántos dólares más necesita Diego.* {We are going to start now with Belén. Let's see Belén, explain to us how many more dollars Diego needs.}

Belén: *Cuatro.* (Four.)

Ms. Arenas: *Muy bien. Belén dice que cuatro, a ver Belén, cómo lo hiciste? Porque lo hiciste más rápido que Ms. Arenas. ¿Cómo lo hiciste?* (Very good. Belén says [he needs] four. Let's see Belén, how did you do it? Because you did [the problem] faster than Ms. Arenas. How did you do it?)

Belén: *Porque primero puse estos* (points to a row of 10 tallies), *y luego puse estos* (points to a second row of six tallies that she has drawn directly beneath the row of 10), *y luego conté estos* (points to the "extra" four tallies in the row of 10). (First I put these, and then I put these, and then I counted these.)

Ms. Arenas: *Muy bien. ¿Cuántos dólares costaba el avión? Miren todos lo que hizo Belén pues.* (Very good. How many dollars did the plane cost? Everyone look at what Belén did.)

Belén: 10.

Ms. Arenas: *El avión costaba 10.* (The plane cost $10.)

Belén: (writes the number 10 on the large white board)

Ms. Arenas: *¿Y cuántos tenía Diego?* (And how many [dollars] did Diego have?)

Belén: *Seis dólares.* ($6.)

Ms. Arenas: Miren todos cuántos tenía Diego. Cuéntelos. (Look [everyone] at how many Diego had. Count them.)

Belén: Uno, dos, tres, cuatro, cinco, seis. (One, two, three, four, five, six.)

Ms. Arenas: Y miren cómo lo hizo Belén. Los fue poniedo abajo de lo que costaba el avión. (Points back and forth between row of 10 tallies and row of six tallies.) Y luego, cómo supiste que le faltaban 4 dólares? (And look at how Belén did it. She was putting them [the tallies that stood for the money Diego already had] below what the toy plane costs. And then, how did you know that he needed $4 more?)

Belén: Porque yo los conté rápido, conté estos (points to four "extra" tallies in the row of 10) que faltaban. (Because I counted them quickly, and then I counted these that were left over.)

In this example, Ms. Arenas began by positioning Belén as a competent problem solver who had something important to explain to members of the classroom community (e.g., "A ver Belén, explícanos… porque lo hiciste más rápido que Ms. Arenas.") She then invited Belén to explain how she figured out that Diego needed $4 more to buy the plane. We found that beginning discussions by first positioning students as competent, and then extending open invitations for students to share their ideas was typical of all three teachers' practice. In fact, questions that invited students to explain their thinking or pushed for more details about something that a student said were the most frequently occurring teacher actions across all of the lessons that we analyzed. However, when students struggled to articulate their ideas, or, when they explained their strategy in a way that may have been comprehensible to the teacher, but not to other students in the classroom, teachers often shifted to more focused, closed questions[2], and then returned to open-ended probes later in the interaction.

For example, Bélen stated that first she "put these" and then "these" and then she "counted these." Understanding her explanation required attending to physical representations she referred to, and even then, her reliance on indexical language somewhat obscured her intended meaning. We suspect that Ms. Arenas understood Belén's explanation. Even so, for the benefit of other students, and Belén, she responded with a series of closed questions aimed at helping Belén clearly articulate the meaning of her tally mark representation. She then modeled for Belén an explanation that highlighted a key aspect of her strategy: lining up tally marks to compare the amount of money Diego had with total cost of the plane (e.g., "*Y miren como lo hizo Belén. Los fue poniedo abajo de lo que costaba el avión.*") It is important to note that even as Ms. Arenas shifted to more closed questions, she continued to assign competence to Belén's ideas by repeatedly inviting other students to attend to her explanation. Ultimately, Ms. Arenas returned to an open-ended probe, asking Bélen how she knew that Diego needed $4 more. At this point, Belén clearly explained how she counted the 'leftover' tallies and got four.

This pattern of shifting from open-ended questions to closed probes and then back to open invitations to explain ideas was apparent in all three classrooms. Since it was particularly evident in interactions where students initially struggled to articulate their reasoning, we see this strategic use of questioning, coupled with an explicit positioning of students' ideas as important, as a notable way that teachers scaffolded students' ability to communicate their thinking. We see these two practices—strategic use of questioning and explicit positioning—as interrelated; they work together to support students' participation in mathematical discourse. Merely shifting between open and closed questions, without positioning students as competent members of the classroom community (e.g., Empson, 2003), would not have been as effective, we contend.

We continue with a second example, from Ms. Field's classroom. In this lesson, students solved a simple addition problem about two girls, Amalia and Natalie, who joined their collections of marbles (3+4=7). After several students described their strategies, Ms. Field invited Amalia, an English language learner, to share her solution with the class.

Ms. Field:	Would you tell us what you did, Amalia?
Amalia:	I put a three and four (points to two sets of squares she has drawn on her white board)
Ms. Field:	(Holds up Amalia's board for other students to see). Everyone look at Amalia's. Cap your marker. Amalia, tell us what you did.
Amalia:	I did two, uh ... square and square. Four squares and three squares (Points again to the two sets of squares she has drawn.)
Ms. Field:	Okay, how did you get your answer? What did you do?
Amalia:	(Pause, no response.)
Ms. Field:	What's your answer?
Amalia:	(No response, she is looking at her white board.)
Ms. Field:	This is good. Did you count, or how did you do it?
Amalia:	I – I count. I count.
Ms. Field:	Show us.
Amalia:	I count. One, two, three. And I count four.
Ms. Field:	And then, what's your answer?
Amalia:	Uh ... (pause)

2 We use *closed questions* to refer to fill-in-the-blank type questions that elicit a one-word response, such as a number or a statement of "yes" or "no."

Ms. Field:	How did you . . . (points to two sets of squares that Amalia has drawn on her white board) – did you count them? Show me, how did you do it?

At this point, Amalia pointed to a hundreds chart on the board, indicating that she used the chart to solve the problem. Ms. Field invited Amalia up the board to show the class how she counted.

Amalia	One, two, three (points to the first three numbers on the hundreds chart as she counts). And that be four. (pause) Because I, uh (pause)
Ms. Field:	So you had three (points to number three on the hundreds chart) and then you knew you had four more. So you counted four more. So count four more. One –
Amalia:	One, two – (points to number four and then five on the hundreds chart)
Ms. Field:	Three –
Amalia:	Three, four (points to number six and seven on the hundreds chart).
Ms. Field:	So what was your answer?
Amalia:	Four – is this one. (points to the number seven on the hundreds chart.)
Ms. Field:	Seven (nods head). Very good, very good. Thank you.

As in the previous example, Ms. Field began by first positioning Amalia as someone who had an important contribution to make, and then inviting her to share her strategy with the class. Also like Ms. Arenas, Ms. Field shifted between open-ended and more focused probes in response to Amalia's apparent uncertainty. For instance, when Amalia did not respond to her invitation to explain how she solved the problem, Ms. Field asked a more focused question, "Did you count?" to which Amalia responded affirmatively, "I – I count. I count." Ms. Field then returned to a more open-ended question: "Show us."

We find it significant that when teachers shifted to closed probes, they used the probes strategically: to confirm their interpretation of a student's action, to establish that a student understood the quantities involved, or to clarify a student's representation. Once a student responded to such probes, and therein clarified some aspect of the strategy, teachers returned to a more open elicitation of the student's ideas. This pattern is significant because it created ongoing opportunities for students to communicate thinking in meaningful ways (i.e., beyond a one-word response), even for students like Amalia who struggled to articulate their ideas.

Also important is that throughout the interaction, Ms. Field continued to position Amalia's ideas as important, and to express confidence in her ability to communicate her reasoning to her peers. Given that Amalia was just beginning to learn English, these repeated expressions of confidence are even more significant. It is relevant to note that other aspects of Ms. Field's instruction, in particular, students' access to a variety of resources, also scaffolded Amalia's participation in this discussion. Specifically, the opportunity to use a hundreds chart to demonstrate for her peers how she started at three, and then counted on four to get seven, was significant.

In summary, we found that teachers scaffolded students' ability to communicate their thinking through a strategic use of questioning and positioning. When coupled with other important aspects of instruction, such as access to multiple resources to communicate ideas, these practices expanded students' opportunities to participate in mathematical discourse.

The previous sections described specific instructional practices that teachers used to support students as they solved and discussed mathematical problems. To further document the impact of these practices, we discuss students' performance on the post-assessment measure.

Analysis of Post-Assessment Measures

As previously noted, 45 students across three classrooms participated in a problem-solving-based post-assessment in May of kindergarten. Relative to the percentage of students who correctly solved particular problems on the pre-assessment, students demonstrated remarkable growth. A significant majority of students successfully solved the most basic join and separate problems (80 percent and 73 percent, respectively), and the join change unknown, multiplication, and partitive division problems were each solved by approximately half of the students (56 percent, 49 percent and 43 percent, respectively). The most difficult problems for students were those involving comparison and division with a remainder (problems g and i in Table 2), and even so, approximately one-fourth of the students solved each problem correctly, and an even greater percentage (33 percent) used a valid strategy. Interestingly, the multistep problem, which one might assume to be too difficult for young children, was correctly solved by 44 percent of the students. Table 3 presents a summary of students' performance on various post- assessment items.

Compared to the Carpenter, et al. (1993) study, kindergartners' performance was comparable on the easier problem types, and somewhat less successful on the more challenging problems (approximately 40 percent to 50 percent solved vs. 60 percent to 70 percent). The most notable difference was on the measurement division with a remainder problem, solved by 27 percent of the students in our study and 64 percent of students in the Carpenter, et al., study. We conjecture that differences on the more difficult problem types may be attributable to differences in students' opportunities to solve these kinds of problems. While the teachers in our study tended to focus on join, separate and basic multiplication problems during their lessons, the teachers in Carpenter, et al., reported that they presented students with a broader range of problem types.

That being said, it is worth noting that students' performance on join result unknown and separate result unknown problems was comparable to that of bilingual, Hispanic *first* graders who solved similar problems in Secada's (1991) study. Moreover, the kindergartners were more successful on a comparable join change unknown problem (55 percent solved, as compared to 29 percent of first graders)[3]. In addition, kindergartners in our study solved a much broader range of problems than the results of a national assessment of 22,000 kindergartners might predict (NCES, 2000). While the NCES study found that only 18 percent of kindergartners demonstrated the ability to solve simple addition and subtraction word problems by the end of the year, and that only 2 percent could solve basic multiplication and division problems, the students in our study solved similar problems at much higher rates. For example, 71 percent of students solved either a multiplication *or* a division problem, and 44 percent correctly solved *both* problem types. While details about the instruction that kindergartners in the NCES study received were not provided, we can assume that most students were not in classrooms that emphasized solving and discussing problems. This once again demonstrates the value of providing young children with *opportunities* to solve problems in ways that make sense to them, and the need to document how teachers structure such opportunities to support all students' learning.

Strategy use.

Similar to the pre-assessment, and consistent with the findings of previous research (Carpenter, et al., 1993), students used direct modeling strategies to solve the majority of problems (84 percent). However, in contrast to the pre-assessment, approximately half of the students (49 percent) used an advanced strategy (e.g., either counting or a recalled number fact) to solve at least one problem. Thus not only did students progress in their ability to solve a

3 Students in Secada's (1991) study did not solve multiplication, division, compare, or multistep problems, and thus a comparison of those problem types is not possible. It is also important to note that the instruction students received was not necessarily focused on solving and discussing problems.

