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Teaching for Excellence and Equity in Mathematics

TODOS
Mathematics for ALL

An Affiliate Organization of the National Council of Teachers of Mathematics

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TODOS and the editors of *TEEM*
express deep appreciation to
PEARSON EDUCATION
for its generous sponsorship of this issue.

From the Editors

This is an exciting time and opportunity for articles with an equity focus. In addition to our *TEEM* journal, TODOS also published the second research monograph and other mathematics education journals are sponsoring "equity focus issues," including a 2011 issue of *Journal of Mathematics Teacher Education* (partnering with the Association of Mathematics Teacher Educators), the *Journal for Research in Mathematics Education* (see www.nctm.org/jrme/equity), and the October 2009 issue of *Teaching Children Mathematics*.

The *TEEM* Editors have also been active in this area as individuals since the last issue of TEEM: **Cynthia Anhalt** published a research paper in the second TODOS monograph with co-authors Anthony Fernandes and Marta Civil. In addition, she participated in a symposium with Julia Aguirre, Sylvia Celedón-Pattichis, and Erin Turner at the 2010 Society for Advancement of Chicanos and Native Americans in Science (SACNAS) national conference. **Larry Lesser** gave a keynote address on equity at his local NCTM affiliate's spring 2010 conference and had equity-releated publications in *Statistics Education Research Journal, Proceedings of the International Conference on Teaching Statistics, Philosophy of Mathematics Education Journal,* and *Journal of the Association of Mexican American Educators.* **Miriam Leiva**'s several featured national and international presentations this year include the Iris M. Carl Equity Address at the 2010 NCTM annual meeting (see http://vimeo.com/11176098) and the keynote address at the United Arab Emirates Higher Education Mathematics Conference in Abu Dhabi. The latter experience yielded a paper in the fall 2010 *Women and Mathematics Education Newsletter*.

This issue of *TEEM* has newly-written, full-length pieces that went through a rigorous double-blind external review process with an acceptance rate of about 30%. *TEEM* gratefully thanks those individuals who donated their time and expertise to conduct these external reviews. Because of the wide range of types of papers submitted to *TEEM*, we are always interested in having more people join our growing database of reviewers (see **www.math.utep.edu/Faculty/lesser/TEEM.html**).

At the above URL is a broadening of our initial call for papers. First, we now accept papers year-round, although the months of November and April are still preferred. Also, to ensure each issue of *TEEM* offers something valuable to all scholars (including teachers and teacher educators), we recently added a call for papers of a second category. In particular, we aim to publish at least one article or paper in each issue specifically written by and for classroom teachers and/or teacher educators. While this feature would still go through peer-review, it need not follow as strictly the structure of regular articles. We welcome queries (by email at **teem@todos-math.org**) on possible topics and here are some suggestions:

- A description, discussion and reflection on what happened while trying to implement a particular strategy or recommendation from the 'excellence and equity' research literature (from a previous issue of *TEEM*, or another publication or source)

- A specific classroom-tested TODOS/*TEEM*-oriented "excellence and equity" activity (or piece of curriculum) accompanied by a camera-ready worksheet for classroom use -- as appropriate, similar to NCTM's *Mathematics Teacher*, *Mathematics Teaching in the Middle School*, and *Teaching Children Mathematics* that have a featured activity article followed by blackline master worksheets.

- An article that focuses on some aspect of the TODOS mission:
  - to advocate for an equitable and high-quality mathematics education for all students;
  - to implement lessons and programs that incorporate the role that language and culture play in learning mathematics;

- to inform the public, including parents, and influence educational policies in ways that enable students to become mathematically proficient; and
- to inform teacher education programs.

TEEM also gratefully acknowledges the support of all the leaders in our sponsoring organization, TODOS: Mathematics for ALL. We hope *TEEM* serves and increases the TODOS membership, and also serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities. We welcome feedback about the journal as well as results of teachers or professional developers putting the ideas (or DARE questions) of the papers into practice.

In this issue of *TEEM*, Kathleen Chval and Rachel Pinnow examine misconceptions held by pre-service teachers based on an ELL's country of origin. Continuing the "international theme," Alejandra Sorto sheds potential insight into students we teach by studying classroom practices in countries that are homelands for some of them. Finally, Margarita Jiménez-Silva, Conrado L. Gómez, and Janel White-Taylor share an example of their creative collaboration with teachers to create mathematical board games to use with ELLs. Enjoy the issue!

Editors
Lawrence M. Lesser, The University of Texas at El Paso
Cynthia O. Anhalt, The University of Arizona

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Miriam A. Leiva, University of North Carolina Charlotte

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students — in particular, Hispanic/Latino students— by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics.

ARTICLES IN THIS ISSUE

"[equity includes] the equitable distribution of material and human resources, intellectually challenging curricula, educational experiences that build on students' cultures, languages, home experiences, and identities; and pedagogies that prepare students to engage in critical thought and democratic participation in society"

-- Pauline Lipman (2004)

Pre-service Teachers' Assumptions about Latino/a English Language Learners in Mathematics

Kathryn B. Chval and Rachel J. Pinnow

Abstract

We share data collected from 51 pre-service elementary mathematics teachers who were asked about teaching mathematics to English Language Learners (ELLs) who moved to the U.S. from Central America and China. We describe three critical misconceptions held by pre-service teachers (e.g., differential treatment of ELLs based on country of origin, isolation rather than community, and outsourcing to meet ELL needs) and discuss implications for teacher preparation and professional development to better equip teachers to teach mathematics to Latino/a students.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Do you make assumptions about how ELLs learn mathematics based on their country of origin?
2. How might these assumptions influence your interactions with ELLs?

Kathryn B. Chval (chvalkb@missouri.edu) is an Associate Professor at the University of Missouri. Her research focuses on (1) effective preparation models and support structures for teachers across the professional continuum, (2) effective elementary teaching of underserved populations, especially Latino/a English language learners, and (3) curriculum standards and policies.

Rachel J. Pinnow (pinnowr@missouri.edu) is an assistant professor of TESOL at the University of Missouri. Her research interests include education for culturally and linguistically diverse learners, second language acquisition, and multimodality (social semiotics).

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Pre-service Teachers' Assumptions about Latino/a English Language Learners in Mathematics

Kathryn B. Chval and Rachel J. Pinnow

Imagine a third-grade teacher in her first year of teaching who has two English Language Learners (ELLs) in her classroom. One student is from China and the other is from Central America. What assumptions might this teacher make about what these children know mathematically? Would the teacher make different assumptions or instructional decisions based on the children's countries of origin? This article shares qualitative data collected from pre-service elementary mathematics teachers who were asked about these potential scenarios. These data indicate that pre-service teachers (PSTs) would have lower expectations for Latino/a students that would hinder their participation and achievement in mathematics classrooms. These data also suggest that we need to strengthen our teacher preparation programs so that PSTs are better equipped to teach mathematics successfully to Latino/a students.

To assess elementary PSTs' beliefs about teaching ELLs from different countries of origin, we undertook a qualitative study (Miles & Huberman, 1994) to determine whether or not country of origin might affect teacher expectations regarding mathematical ability and instructional needs of ELLs. We collected written responses from 51 PSTs using two writing prompts on a one-page document (see Appendix). The first prompt stated, "Imagine it is your first year of teaching. You have taken a position as a third grade teacher. One of your students has just moved to the United States from Central America." The second prompt was identical except that "Central America" was replaced with "China." For each of the two prompts, PSTs wrote individual responses