Table 3. Post-Assessment Results: All Three Classrooms

Percentage of children correctly solving each problem and using a valid strategy (n=45)			
Problem Structure	**Problem**	**Correct Answer**	**Valid Strategy**
Join Result Unknown	a. Julio has six candies. His sister gives him six more candies. How many candies does Julio have now?	80%	91%
Separate Result Unknown	b. Karla had 13 cookies. She ate five of them. How many cookies does Karla have left?	73%	86%
Join Change Unknown	c. Francisco wants to buy a toy plane that costs $11. Now, he only has $7. How many more dollars does Francisco need so that he can buy the toy plane?	56%	69%
Multiplication	d. Sara has three bags of marbles. There are six marbles in each bag. How many marbles does Sara have altogether?	49%	64%
Partitive Division	e. Estevan had 15 marbles. He shared the marbles with three friends so that each friend got the same number of marbles. How many marbles did each friend get?	42%	49%
Measurement Division	f. Alan had 10 cookies, and some little bags. He wants to put two cookies in each bag to give to his friends. How many bags can he make?	40%	53%
Compare	g. Fernando has 12 toy cars. His sister Anabel has nine toy cars. How many more toy cars does Fernando have than Anabel?	24%	33%
Multistep	h. Javier has two bags of candy. There are four candies in each bag. Javier gets hungry and eats three of the candies. How many candies are left?	44%	49%
Measurement Division with Remainder	i. In art class, 15 children are going to paint. To paint, they need to sit at tables. Only four children can sit at each table. How many tables do they need so that all the 15 children can paint?	27%	33%

variety of problems, they also began to use more advanced strategies, particularly on the easier problem types.

Students' explanations.

Whereas students' explanations at the beginning of the year were often vague or incomplete, many students produced clear, and more mathematical descriptions of their thinking on the post-assessment. For example, Dalia solved a join change unknown problem (Ana has $7, how many more does she need to have $11?) by drawing 11 tally marks, crossing out seven, and then counting those that remained to get four. She then explained her thinking:

Dalia:	Primero Ana quiere comprar un avión que cuesta 11 dólares. Y le faltan.. (counting the tally marks she did not cross out) 1, 2, 3, 4.
Interviewer:	¿Cómo supiste que le faltan 4 [dólares]?
Dalia:	Puse 11, y quité los 7 (points to the seven tally marks that she crossed out), y luego le faltan 4 (points to the four remaining tally marks).
Dalia:	*First Ana wants to buy a plane that costs $11. And she needs (counting to the tally marks she did not cross out) 1, 2, 3, 4.*
Interviewer:	How did you know that she needs $4 more?
Dalia:	*I put 11, and then I took away seven (points to the seven tally marks that she crossed out), and then she needs four [more] (points to the four remaining tally marks).*

Dalia's explanation is typical of the explanations students provided on the post-assessment in that she restated key features of the story, reiterated the steps of her strategy ("Puse 11, y quité los 7), and used language that described relationships between quantities ("Le faltan 4").

We also found an increased metacognitive awareness in some students and an eagerness to discuss their thinking about their problem solutions. For all students there was a greater ease in describing their solution strategy as they used the events of the story to structure their discourse (Celedón-Pattichis & Marshall, 2007).

Analysis of pre- post-assessment gains.

While a problem-by-problem comparison of students' performance on the pre- and post- assessments is complicated—the post-assessment included a broader range of problem types, and problems that were similar in structure involved larger numbers—for the 21 students who participated in both assessments, we compared their success on seven "matched" items. In all cases but one, the matched items were identical in structure and similar in context, but the post- assessment version included larger numbers (e.g., three pockets with two pennies in each [3x2] versus three bags with six marbles in each [3x6]). While students solved an average of only two of the seven items correctly on the pre-assessment, they solved an average of four of the matched items correctly on the post-assessment. In other words, even though the items were slightly more difficult, students solved *twice* as many. It is also significant that these gains were apparent across students who began the year at different levels of mathematical proficiency. In fact, the six students who demonstrated "limited number concepts and skills" on initial kindergarten screenings achieved the greatest gains, solving an average of 3.2 of the seven problems on the post-assessment, as compared to an average of less than one problem on the pre-assessment (a 45 percent gain). We find this result significant, as it demonstrates that students who begin the year with low levels of number-related skills can benefit substantially from reform-oriented mathematics instruction that is organized around solving and discussing problems.

CONCLUSION

We think the issue for Hispanic students in mathematics education is the opportunity they have during classroom instruction to engage in tasks that allow them to make sense of the mathematics and learn with understanding (Hiebert & Carpenter, 1992). When low-income Hispanic students have repeated opportunities to solve and discuss mathematical problems, in classroom environments that draw upon cultural and linguistic resources to support their understanding, the gains are impressive, as this study documents. The aim of our research was not only to document students' learning, but also to better understand the specific instructional practices and cultural and linguistic resources that teachers employed to support these young students.

Furthermore, we acknowledge that class is an issue in mathematics education. Specifically, some researchers have argued that children of low SES backgrounds experience difficultly with the open-ended nature of problem solving, reasoning and communication (Lubienski, 2000). While not explicitly promoting a deficit orientation, the idea does assume that some students lack the richness of outside experiences and the sense of personal efficacy necessary to deal with the ambiguity inherent in problem-solving situations. Boaler (2002a) counters these arguments and refocuses our attention on the critical role of teachers' practices. She notes that students' interactions with problem-solving-based curricula are mediated by teacher moves that either support student understanding or leave students at a loss for what to do next. Because of the crucial mediating role of the teacher in promoting equity, Boaler argues that the field of mathematics education "is in need of additional examples of particular teaching practices that reduce inequalities" (p. 240). She notes that there is a lack of research on teaching practices with low SES students that help them develop the sociomathematical norms necessary to be successful in mathematics classrooms that are structured around solving and discussing problems.

We agree with the urgency of Boaler's call, and present this study as one example of research that documents teaching strategies that support the successful participation of Hispanic students, many of whom are English language learners and/or come from low socioeconomic backgrounds, in reform oriented mathematics instruction. Not only do we contribute a detailed analysis of how particular instructional practices may have supported students' learning, but we focus explicitly on practices that attend to the critical role of culture, language, and participation. While we can only conjecture about the impact of a specific practice, the learning gains that students demonstrated on post-assessment measures suggest that repeated opportunities to participate in classrooms organized around these practices were consequential.

Finally, we conclude with several suggestions for future research related to young Hispanic students' mathematics learning. Future studies might investigate the following issues: 1) Differences in Hispanic students' mathematics performance given different linguistic environments (i.e., classrooms in which mathematics is taught mainly in Spanish versus classrooms where mathematics is taught in English only); 2) The longitudinal impact of meaningful problem-solving experiences in kindergarten as Hispanic students progress through the elementary grades, and 3) The impact of repeated opportunities to solve and discuss problems on young children's mathematical identities (their relationships to the discipline) and their emerging sense of mathematical agency.

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Understanding English Through Mathematics: A Research Based ELL Approach to Teaching All Students

Joyce Fischer • Texas State University-San Marcos
Robert Perez • Homer Hanna High School

INTRODUCTION

This paper is a qualitative and quantitative study of research methods that have been "discovered" and are being used by some in-service teachers of English Language Learners (ELLs) in the Texas Rio Grande Valley in everyday classroom situations. In a focus group study, these teachers were asked if they had any formal professional development training in working with English Language Learners, and all of them replied that they had not. This paper focuses on the work of one high school teacher as an example of the professional quality of all teachers, in general, who work hard—in the absence of formal training—to "discover" effective teaching methods and strategies simply because they are committed to promoting high achievement and success for each and every student. Robert Perez, the high school teacher described in this paper, has had remarkable success in mathematics with ELL students in Texas at the high school level. The quantitative part of the study describes Robert's TAKS test scores representing student success rate compared to his school colleagues success rate. Robert's approach is unique, in that, while others are attempting to teach mathematics in English, Robert teaches English through the mathematics concepts that the students already know and the ones they are learning.

Joyce Fischer, a university researcher, will point out the links to the body of research that support the methods and strategies (Joyce likes to call them "teachniques") used by Robert, who as a teacher was never made formally aware of this body of research. The researcher has been working with teachers in the Texas Rio Grande Valley region for several years and was very impressed that widely recognized research methods and strategies were being "discovered" and applied by teachers in classroom environments. Robert was a graduate student at a summer program in the Valley and, as a result of his teaching success in this program, became a master teacher in the math camps that were being conducted each year by Dr. Fischer.

Teachers' success, in Texas, is measured by their students' success in the courses. Due to the sensitive nature of teacher and student test results for individual schools and restrictions on reporting these results, specific data cannot be presented here. Instead, student success will be discussed in general terms over a relatively broad span of years without specific identifying factors. The sections below describe the background related to the problem ELL students have as the lowest performing group of students in mathematics in Texas, provide specific language learning practices for ELL students, introduce the featured teacher and his students, discuss pertinent teaching strategies, and conclude with lessons learned throughout this action research project.

BACKGROUND

According to national estimates released by the United States (U.S.) Census Bureau in May 2006, the nation's minority population totaled 98 million in 2005, or 33 percent of the country's total of 296.4 million people. Hispanics continue to be the largest minority group at 42.7 million (14.4 percent of the total U.S. population and 43.6 percent of the nation's minority population) with a 3.3 percent increase in population from July 2004 to July 2005. Hispanics accounted for almost half (1.3 million people, or 49 percent) of the national population growth of 2.8 million and are the fastest growing ethnic group in the U.S. Of the increase of 1.3 million Hispanics, 800,000 was due to natural increase (births minus deaths), and 500,000 was due to immigration (U.S. Census Bureau, 2006). The Hispanic population in 2005 was much younger than the national population with a median age of 27.2 years compared to that of the population as a whole at 36.2 years. About one-third of the Hispanic population was under 18, compared to one-fourth of the total population.

In August 2005 in Texas, minorities represented 50.2 percent of the state's population; Hispanics were the largest minority group with a growth rate increasing at a much faster rate than any other group in the state. As the Hispanic population increases, the group of students in Texas schools that are classified as English Language Learners (ELLs) or Limited English Proficiency (LEP) learners also grows. According to the U.S. Department of Education's Office of English Language Acquisition (OELA) (www.ncela.gwu.edu/stats/3_bystate.htm), the rate of growth in numbers of ELLs in Texas exceeds the rate of growth for all students (see Figure 1).

Growth from 94-95	LEP Enrollment	Growth from 94-95
0.0%	457,437	0.0%
5.1%	478,297	4.6%
6.4%	513,634	12.3%
5.1%	507,262	10.9%
4.4	533,351	16.6%
5.4%	554,949	21.3%
7.2%	570,022	24.6%
9.0%	601,791	31.6%
12.4%	630,148	37.8%
14.2%	660,707	44.4%
16.3%	615,466	34.5%

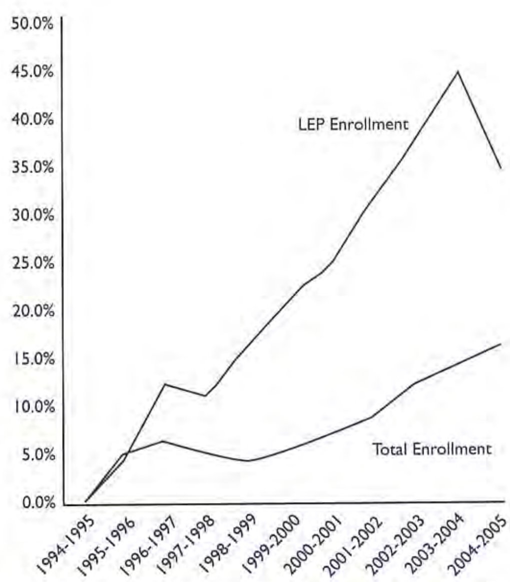

Figure 1: Growth of LEP Enrollment in Texas from 1994–1995 to 2004–2005

The percentage increase of LEP enrollment over this 10-year period of time was more than twice that of the total enrollment in Texas schools.

The 2000 census showed that 81.5 percent of the Hispanic population in Texas spoke a language other than English at home. ELLs consistently perform lower on the TAKS test than any other groups of Texas students, lower than economically disadvantaged students and even lower than special education students (see Figure 2).

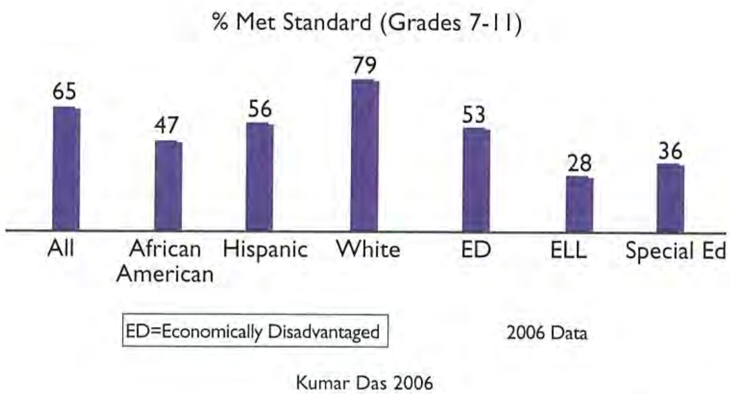

Figure 2: Categories of Students Meeting TAKS Standards: 2006 Data

LANGUAGE LEARNING AND ELLS

Although second language acquisition involves many aspects, some of these aspects are more difficult, if not impossible, to learn or change, especially at the later stages of the educational process, such as auditory memory, personality, and auditory discrimination (Collier, 1987; Cummins, 1996). When ELLs arrive in Texas at a later stage in the educational process, especially at the secondary level (grades 9-12), they face an even greater challenge to learn academic English and course content simultaneously (Short & Boyson, 2004). With ELL students, prior knowledge is totally embedded in the Spanish language: so, at the secondary level of education, success in mathematics is very dependent on students' ability to connect the English words used in the classroom to their knowledge of the Spanish language in order to retrieve the already known math concepts. Current Texas Education Agency policy states that all exit level LEP students classified as recent immigrants may receive a "LEP postponement" from taking the exit level TAKS test in English for one year but must take the test in English the second academic year that they are in the country. A worst case scenario that demonstrates the emphasis and importance put on language acquisition is illustrated by a student who has no prior knowledge of the English language, comes to the U.S., and enters school in May of a given year. That student would have to take the TAKS test in English the following year with only one school year of actual English language background. If that student was taking the 11th-grade exit test and did not pass it, he/she would be placed in TAKS remedial courses during the senior year.

Maria de Lourdes, a Hispanic teacher with whom the researcher has collaborated in several projects and who is now bilingual in English and Spanish, struggled with going to school and learning English herself because she entered the educational process at a later stage. She describes the process this way: "The students have to carefully listen to the question that the teacher is asking in English, translate that into Spanish (mentally), think about what the question means and answer it in Spanish (mentally), translate that answer into English (mentally), and then finally say this answer. By the time they have finished this process, many teachers have already moved on and the student is left behind." (Video interview, June, 2004.) Considering the amount of work involved in this process, it is no wonder that researchers in the field believe that ELL students should get extended wait time (Stahl, 1994; Green, 2005), more time on task (National Research Council, 2000, 2003), smaller class size (Achilles, 1999; Pedder, 2006), and extra-literacy skill building strategies like those used in the sheltered instruction programs

(Echevarria, Vogt, & Short, 2004; Echevarria, Short, & Powers, 2006), such as graphic organizers (Lesaux, Koda, Siegel, & Shanahan, 2006), especially for problem-solving situations. Improving the academic achievement of the ELL population of students is an essential part of raising the mathematics achievement level of all Texas students. Stephen Murdock, the official demographer of the state of Texas, believes that "the successful education of ELL students is essential for the future economic well-being of Texas." (Video interview, January 2007.) To be successful, ELLs need classes that have goals aimed at both language and content.

One of the instructional strategies that addresses language and content requirements is known as content-based literacy or content-based instruction (CBI). CBI is defined as "curriculum concepts being taught through the foreign language ... appropriate to the grade level of the students" (Curtain & Pesola, 1994, p. 35). Many researchers such as Papai (2000), Hammrich and Ragins (2002), and Short and Boyson (2004) have long argued for this basic well known approach to teaching ELLs. Research results have shown that CBI results in language learning, increased motivation and interest levels, content learning, and even greater opportunity for employment (see Met, 1999; Stoller, 2002) and works especially well for the linguistically harder to teach population of adult learners (Byrnes, 2000).

Classroom implementation of CBI can take many forms. One paradigm that has proven to be effective for students is a form of the mastery learning model (MLM) (Green, 2005), which delivers the content matter in the following order: presentation, guided practice, independent practice, review, assessment, re-instruction (when necessary), and reinforcement. Combining teachniques such as the MLM with a teacher delivery model called reciprocal teaching (Hernandez, 1991; Klingner & Vaughn, 1996)—which is a scaffolding approach that involves using small groups of students (National Research Council, 2000, 2003)—extends and builds on student content knowledge (Evan & Lappin, 1994; Cummins, 2000) through questioning, clarifying, predicting, and summarizing. When these two methods are integrated with contextually based problem solving (Barron, 1991; Daniell, 1999; Netten & Germain, 2000), the learning results strengthen and deepen the ELL student's cognitive mathematical ability that is essential for the development of higher order thinking skills (Zohar, 2003, 2006) while teaching each student to analyze and reflect on his/her own learning (Hillocks, 1995; Halpern, 1998).

THE TEACHER AND THE STUDENTS

Robert Perez is a high school teacher who has been teaching mathematics for 12 years. He started teaching mathematics at the middle school level and for six years took turns teaching seventh- and eighth-grade mathematics as well as pre-A.P.; pre-algebra; and pre-A.P. algebra I. He then moved to the high school level, teaching such subjects as algebra I, algebra II, geometry, math models, and pre-cal. He is currently teaching algebra I and TAKS review classes (these classes are designed to help students who have previously scored poorly or are at risk of scoring poorly on the TAKS test). The schools where he teaches now and has taught in the past are all located in Texas, in a city on the border with Mexico, in which the ESL population in the city's schools is in the eighty to ninety percentile range. On average, anywhere from one-fourth to three-fourths of these students spend their weekends and holidays in Mexico, and some of them actually live and commute daily or weekly from Mexico. On average, one-third of the student population in the city is classified as ESL I or ESL II, and this is the population that is at the highest risk for not completing their education or for dropping out early (Cortez, 2007). These are the students that Robert has been teaching for most of his 12 years as a teacher.

Being an ESL student himself, Robert speaks Spanish as his native language. Since he only heard and practiced English at school, he has been able to empathize with his students and use techniques that he developed himself when he learned English. When he went to school, the only technique used to teach English to ESL students was a total immersion approach. It was thought at that time that by saturating students with English only, they would have to learn the language. Because this method did not work for him, he had to develop strategies on his own.

The first six years of his teaching career, Robert worked with an academic team composed of the five core area teachers who were all teaching the same group of students. This team worked together primarily to deal with discipline problems, and Robert worked, by himself, striving to come up with ideas that he believed would ensure academic success for the students. Later, after Robert transferred to the high school level, he got together with two other mathematics teachers to endeavor to improve the overall pass rate for students taking the algebra end-of-course exam. One of the teachers was a proponent of dual language instruction. Until then, Robert had always taught exclusively in English and met with students who did not understand at a later time and repeated instruction in Spanish. The other teacher was also a proponent of peer tutoring. These teachers combined all of their practices, spliced them together, and discussed the classroom results to come up with what became a very successful program for their students. Robert continued to refine the program himself over a three-year period of time, watching student pass rates on the algebra end-of-course exam climb from the low teens to the mid-seventies and hold at this level or above for the next three years.

Working with his colleagues, Robert was able to develop the program that he now uses in his classroom, part of which is presented in this paper. His program has been successful to the point that his students' pass rates for the TAKS test have held steady in the 70 percent – 90 percent range compared to an average pass rate of 15 percent – 25 percent at his school. He uses this program successfully in all of his classes at all grade levels. He has further discovered that even if the students are native English speakers, they can still be weak in connections from English to mathematics and that this program supplies the needed specialized reinforcement in these content areas.

STRATEGIES

Prior Knowledge

One critical strategy that Robert uses when working with ELLs is to make use of their prior knowledge. Students come to the classroom with skills, abilities, and knowledge from cultural, familial, educational, and environmental backgrounds. Since all students are unique, classroom teachers must find ways to guide their learning by looking for common threads in their prior knowledge. The importance of accessing prior knowledge was first visualized in 1975 when the linguist Roger Shuy proposed an iceberg metaphor to represent Chomsky's (1966) "deep to surface structure" analogy to describe the language-learning method used by students who were struggling with learning a second language by translating knowledge from their first language (L1) to the second language (L2). This metaphor depicts two icebergs (L1 and L2) that from above the surface of the water look disconnected and unrelated. However, when viewed from beneath the surface of the water, they have a huge overlapping base. This base represents all of a student's content knowledge that can be thought of as trapped, frozen, or fossilized in a former language (L1) (see Figure 3).

Jim Cummins, a well known expert on second language acquisition, used the Shuy iceberg metaphor to talk about language proficiency in general. He extended the iceberg model by defining the overlapping base of his two icebergs (L1 and L2) as the Common Underlying Proficiency (CUP) (see Figure 3) and stated that any language must operate through this central processing system. He then subdivided that region into two broad categories of language proficiency: Conversational language and Academic language. Conversational language is the language needed to perform interpersonal communication skills in daily life, which Cummins refers to as the Basic Interpersonal Communicational Skills (BICS) or peer-appropriate language, and Academic language is the language needed to pursue formal academic schooling, which Cummins refers to as the Cognitive Academic Language Proficiency (CALP) or classroom- appropriate language (Cummins, 1980). He finds that BICS can be achieved by most students in about two years, while CALP requires five to seven years of study.

One technique that Robert remembers using as an ELL was starting with a simpler language and working his way up (progressing from BICS to CALP). He learned how to read by reading comic books, so he introduces those into his classrooms followed by novels with pictures and only then going to full text novels. He always tries to have high interest topics (contextual applications) for students to read and provides a library for them with all the current highest interest books both in English and in Spanish. This library represents a foundation of words that the students can access to increase basic language skills and build vocabulary. Some examples of words that the students might encounter in this body of literature and their definitions in the two languages are: *factor* 'factor'—this word means the same in Spanish and English and refers to a reason or a contributing cause as to why something occurred; *tabla* 'table'—means a piece of wood in Spanish and a place to eat in English; *razon* 'ratio'—in Spanish, this word means reason, or a reason why and in English, a proportional relationship between two different quantities.

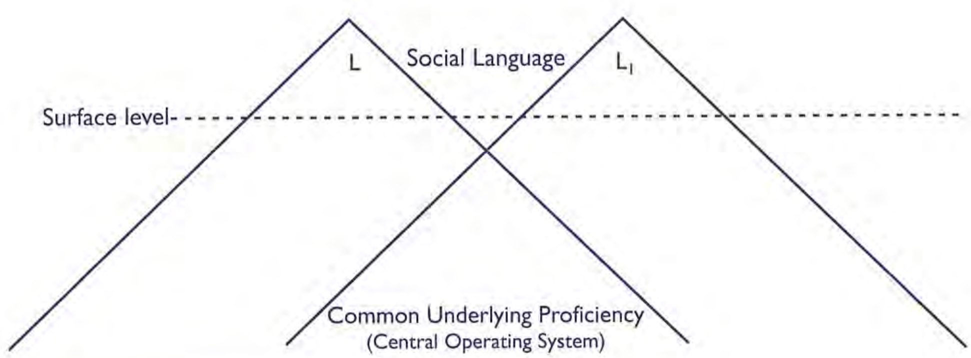

Figure 3: Shuy's Iceberg Metaphor

Techniques that retrieve ELL students' prior knowledge include using language cognates, brainstorming, and contextualizing the content.

Cognates

One way to teach vocabulary and reading that has been promoted for many years is to begin with language cognates (Gallegos, 1979; Nagy, 1992; Martinez, 1994). Language cognates are words that have a common origin, usually in some earlier language, so connections between the word in L1 and the word in L2 are easier to make (Garrison, 1990; Carroll, 1992; Kroll & Dussias, 2004). For instance, 'night' in English and *noche* in Spanish are cognates. Researchers estimate that as many as one out of three words in English and Spanish are cognates and that using cognates is very beneficial to student learning (Shillaw, 1995; Laufer, 2003).

The first technique that Robert can remember using to learn English as a youngster was to look for all the words that looked the same and in many instances sounded the same (cognates). This technique represents the initial step that he uses to build the foundation for his teaching method goal, using mathematics to teach English. Robert says that the use of cognates in the classroom is one of the most important approaches that can be used; it allows students to connect with prior knowledge in their first language. By allowing the students to see the cognates, some of the fear of the mathematics language and the English language is eliminated (see Table 1).

Table 1: Cognates: Cognatos

sum : sumar	addition : adicion
minus : menos	equal : igual
multiply : multiplicar	total : total
perimeter : perimetro	area : area
diameter : diametro	cube : cubo
horizontal : horizontal	vertical : vertical (la)
velocity : velocidad	distance : distancia
circumference : circumferencia	solve : resolver

Table 1 represents just a small list of the cognates that is presented to the students, and the students are encouraged to go out and look for cognates on their own, whether on the Web or in books. Robert reports that this strategy also helps students that are not ELLs because it allows them to connect with their own prior knowledge and broadens their language base.

Brainstorming

Another important technique that works well for all students, and especially for ELLs, is brainstorming. Brainstorming is a method that allows students to come up with ideas spontaneously and then reflect on and refine the ideas later without permitting negative comments or judgments. Brainstorming is a dynamic way to stimulate originality and creativity among students (Chirumbolo, Mannetti, Piero, Areni, & Kruglanski, 2005). As with most strategies, there are ideal ways to get the most effective results such as careful choice of group size (Lowry, Roberts, Romano, Cheney, & Hightower, 2006), initial individual input (Brow, Tumeo, Larey, & Paulus, 1998; Barki & Pinsonneault, 2001), and ordering and combining brainstorming with convergent and divergent exercises (Coskun, 2005). The idea of combining brainstorming and limited group size has flourished in multidisciplinary and mentoring arenas resulting in the formation of specialized groups that are known by many names such as focus groups (Goodfellow & Sumsion, 2000), virtual teams, (Martins, Gilson, & Maynard, 2005), and group support systems (Rains, 2005).

After the initial step of discussing cognates, the second step in the plan is to practice translating English phrases into mathematical expressions and equations (BICS to CALP). Robert begins the process by placing an operation symbol on the board and asking the students as a class to brainstorm and come up with words that it represents. The answers may be in English or in Spanish. Table 2 shows an example of one class's brainstorming for addition.

Table 2: Addition (+): Brainstorming the Concept of the Operational Symbol

mas : plus	total : total
suma : sum	perimetro : perimeter
mas que : more than	incrementar : increase
mayor que : greater than	ganar : gain
subir : rise	entero : whole
mayor : older	todo junto : all together
deposito : deposit	mas alto : higher

As the answers are given by the students, Robert provides either the Spanish or the English translation. This second step is continued the next day or several days later with the minus symbol (see Table 3).

Table 3: Subtraction (–): Brainstorming the Concept of the Operational Symbol

restar: subtract	sustraccion: subtraction
menos : minus	quitar: take away
sacar dinero : withdraw	lo opuesto: opposite of
remover : remove	mas chico: younger
menos que: less than	caer : fall
perder : lose	diferencia : difference
vaciar: empty	bajar : lower

A table is constructed for each of the basic operations (addition, subtraction, multiplication, division) and for the concept of equality. As a class, the students practice repeating each table about every two days. Each table is copied by the students into their notebooks so that they can refer to it anytime they need it. This brainstorming approach allows the students to see the words that signify that particular operation and thus gives them the word recognition that will be necessary for them to be successful in decoding word problems involving that operation.

Robert has found that there are many native English-speaking students who do not know all the words that are presented here and, therefore, benefit from this exercise as much as the ELL students. After the four basic operations and the equals sign have been covered (usually in the span of a week or two), then all the words are gathered and organized into a tree diagram in English only (see Table 4).

The students copy these words into their notebooks for future reference and for any future work with word problems.

Table 4: A Translations Tree Diagram

Translations

+	–	X	÷	=
add	substract	multiply	divide	equals
sum	minus	times	into	same as
total	difference	product	quotient	results in
greater than	less than	by	half (÷2)	as good as
increase	decrease	double (X 2)	share	equivalent to
deposit	withdrawal	square	split	comparable to
increment	diminish	half (X 1/2)	partition	
more than	narrow	triple (X 3)	segregate	
grow	fewer	quadruple	take apart	
accumulation	smaller	dual	divided by	
plus	shorter	binary	break up	
larger	take away	twice	separate	

Multiple Meaningful Contexts and Learning Strategies

When working on vocabulary learning, there are two basic ways to teach word meanings: through meaningful context such as pictures (Nagy, Herman, & Anderson, 1985; Sternberg, 1987) and through learning strategies, but a combination of both methods is the most effective (Carlo, August, & Snow, 2005; Echevarria, Short, & Powers, 2006). Contextual teaching and learning strategies in mathematics are manifested through contextually based problem solving.

The next and more advanced learning level in Robert's method involves the practice of translating mathematical expressions and equations into English phrases, so he adds to the words in Table 4 three "special" words that he gives to the students. These words are presented as being "special" or "magical" because they change the order of a mathematical expression and place the beginning words at the end of the expression and the ending words at the beginning. These words are: than, from, and into. The students are taught that when these words are encountered in a word problem, the phrase is reversed when written as a mathematical expression. The example below illustrates this process:

Five is subtracted from an unknown number
X – 5

Although the five comes first in the English word expression, it is placed at the end when written in a mathematical expression. After mastering this step, all of the students are able to translate simple word expressions into the proper mathematical representation such as:

Five less than an unknown number
N – 5
Ten more than twice x
2x + 10

Even though the expressions are simple, Robert takes care to make sure that the complexity of the problem grows. He always starts at the most basic level and increases the difficulty as he progresses through the content objectives. Paying special attention to how the problems are presented and making the beginning ones easy enough for all of the students to understand, he builds the complexity so that even the most advanced students will be challenged by the problems towards the end of the semester (see Figure 4).

Figure 4: Robert checks each student's work for understanding

After the students have gotten proficient at translating word problems into mathematical terms, they are then given a simple mathematical expression and asked to translate it into an English phrase. At first, they may need a little prompting, but soon they should be able to provide six or more ways of translating any mathematical expression. The following are examples of the students' work with the operations of addition and subtraction:

4x + 7
Four times a number plus seven
Four times an unknown number increased by seven
The sum of four times an unknown number and seven
The total of four times an unknown number and seven
Seven more than four times an unknown number
Seven added to an unknown number quadrupled
X – 3
A number x minus three
The difference of an unknown number and three
Take away three from an unknown number
Three less than an unknown number
Three subtracted from an unknown number
An unknown number decreased by three

As can be seen, even here, the complexity of the mathematical expressions is growing which the students recognize themselves. Robert asks for at least six ways of stating the same idea, makes sure that the students use many different symbols, and encourages the students to provide at least three different ways of stating each concept in their daily practice problems.

While students are engaged in this learning process, they must also be introduced to inequalities. Inequalities represent a challenge for the students, and Robert takes the time to explain how much difference one word in a sentence makes in the translation of a mathematical expression. The first time the students encounter this problem is when they have to differentiate

between	Five less an unknown number $5 - x$
and	Five less than an unknown number $x - 5$

This concept is difficult for the students to comprehend, and the teacher has to take time with individual students to make sure that they understand it (see Figure 5). This challenge is compounded by the fact that the wording used in the second expression has no direct translation in the Spanish language.

When presenting inequalities, this problem arises again. For example,

	Five less an unknown number $x - 5$
and	Five less than an unknown number $5 < x$

Figure 5: Robert examines a student's written work for precise word usage

Once again, Robert must take the time to make sure that the students understand the concept. The process presented here takes place over an entire year and is integrated with the regular objectives that the curriculum requires.

Robert pays close attention in his instruction to the progressive intricacy of the problems when presenting them to the students, and students must be guided to think deeply about them (see Figure 6) in order to achieve the next difficulty level, which involves being able to break the mathematical word problems into simpler parts and solve them. As the process evolves, complexity continues to be added. For example,

The sum of a number and four is twenty.
Find the number.

If five times a number is decreased by twelve, the difference is eighteen.
Find the number.

Another concept that is woven into this process at this stage is geometry.

The length of a rectangle is four times the width. If the length is 15 feet, what is the width?

The length of the rectangle is two times the width. If the perimeter is 18 feet, what is the width?

As can be seen in the examples above, the complexity grows, and no objective is neglected. In the next set of examples, decimals and fractions are also included in the problems.

If three-fourths of a pound of coffee costs $3.75, what does one pound of coffee cost?

The sum of three consecutive odd integers is –79. Find the integers.

Separate 49 into two parts so that one part is one more than twice as much as the first part. Find each part.

Figure 6: Robert questions a student, guiding her to reflect on her own work

Each day the difficulty of the problems grows, but by this time some students can start to lose interest in these types of problems. Robert's solution is as simple as adding the students' names to the problems (personalizing the math).

One example is:

Maria has four more CD's than twice as many as
Julio has. Altogether, they have sixty four CD's.
How many CD's does Maria have?

Another idea that Robert uses to offset boredom is to have the students create their own word problems with solutions (see Figure 7). He then chooses five of the best problems and creates quizzes to use with the students, making sure to give the students credit for their problems. He has found this to be a very successful way to keep students motivated and learning.

Figure 7: A student works at creating personalized mathematics word problems

These last four contextualized examples given here would be appropriate word problems for any challenging standardized test. Robert presents these problems to the students toward the end of the program, and by this time they are able to solve them.

The sum of $145 was divided among 3 people so that the second received $5 less than twice as much as the first, and the third received $5 more than the second. How much did each person receive?

Juan works at a fast food restaurant and received $8.75 in tips one afternoon, all in quarters, dimes, and nickels. There were five fewer dimes than quarters and ten fewer nickels than dimes. How many of each coin was there?

The sum of the degree measure of two supplementary angles is 180°. If the measure of one of two supplementary angles is six degrees less than twice the measure of the other, find the measure of each angle.

The sum of the degree measures of the angles of a triangle is 180°. Two of the angles of a triangle have the same measure and the third angle measures 45° more than each of the other two. Find the measure of each angle.

Robert's views are that traditionally mathematics has been taught independently of any other discipline; that most people think of it as being a universal content topic, a language unto itself that is understood and taught in most countries in almost the same style. He has found that by using mathematics and the concepts that most ELL students already understand, teachers can enhance English language recognition and retention. From personal experience Robert has learned that this process improves students' understanding of the English language and their standardized test scores.

CONCLUDING REMARKS

The authors believe that there is an "academic gap" in the education system, that is, a "connection of ideas" gap between teachers at different grade levels, especially college and PreK-12 in-service teachers, that needs to be remedied by a mutual exchange of educational information, such as research and classroom practice, through established strategies that promote much needed mentoring and communication among teachers at all grade levels: a P-16 collaboration (see Figure 8).

Figure 8: Joyce applies research to problem solving with in-service teachers and students

To strengthen the bond between PreK-12 teachers and university researchers, the Texas State University System Mathematics for English Language Learners (TSUS MELL) Initiative team has produced a research-based document, the Classroom Practices Framework (CPF) (2005), that addresses the specialized needs of this rapidly growing group of students and describes a framework for best practices in the classroom that contains ELLs (see the abbreviated version of the CPF in the appendix to this paper). Robert's teaching approach is a concrete demonstration of the research embodied in the CPF. His teaching style reflects this research naturally, and other teachers will benefit from seeing the body of research effectively applied to actual classroom settings from a practicing teacher's viewpoint.

When we recall Maria de Lourdes' description of how ELL students approach questions asked by the teacher, it is easy to understand the difficulties involved in connecting content when that content is embedded in two different languages that must operate through the same universal processing mechanism (see also Rossell & Baker, 1996; August & Hakuta, 1997; McQuillan, 2005). For this reason, ELL students must progress academically in both languages, professionally known as facilitation theory, in a bilingual additive learning environment to ensure maximum learning capacity (Cummins, 1979; Tinajero, 2005). In fact, years of longitudinal studies across the U.S. with diverse populations of ELLs by Wayne Thomas and Virginia Collier have demonstrated that not only is dual language the best instructional delivery method possible for ELLs, but it is also a very effective way to teach all students (Collier, 1989; Collier & Thomas, 2002, 2004; Thomas & Collier, 1997a, 1997b, 2003, 2005).

An especially effective way to close these existing academic gaps is for in-service teachers at all grade levels and university researchers to work closely together through action research projects. It is our belief that this sharing process will enable all educators to find, discuss, apply, disseminate, and implement best practice strategies, such as the teachniques presented in this paper that have been shown to be effective with ELL students, and to guide all students to successful learning of mathematics as well as other academic disciplines.

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APPENDIX

MELL Classroom Practices Framework

Participating TSUS Institutions Include:

Angelo State University
Lamar University
Sam Houston State University
Sul Ross State University
Texas State University

for the Math for English Language Learners Initiative
the Texas State University System and
the Texas Education Agency Collaborative

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LEARNING ATMOSPHERE AND PHYSICAL ENVIRONMENT

A caring classroom atmosphere of mutual respect and support is facilitated by the teacher who:

- Knows each child as an individual
- Embraces languages, customs, and cultures of ELL students
- Provides culturally rich learning materials
- Encourages self-expression and provides positive recognition
- Builds student confidence and esteem
- Fosters an emotionally safe environment that allows students to feel secure and to take risks.

The classroom is visually rich to support student learning. It:

- Incorporates displays of student produced work, whenever possible
- Is colorful and thought stimulating
- Contains pertinent, real-world information and applications
- Reinforces math-specific vocabulary and concepts
- Provides color-coded learning supports when appropriate.

Room arrangement facilitates student interaction and group work.

INSTRUCTIONAL PRACTICES

Instructional practices foster cooperation and collaboration.

Concepts are presented accurately, logically, and in engaging ways.

Multiple representations incorporate mathematics learning levels: concrete, semi-concrete, and abstract.

The teacher employs student-centered instructional practices:

- Approaches content from a concept-oriented constructivist method
- Surrounds students with different modalities (e.g., aural, visual, kinesthetic)
- Connects new concepts to prior learning
- Encourages students to refine and reflect about their own work and verbalize concept understanding "in their own words"
- Chooses homework to optimize individual content development
- Provides extra help and resources on an individual basis.
- Students are frequently partnered with peer learners to enhance learning opportunities:
- To develop math content
- To aid English language development
- To insure sustained active participation in the class
- To welcome new students into an established learning community.

Instructional activities are varied and support diverse learning styles and multiple intelligences, including for instance:

- Frequent use of models
- Music as a motivator and anchor
- Mind maps, poster-walks, and word walls
- Key vocabulary and cognates presented in different forms
- Vivid adjectives.

MATHEMATICS CONTENT AND CURRICULUM

Glossary of mathematical terms is always available for reference.

Content is aligned to appropriate grade-level, mathematics TEKS and professional standards.

Content is based on diagnosed student needs.

Content is systematically designed to incorporate sound learning principles.

- To incorporate increased complexity
- To present a cohesive big-picture through chunking
- To connect concepts through bridging and scaffolding
- To emphasize multidisciplinary understandings
- To reflect on inherent patterns by comparing and contrasting concepts.

Curriculum is challenging, relevant, age-appropriate, and well-paced:

- To include contextually-based problems
- To incorporate student realities
- To involve interactive problem solving.

LANGUAGE PRACTICES

Language support is offered without supplanting English instruction.

Support is aligned with student's diagnosed language needs.

Language used is appropriate to age and grade level and presented in a socially meaningful context.

Mathematics-specific vocabulary is explicitly taught and reinforced through repetition.

Teachers are knowledgeable about the second language acquisition theories and best practices embodied in Texas Administrative Code, Title 19, Part II, Chapter 128.

Ideally, dual language instructional support should be offered.

When dual language teachers are not available, sheltered instruction should be utilized to provide strong language support by addressing content through ESL.

FAMILY AND COMMUNITY INVOLVEMENT

Schools connect to student's family-life by embedding contextual experiences and skills in teaching and curriculum.

Projects are relevant and promote family interaction.

Opportunities are available for English-speaking higher grade-level students to mentor ELL lower grade-level students either in an in-school or afterschool program, as appropriate.

Teacher engages in frequent communication with families:

- About activities and events in which parents can participate
- About student progress.

Teacher utilizes services provided by a community liaison and is knowledgeable about community resources.

Parents are informed about the benefits of using their most cognitively advanced language at home.

ASSESSMENT OF STUDENT LEARNING

Classroom assessment is designed to foster student success.

Assessment methods allow students frequent opportunities to demonstrate mastery in a variety of ways.

Various assessment techniques are used to measure student understandings.

Grades are oriented to promote and emphasize valid step-by-step logical reasoning processes.

Assessment data and results shape instructional planning.

Flexible time allotments are given to demonstrate concept mastery.

The Travieso Activity

Carl A. Lager • University of California, Santa Barbara

INTRODUCTION

To address the achievement gaps in the United States (U.S.) between Hispanic students and White and Asian students in mathematics, contributing factors must be identified, understood, and counteracted. One overarching factor for Hispanic English Language Learners (ELLs) is language. Because mathematics instruction, curriculum, and assessment are predominately English-only throughout the U.S., students with lesser experience and facility with English natural language, English mathematical language, and mathematical discourse in English often encounter greater difficulty making and communicating mathematical meaning than their non-ELL counterparts, regardless of ethnicity (Lager, 2006; Brenner, 1994).

In fact, the further one moves toward grade 12 along the K–12 mathematics continuum, the greater are the language demands in terms of lexical complexity, density, and verbiage to generate and share new mathematical understanding. Unfortunately, despite the increasing language load, in my professional development experience (as a participant and facilitator) most secondary mathematics teachers tend to be less inclined than their elementary colleagues to acknowledge and explicitly address the language needs of their students. Being unaware of the depth and complexity of their students' needs (Richardson & Wilkinson, 2006), inadequately trained to address them (Gándara, Maxwell-Jolly, & Driscoll; 2005), and neutral toward their students' plight (Reeves, 2006) are three common contributors to many secondary teachers' collective reticence.

As a result, many mathematics teachers of Spanish-speaking ELLs in the United States are ill-equipped to identify and meet the unique needs of their students (Khisty, 1995). Because most of these secondary mathematics teachers have never had to learn mathematics content in a non-English speaking classroom, they do not have firsthand knowledge of how ELLs engage with mathematics instruction or textbooks. For example, though aware of the need to teach mathematical vocabulary (via word walls and the like), teachers often do not fully grasp that to access mathematics problems, ELLs must also make meaning from everyday language expressions, word order, and the mortar that connects distinct language chunks into a coherent text (Fillmore & Valadez, 1986).

Unfortunately, this lack of language awareness sometimes leads well-intentioned teachers and mathematics educators not only to underestimate ELLs' mathematical abilities and learning challenges, but wrongly view ELLs through a deficit lens. For example, though Jasper & Huber's (2005) ELL mathematics teaching guide provides some specific assistance, the given English language development strategies stay on the surface, focusing almost exclusively on defining/showing mathematical vocabulary terms (in English and Spanish) and using sample mathematics items that use Hispanic first names and "culturally relevant" situations (lawn-mowing business, selling sodas, etc.), not on modeling meaning-making strategies. Further, stating that "Many Hispanic children come to school with less developed scholastic skills" and that "some Hispanic parents exhibit low educational expectations for their children and do not emphasize academic achievement" (section 2) reinforce existing negative stereotypes.

Though some professional development activities already exist that speak generally to the language and affective needs of ELLs, very few are specific to mathematics and ELLs at the secondary level. Of those few, almost none discuss the mathematics register (Halliday, 1974; Cuevas, 1986; Spanos, Rhodes, Dale, & Crandall, 1988) or how multilingual students often shift back and forth between different language registers when doing mathematics (Moschkovich, 2000). Therefore, to promote ELL learning and achievement, a 90-minute, interactive professional development session was designed and facilitated to enable secondary mathematics teachers to encounter and feel, albeit temporarily, just some of the experiences and emotions that ELLs have on a daily basis in mathematics classrooms. Asked to solve one mathematics problem in a Spanish-only environment, participants systematically engaged with activity tasks that were intentionally created and sequenced to optimally parse out different language and affective aspects during the simulation and highlight their respective pedagogical implications in the concluding whole group debriefing.

The Travieso activity was initially created by a team of three professional developers[1] from the CMP[2]-funded English Language Development – Mathematics Content (ELD-MC) project, then later revised by a second team of developers[3] from the NSF- and USDOE[4]-funded Vermont Mathematics Partnership (VMP), then revised later solely by the author. Though each contributor brought classroom teaching and professional development experience to bear on the iterative development, facilitation, and self-evaluation of this activity, no one explicitly used either a theoretical framework to create it or an analytic framework to evaluate it. However, in hindsight, the developers collectively were aware of and addressed many of the concerns and components explicated in a professional development design framework (Loucks-Horsley, Hewson, Love, and Stiles, 1998; Loucks-Horsley, Love, Stiles, Mundry, and Hewson, 2003) and a professional development evaluation framework (Guskey, 2000). Therefore, ex post facto, both frameworks are used in this article to deconstruct and analyze the activity to illuminate findings, generalize lessons learned, and ground future work in this area.

Due to the lack of signed human subject waivers, there are no standard empirical analyses included in this work. However, Blau's (2003) novel classroom experiment methodology is used to reconstruct a typical Travieso facilitation, share participant reaction and learning, and voice facilitator commentary. The analysis and findings are drawn from the 15 facilitations to hundreds of elementary and secondary mathematics teachers, paraprofessionals, administrators, and policymakers across the country over the past six years.

Theoretical Framework

Updating their earlier work (Loucks-Horsley, Hewson, Love, and Stiles, 1998), Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003) formulated a linear, six-step professional development design framework for the development of mathematics and science teachers. The six steps are:

1. Commit to vision and standards
2. Analyze student learning and other data
3. Set goals
4. Plan
5. Do
6. Evaluate.

They posit that, in a one-to-one correspondence with steps 1–4, four key inputs should be addressed: knowledge and beliefs; context; critical issues; and strategies.

Loucks-Horsley et al. (2003) state that knowledge and beliefs are two key inputs into the professional development designing process. They define knowledge as research-supported information and beliefs about what a person thinks he or she knows or is in the process of knowing based on new information. They identify five key knowledge areas and beliefs that directly impact their framework:

1. Learners and learning
2. Teachers and teaching
3. Nature of science and mathematics
4. Professional development
5. Change process.

All five are reflected in the development and facilitation of the Travieso activity.

More broadly, best practices are generated, in part, from this knowledge base. However, best practices do not exist in a vacuum, but within contexts. Context is made up by the economic, social, and political realities at the school site and local, state, and national levels in which the professional developers operate. Because no two contexts are exactly the same, professional development should be tailored to meet the unique needs of each setting. Of the nine context clusters outlined in the framework, these three garnered the most attention during this activity's development: (1) Students, standards, and learning results; (2) teachers and teachers' learning needs; and (3) curriculum instruction, assessment practices, and the learning environment.

Across contexts, professional developers often face critical issues that affect the design and efficacy of their work. Of the critical issues Loucks-Horsley et al. (2003) consider when designing professional development, the ensuring equity principle was the focus of the Travieso activity. Challenging participant beliefs about the capacity and manner of ELLs' mathematical learning, heightening participant sensitivities to ELLs' challenges and strengths when learning mathematics, and facilitating full participant engagement and learning were explicit equity objectives for the activity. Because simple solutions don't exist for such equity issues, they often must be continually addressed throughout the professional development design cycle by iteratively planning, designing, implementing, and evaluating the activity.

Throughout this development cycle, just as effective classroom teachers regularly draw from the many instructional tools in their "toolboxes" to facilitate student learning, so too do professional developers use different strategies to facilitate participants' acquisition of activity goals. Of the 18 strategies for professional learning outlined in the professional development framework of Loucks-Horsley et al. (2003), workshops and immersion experiences were the two used for the Travieso experience. The Travieso workshop experience was the vehicle used to facilitate an immersion experience to raise teachers' awareness of language and affective issues that impact ELLs' mathematical learning, and prompt them to begin reexamining their beliefs about how their ELLs experience doing mathematics in their classrooms. Both strategies have their own key elements and implementation requirements.

Workshop strategy

According to Loucks-Horsley et al. (2003), the workshop strategy has three key elements and five implementation requirements. Clearly communicating the stated goals to the participants, ensuring that a leader facilitates participant learning, and providing group structures that foster a collegial learning environment are the three key elements. The five implementation requirements are: (1) Expert leader knowledge; (2) time away from the workplace for the participants; (3) an explicit curriculum; (4) access to resources and materials; and (5) incentives to attend. All key elements and implementation requirements are part of the Travieso activity.

Immersion strategy

The immersion in problem solving in mathematics strategy is undergirded by three assumptions, three key elements, and two implementation requirements (Loucks-Horsley, Love, Stiles, Mundry, and Hewson; 2003). The first assumption is that process and content are both components of mathematics. The second assumption is that teachers benefit from experiences as learners that are based on the same principles that they are expected to implement with their students. And the third assumption is that teachers need to possess an in-depth understanding of mathematics content and processes. Missing from, but related to, all three assumptions is one of the activity's foci, language. Because language is a component of mathematics content as well as learning and communicating processes (e.g., Pimm, 1987; Esty & Teppo, 1996; Garrison & Mora, 1999; Esty, 2000; NCTM, 2000; Chval and Khisty, 2001, Lager, 2006), teachers need a deep understanding of language, too. Therefore, this activity was purposefully designed to be a language immersion experience also.

Of the immersion strategy's key elements, the first is that the immersion is an intensive learning experience. Learning how students learn mathematics is the second. Working toward informing teachers' conceptions about mathematics and teaching is the third. Again, language, a critical component, is missing from elements two and three. Learning how students engage with language in mathematics and having teachers' conceptions about language in mathematics change are two key elements of this activity. Therefore, because almost any group of potential participants has a wide range of Spanish reading, writing, reading comprehension, listening, and speaking proficiencies, similar to the spread of their ELLs' English proficiencies, Spanish is the language of the activity. For an example of a similar professional development activity where nearly all the participants have little or no language proficiency in an immersion experience, see Anhalt, Ondrus, & Horak's (2006) Chinese example.

Having qualified facilitators and engaging in long-term experiences are the two implementation requirements (Loucks-Horsley, Love, Stiles, Mundry, and Hewson; 2003). Though the first requirement was met, the second was not. However, this activity would be an excellent introductory session to a long-term professional development series about identifying and addressing the language and affective needs of ELLs in secondary mathematics classrooms. In fact, both the ELD-MC and VMP projects have used it in this manner.

Analytical Framework

Guskey (2000) has created a five-level model for evaluating professional development. Moving from the most simple to the most complex, the levels are: (1) Participants' reaction; (2) participants' learning; (3) organizational support and change; (4) participants' use of new knowledge and skills; and (5) student learning outcomes.

At Level 1, participants' reaction, the evaluator gauges how participants considered their professional development experience, in this case the Travieso activity. Content, process, and context questions (National Staff Development Council, 1994, 1995b) are asked to appraise participant responses. Inquiring whether the explored ELL issues were relevant to professional responsibilities, if materials enhanced learning, and about the adequacy of the seating arrangements are examples of each of these types of questions.

At Level 2, participants' learning, the evaluator wants to know if participants attained the intended cognitive, affective, and psychomotor learning goals. Understanding how secondary ELLs might engage with a new mathematics problem, temporarily feeling their frustration, and attempting to adjust future instruction to meet their newly recognized needs are examples of each of these types of goals as well as their interconnectedness. Due to the self-contained nature of the intervention, as well as the kinds of available data, this self-evaluation will focus only on levels 1 and 2. Though the lack of a follow-up on levels 3 through 5, a formal evaluation plan, and an outside evaluator are limitations of this work, the activity is still powerful and the lessons learned are still viable and applicable to future facilitations of this and similar professional development.

Research Questions

To begin addressing and changing teachers' perceptions of their students'—and especially their ELLs'—mathematical abilities, language challenges, and attitudes toward learning, the following research questions are asked regarding the Travieso activity:

1. If a 90-minute intervention could be created to allow participants to temporarily experience a mathematics word problem like an ELL, what would it look like? What components would be included and why? How would it be facilitated?
2. If such an intervention was facilitated, how did the participants experience the intervention? What types of language, mathematics, and affect challenges did they experience? How were these challenges intertwined? How did they respond to those challenges?

Methodology

In his book, *The Literature Workshop*, Blau (2003) uses a classroom experiment methodology to systematically explicate, comment on, and analyze his prototype professional development exercises for literature teachers. His readers are guided through and invited to participate in his exercises as if they were there. He explains:

> *The text representing my voice in the workshop as well the voices of the participants is drawn from notes and handouts I use in actual workshops as well as from memory and videotapes of numerous workshops I have conducted with students and teachers. But it is a reconstruction of a typical workshop rather than a transcription of any single one, and is designed to create a new workshop experience for readers of this text. (pp. 34-35.)*

Similarly, within this manuscript I will apply Blau's methodology to reconstruct and deconstruct, comment on, and analyze a typical Travieso professional development session. Though snippets of actual dialogue are not included, the voices of the author, the development team members, and the participants are shared throughout the work. The data sources are the notes and handouts from the professional development session itself, DVDs and videotapes of sessions, written evaluations, personal communications with participants before, during, and after the sessions, and memory. Though one cannot appropriately experience the Travieso activity simply by reading about it, an understanding of the activity's "nuts and bolts," "aha" moments, and lessons learned can be acquired. In addition, the work and accompanying materials at www.traviesoactivity.com[5] can be used to organize, adjust, and lead this activity for others.

Discussion

The rest of this paper is comprised of three sections. The first section documents the activity's tasks and their facilitation, the second shares the findings by activity stages and participant/facilitator interactions, and the third is the conclusion.

The Travieso Activity

Answering the why/Setting the context (10 minutes). The facilitator begins the activity by sharing a Powerpoint™ presentation that lays out a bare bones argument for the need for the activity. Included are data on the number, composition, and large-scale assessment mathematical performance of English Language Learners (ELLs) in the state and nation, the NCTM shift from a deficit thinking (Schorling, 1926) to inclusion model (NCTM, 2000) for ELL instruction and assessment, and some of the injurious consequences of ELL instructional policies and practices that have yet to satisfactorily meet their needs.

Set-up (5 minutes). Participants are told that they will be engaging with one mathematics problem in Spanish for the next 40 minutes and taking part in a whole group debriefing afterward for 35 minutes. In English, the facilitator explains that there will be seven facilitation stages comprised of different problem presentation and problem solving modalities and that at the conclusion of each stage participants will be asked to fill out an Affective Effective Scorecard (AES). Within each AES, participants briefly answer three questions in writing (*How are you feeling about solving this problem? What do you understand about this problem? What would help you solve this problem?*) and on a 4-point scale mark their confidence level in their answer to the mathematics problem. Because many participants feel uncomfortable doing mathematics in front of their colleagues, especially in a language with which they have varying levels of proficiency, laying out the structure of the activity ahead of time helps them be open to experiencing sufficiently valuable cognitive dissonance without discounting the entire experience altogether.

Facilitation (40 minutes). The facilitator and participants immediately transition to reading, writing, speaking, and listening to Spanish, and no other languages, for the implementation of the immersion activity. At the beginning of each stage, the facilitator hands out that stage's AES. During the final two minutes of each stage, participants are asked to fill out the accompanying AES. The introduction and stages follow.

Introduction (2 minutes). After wishing the participants a "Buenos días, clase" (Good morning) or "Buenas tardes, clase" (Good afternoon), the facilitator emphasizes he will not translate any part of the problem, verify a participant's understanding of the problem, or answer any mathematics-related questions. The overview of the activity is then restated, in Spanish, to help participants turn on their internal Spanish language centers and transition to Stage 1.

Stage 1 – Reading the problem (5 minutes). The facilitator reads the problem out loud one time with similar speed, tone, and inflection to how he or she would read an English translation of the problem in an English-only classroom.

> *En un rancho, hay un caballo que se llama Travieso que camina dentro de su corral cuadrado. Con una soga, él está amarrado al poste que está en el centro del corral. Siempre él escoge a caminar en un sendero circular más largo que posible. Si un lado del corral mide s unidades, escribe una expresión algebraica para representar la distancia caminada por Travieso después de dar tres vueltas y un quinto adentro del corral*[6].

Participants work silently and independently[7].

Stage 2 – Projecting the problem (5 minutes). The typed out version of the problem is projected on the LCD screen. A volunteer reads the problem out loud. Participants work silently and independently.

Stage 3 – Receiving a hard copy (5 minutes). The facilitator hands out a hard copy of the typed out problem to each participant. Participants work silently and independently[8].

Stage 4 – Working in pairs (6 minutes). Participants can work in pairs but must speak Spanish with each other[9].

Stage 5 – Seeing the image (5 minutes). With an accompanying image (See Figure 1), the written version of the problem is projected on the LCD screen and handed out in hard copy form to the participants. Participants can continue working in pairs.

Stage 6 – Using the dictionary (5 minutes). Paperback pocket Spanish/English dictionaries are handed out. Participants can work in groups.

Stage 7 – Speaking English (5 minutes). Participants can speak English with one another and continue working in groups[10].

Debriefing (35 minutes). In English, the facilitator orchestrates a whole group debriefing to help participants share their feelings about the experience as a whole and at the different stages, the lessons learned, and next steps to better address ELL needs in their classrooms. For those participants who don't feel comfortable sharing publicly, providing time for a quick write reflection works well too (C. Schneider, personal communication, August 19, 2005), though I have not employed this strategy. The overarching goal is to help participants explicitly connect their small-scale affective and cognitive experiences accessing and solving one mathematics problem as Spanish language learners to the daily, large-scale mathematics classroom experiences of their ELLs.

Findings

The findings are organized into two categories, stage-by-stage analysis and participant/facilitator interactions. The first category documents how participants interacted with each stage, while the second explicates how the participants interacted with each other and the facilitator across the stages. Throughout both categories, the findings are consistently connected back to typical ELL behaviors, experiences, and realities in mathematics classrooms.

Analysis by stages—

Stage 1 - Reading the problem provides the least linguistic support, yet models a significant portion of current mathematics instruction and explanation. Multiple difficulties occur during this stage. Due to the facilitator's imperfect enunciation and rapid rate of speech, distinguishing one spoken word from another can be very challenging, making decoding and meaning-making difficult. The lack of a permanent record of what was said is a second cause of comprehension problems. For example, if the decoding and translation of a word or phrase is not immediate, a participant will spend a few seconds thinking about the meaning of that word or phrase. Yet, during that interval several other spoken words go in one ear and out the other, without being processed and without being provided an avenue to revisit them later. Iteratively revising one's meaning-making is nearly impossible at this stage. Without any lexical, visual, physical, or interpersonal supports, participants with beginning or no Spanish proficiency understand very little about the problem and lack any meaningful way to help themselves.

Stage 2 - Projecting the problem helps match the aural input (the spoken word) with the lexical (the written word). Providing a temporary written record of the problem on the LCD screen allows participants, especially those with intermediate to advanced Spanish language proficiency, to decode and translate words/phrases they could not before. Identifying cognates is the most common reading comprehension strategy employed at this stage for participants with the least Spanish language knowledge. Though they easily recognize true cognates (e.g., *corral, rancho, algebraica, expresión, representar, distancia*) they also unknowingly find and accept false cognates, hampering problem comprehensibility. Confusing *llama* (named) with the camel-like animal and *vuelta* (turn or revolution) with vulture are two such examples. Notice how both misinterpretations accommodate the ranch context.

Some participants with intermediate-advanced Spanish language proficiency mix up one Spanish word with another of similar pronunciation, spelling, or meaning. Confusing *escoge* (chooses) with escoba (broom) and *quinto* (one-fifth) with quince (fifteen), cuarto (one-fourth), and medio (one-half) are examples. Notice how a misinterpretation or non-understanding of one critical word (e.g., *quinto* in this problem) can lead to a logical, but incorrect answer (See Appendix for an English translation of the problem). In addition, sometimes *soga* (rope), *quinto*, and *unidades* (units) are unknown vocabulary because they are known only by another name (e.g., cuerda instead of *soga*) or are part of a Spanish mathematics register (Moschkovich, 2000) that was never taught to them. These kinds of translation challenges reflect the fluidity and regionality of language as well as likely downsides of English-only mathematics instruction for ELLs.

When the problem is projected, many participants often begin by transferring information from the screen to their papers. Some start copying the problem verbatim, while others choose specific parts to copy, translate, or jot down notes about. Such efforts take time and can lead to copy errors being made. In the classroom, not leaving the problem up on the board/screen long enough forces students to spend extra time checking with classmates to verify accuracy and fill in gaps instead of working on the problem. Further, if the teacher gives additional information/directions orally while students are still copying (a common occurrence), ELLs may miss key information being given through the spoken medium. Acquiring and processing information from two modalities at once in a well understood language can be challenging, but being forced to do so in a language with which one has a nascent proficiency is both an unsound pedagogical practice for and a microaggression (Solorzano, 1998) against these students.

Stage 3 - Receiving a hard copy provides participants a permanent, word-processed record of the problem. Employing kinesthetic reading comprehension tools, participants often begin to underline and circle words as well as write English translations of words in the margins or on top of or beneath the words themselves. These types of annotations are like those typically used by active readers when engaging with a new piece of writing (e.g., Boke & Hewitt, 2004). Unlike most K–12 school textbooks, from which students are typically prohibited from writing on or in, this format allows, even encourages, students to record notes, scribbles, drawings on the problem itself to assist them in understanding and solving the problem. In México, such work would go into a student's well kept course notebook to be referenced for years to come (Gutierrez, 2006). In addition, unlike some difficult to decipher handwritten problems, this format provides a legible, common text with which pairs or groups of students can interact.

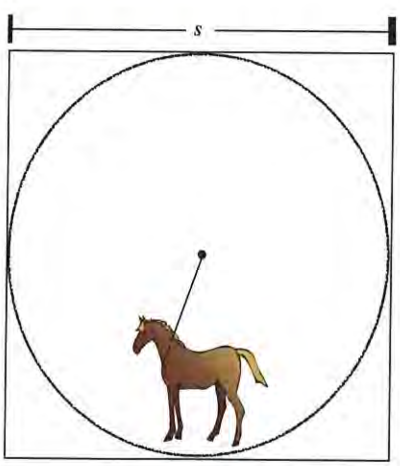
In Stage 4 - Working in pairs, each participant can begin collaborating with a partner. Mimicking many of their own beginning ELLs' behaviors in English only environments, participants with the least Spanish proficiency often use combinations of nonverbal and verbal strategies to communicate with each other. Gesturing, pointing, making facial expressions, and drawing pictures are four such common nonverbal approaches. Verbally, they often grunt almost unintelligible utterances and repeatedly start, stop, then start again, generating broken phrases as they desperately try to communicate with each other. Nervous smiles and laughter follow almost immediately because they feel and empathize with each other's frustration and anxiety as they attempt to communicate all they want to share about the problem, but cannot get across. As a result, these participants often draw pictures and point to specific nouns when trying to match Spanish labels to English referenced concepts or check their initial understanding of a word or phrase. For example, sketching a horse, pointing to it and saying *caballo* or drawing an arrow from *caballo* to a sketch of a horse is preferred to describing a horse in Spanish. Intermediate to advanced

Spanish speakers employ similar strategies, but with an accompanying spoken conversation.

Stage 5 - Seeing the image provides a visual interpretation of the problem context. Checking their mental image against the one provided helps participants evaluate their current understanding of the gist of the task and focuses their content knowledge and problem solving strategies on similar looking mathematics problems they have encountered before. By the end of this stage, almost all participants hypothesize (if not know) that they must calculate at least one of the following:

1. The perimeter of the square (the corral)
2. The area of the square
3. The circumference of the circle (Travieso's path)
4. The area of the circle
5. The difference between the area of the square and the area of the circle
6. The area swept out by Travieso as he walks around the circular path multiple times
7. The distance Travieso travels as he walks around the circular path multiple times

Figure 1. Stage 5 Travieso visual.



Notable about these seven possibilities are their viability and interrelatedness. Notice how possibilities 1, 3, and 7 are about distance and 2, 4, 5, and 6 are about area; possibilities 1, 2, and 5 relate to a square and 3, 4, 5, 6, and 7 relate to a circle.

Integrating this new information with their previous conceptions of the problem improves the participants' translation and understanding of it. For example, seeing the side of the square labeled *s* helps them translate *s unidades* to "s units," because prior problem solving experience has taught them to look for a distance's unit of measure. Further, the introduction of the image helped disavow participants from logical, but unintended pragmatic concerns related to the problem, such as how the rope is tied to the post.

Before seeing the visual, participants have inferred that the rope is initially taut and affixed to the post and horse so that as the horse walks around the corral the rope wraps around the post, thereby continually shortening the length of the rope. This conception is logical and possible, especially given the lack of information in the problem regarding the shape, dimensions, and rotation ability of the post, how the rope is affixed to the post, and the initial length of the rope. In this interpretation, for each successive revolution inside the corral, the largest circular-like path possible would cover a shorter distance than the previous revolution, thereby creating a tightening, Archimedes spiral. Because calculating this arc length would require more known data (e.g., the height, width, and shape of the post) and significantly more mathematical knowledge (http://mathworld.wolfram.com/ArchimedesSpiral.html), accomplishing this task for the spiral would be much more difficult than doing so for the intended circular path.

These kinds of unintended semiotic and pragmatic challenges highlight a common spec-in-the-head difficulty (Davidson & Lynch, 2002). Though this activity was conceived by the developers so that the horse is tied to the top of a rotating post with a rope that is at least $s/2$ units long, these implicit notions were not made explicit in the problem. Notice how these implication/inference disconnects between problem writer and problem solver can significantly impact one's conception of the problem and its perceived level of difficulty. On a greater scale, these types of unintentional ambiguities can lead students to generate incorrect solutions, give up on a problem, or lose problem solving confidence. Regarding this activity, participants repeatedly noted that while they were not surprised by their initial lack of confidence, they did not expect to feel so uncertain about their interpretation of and answer to the problem throughout the later stages.

This stage's meaning-making tensions also raise larger mathematics education questions. Though providing a visual can help a learner check his or her mental image with the "correct" image of the problem or visualize what could not be imagined, is such a scaffold a linguistic support or crutch? Is part of mathematics competence being able to generate the exact meaning and image intended by the problem writer from text only? If providing a visual is good practice in certain problem solving situations, what are the benchmarks to distinguish appropriate from inappropriate visuals with respect to the amount of "mental lifting" a student is expected to do? Deciding when and how to include an accompanying visual in classroom instruction, a textbook, or an assessment item and for what linguistic support and problem solving purposes merits further discussion and research.

Stage 6 – Using the dictionary provides access to paperback pocket dictionaries that ELLs typically carry around in their backpacks. Like their students, participants quickly and easily decipher common nouns, such as *caballo* (horse) and *soga* (rope). However, they also encounter certain challenges as well, like conjugated verbs *amarrado* (tied) and *mide* (measures) which cannot be found verbatim. If the infinitive form (e.g., "medir" – to measure) is unknown to the participant, the most similar looking listed words are typically tried first. For example, the most similar available words to *amarrado* are "amarra" (a mooring line for a ship or boat) and "amarrar" (to moor a boat, to tie up, to tie down or to fasten.) (Webster's, 2003). In addition, "amarrado" has been mistaken for the conjugated form of the infinitive "amar" (to love).

Still, the unit of meaning is not always a single word. Phrases such as *sin embargo* (however) and *lo mas largo posible* (the largest possible) are units of meaning made up of groups of words. Such phrases often are not listed in their entirety in a pocket dictionary. Attempting to deconstruct them word by word to then reconstruct the entire translated phrase can lead to translation errors as well. For example, translated word for word, "sin embargo" becomes "without embargo." This grossly incorrect translation introduces a commerce context that has nothing to do with the problem.

Because mooring boats, love, and embargos have nothing to do with horses and corrals in this problem, participants have become temporarily confused. These stark, contextual contrasts have helped some participants recognize and correct a mistranslation, but further confuse others who tried to recalibrate their incomplete understanding of the problem to accommodate a new, incongruous context. Working collaboratively is one strategy that has consistently helped participants filter out such noise and hone in on the correct meanings.

As soon as *Stage 7 – Speaking English* starts, the collective volume and energy in the room increase immediately. The group discussions are now marked by greater numbers of participants talking and faster rates of speech. For many, frustrated frowns and nervous smiles are replaced by happy, warm grins. Participants eagerly check their interpretation of the problem with other group members and revise their work as necessary. Often, they begin sharing with each other how they felt during the first six stages of the activity even before the official debriefing has started. The sense of relief is palpable, especially for those who had "checked out" earlier and now come back to the fold.

Participants quickly recognize the value of being able to speak with each other in the language with which they have the most fluency, even if it is not the language of instruction. They recognize that they were not off-task or talking about the facilitator, two common concerns many participants have when considering allowing or encouraging their ELLs to converse with each other in non-English languages in their classrooms.

Purposeful absence of specific linguistic cognitive footholds. One of the topics often discussed among participants during Stage 7 is the lack of certain types of linguistic footholds in the problem itself. The Travieso problem was purposefully written in words with as few symbols as possible to reflect the language dependent, problem solving situations in many of today's curricula and assessments that ELLs encounter. In addition, specific decisions were made in terms of the problem's vocabulary and format to provide as few linguistic clues as possible. Three types follow.

First, because students look for numbers to help them in unfamiliar situations, the measurement of one side of the square corral was chosen to be "*s* unidades" (*s* units) so that no common cardinal number (e.g., 6, 10.3) or standardized unit of measure (e.g., feet, meters) would be stated. Similarly, the $3\frac{1}{5}$ revolutions were not written out in Arabic numeral form, but in words ("tres vueltas y un quinto"). Second, because students regularly look for the sentence with the question mark at the end of a problem situation to cue the problem's objective, the sought after algebraic expression was asked in the form of an imperative ("Write out the algebraic expression to represent the distance Travieso walked"). Third, because requisite mathematical concepts are not always referred to by name in a problem, "circumference" and "radius" were not explicitly stated, but implied in the sentence "...con una soga él está amarrado al poste que está en el centro del corral, siempre él escoge a caminar en un sendero circular más largo que posible..." (...with a rope he was tied to a post in the center of the corral, he always chooses to walk the largest circular path possible...). Eliminating this set of cognitive footholds was not done to trip up the participants, but to tacitly prompt them to notice their absence and recognize their utility when engaging with any mathematics problem.

Types of participant and facilitator interactions. In addition to engaging with the written text, participants engage with the facilitator and each other. These human-human interactions can be grouped facilitator-participant, participant-facilitator, and participant-participant.

Though the facilitator previews holistically what will take place during the activity before its commencement, he does not prepare the participants for his purposeful differentiated behavior toward them throughout the activity's facilitation. Once the facilitator starts speaking in Spanish, he immediately warms to the most proficient Spanish speakers and participants who are on task, rule abiding, and hard working (regardless of Spanish proficiency); he ignores or becomes angry with the participants who are off task, rule breaking, and recalcitrant. Smiling at, conversing with, and giving encouragement to participants are three typical explicit facilitator actions that mark the pleasant facilitator-participant interactions; verbally and corporally scolding (e.g., placing the offender's open hand flat on a desktop and softly hitting it with a yardstick) typically mark the unpleasant ones. The facilitator acts in this calculated manner to impose his power and classroom culture upon the participants and mirror how some English proficient mathematics teachers knowingly and unknowingly interact with their ELLs and non-ELLs. As if on cue, participants quickly and unwittingly adopt roles and behaviors of different types of students from their own classrooms.

Some participants want to access and solve the problem so badly they either do not realize they are breaking the rules or purposefully do. Though the "offenders" know from the AESs that the first three stages of the activity require them to work silently and independently, some cannot or refuse to follow the problem solving modality during that interval. Vocalizing the Spanish words out loud as they read the problem, pointing to a word and asking a neighbor its meaning in hushed tones (in Spanish or English), or sharing work with another participant are three common participant "transgressions." The first occurrence or two, the facilitator will look directly at the offending participant and with a smile on his face, silently put his right forefinger to his lips or just say out-loud to all participants "en silencio, por favor," with an emphasis on *silencio*. The facilitator's response typically draws laughter from the participants, as they hear and see themselves (as teachers) in the facilitator's admonition of them. For some participants, that gentle, funny reminder is enough to get them to tow the line. For others, their resistance has just begun.

Resistors purposefully break the rules to not do what the facilitator has asked them to do. Usually, they resist because they cannot understand or solve the problem and cannot communicate effectively with or receive help from the facilitator or a similarly challenged neighbor. Silently sitting with arms crossed, reading a newspaper, or putting their heads down on their desks are three typical actions that mark their solitary, unobtrusive disengagement. Other resistors try to deflect attention from their own challenges and discontentment by sabotaging the activity for others. Loudly talking in English about a non-mathematics topic, mimicking Spanglish food advertising (e.g., "Yo quiero Taco Bell", "Yo quiero cerveza"), or responding to the facilitator in a non-Spanish language are typical "misery loves company" approaches. A few have even walked out in the middle of the activity or flatly defy the facilitator's request to get back on task. Though the aforementioned participant-facilitator interactions are telling, so are the participant-participant interactions between fluent Spanish speakers and less fluent Spanish speakers.

The fluent Spanish speakers gain instant power and prestige over their less fluent Spanish colleagues due to their stronger Spanish proficiency and the conspicuous praise bestowed upon them by the facilitator. Many finish quickly and well ahead of the less fluent Spanish speakers. Because many fluent Spanish speakers are familiar with the language and affective challenges the less fluent Spanish speakers are experiencing, these Spanish experts almost always patiently and graciously help their fellow participants overcome them by explaining and teaching in Spanish. This welcoming and helpful behavior is in direct contrast with how U.S. mainstream English media (e.g., movies, television shows) consistently demean nonnative (heritage) ELLs, using them as comic foils by cruelly making fun on their accents, word choices, speech patterns, and cultures.

Though most fluent Spanish speakers enjoy being the expert in this context, some report that they are uncomfortable with this role because less fluent Spanish speaker resistors often chide them for being de facto teacher's pets. On the other side, some less fluent Spanish speakers get so frustrated with their own inability to understand a fluent Spanish speaker's explanation, and their resultant belief that their colleagues are perceiving them as slow or stupid, that they sometimes feign understanding when they really did not understand. This behavior also takes place when the less fluent Spanish speaker is cognitively exhausted.

In fact, many less fluent Spanish speakers report that being forced for 40 minutes to use Spanish to engage with the problem and their fellow participants temporarily overtax their brains and give them headaches. Participants who initially consider their rudimentary Spanish proficiency sufficient to successfully understand the mathematics problem or discuss it with a colleague, quickly realize that Spanish academic language discourse in mathematics is distinct from and typically less known than conversational Spanish. Usually, a participant will point out that in contrast to many of their ELL students, they already know the mathematics and can draw on greater reading comprehension and problem solving experience to understand and attack the problem. These language-concept challenges mirror those that Garrison and Mora (1999) have already expounded on, organized, and made pedagogical suggestions to address.

Conclusion

Because of the intentionally disorienting and disconcerting nature of the Travieso professional development activity, most participants temporarily experience a little bit of the isolation, frustration, and depression ELLs often feel in their mathematics classrooms. Participants, especially those who are monolingual English speakers, not only are surprised at the existence and intensity of their feelings, but begin to realize they have also underestimated the number, type, and complexity of language challenges that accompany mathematics learning and performance. According to Thompson & Zeuli (1999), such cognitive dissonance is one of five requirements for transformational learning, when participants uproot deeply held beliefs, reorganize their knowledge, and restructure their core principles for learning (Mezirow, 1991, 1997; Thompson & Zeuli, 1999). When participants connect their realizations to the classroom realities that ELLs effectively and cognitively experience several hours per day, 180 school days in a school year, a collective, but sobering "aha" moment is experienced. Almost immediately, they begin reflecting on their previous instruction and assumptions about mathematics and language learning. Hopefully, a transformational process has begun for them that future professional development can support by modeling specific instructional strategies to meet the classroom realities highlighted by the Travieso activity.

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APPENDIX

English Translation of Travieso Problem

On a ranch, there's a horse named Travieso who walks inside a square corral. He is tied to a post in the middle of the corral. He always walks the largest circular path possible inside the corral. If one side of the corral measures s units, write an algebraic expression to represent the distance Travieso covers after walking 3 1/5 revolutions inside the corral.

FOOTNOTES

[1]Tere Hirsch, Silvia Llamas-Flores, and author

[2] California Mathematics Project

[3] Leslie Ercole, Bob Laird, and author

[4]National Science Foundation and United States Department of Education – MSP Award #EHR0227057

[5]To best view and manipulate the Powerpoint™ presentation, click the "Home" tab so that playback buttons appear in the lower right-hand corner of the screen.

[6]The wording of this problem has been revised several times by the development teams working with native Spanish speakers from different countries. The most commonly used version is shared in this article. One sample alternative version is:

En una granja, hay un caballo llamado Travieso que camina dentro de un corral cuadrado. Sin embargo, debido a que él está atado con una cuerda al poste central del corral, él siempre escoge caminar el trayecto circular más largo posible dentro del corral. Si un lado del corral mide "s" unidades, escriba la expresión algebraica que represente la distancia caminada por Travieso después de dar tres vueltas y un quinto dentro del corral.

[7]At this first stage, many non-Spanish speakers immediately begin filling out the first AES, as it is the only task they can carry out at that time. Allow them to do so, but recognize that they are too linguistically overwhelmed to even begin attacking the problem.

[8]Though participants often start writing English translations of certain words on their hard copy of the problem, the facilitator does not penalize them because they are on task.

[9]An extra minute of work time is provided at this stage because participants quickly become engrossed in working together and trying to communicate with each other.

[10]The elapsed time of a stage can be lengthened based on the needs of the participants. For example, if at Stage 4 the participants are still fully engaged when four minutes have passed, the facilitator can make the on-the-fly determination to extend the stage by one to two more minutes. However, to ensure participants feel enough frustration, it is not advised to shorten a stage's length unless participants have finished or have become so unruly as to prematurely end the activity.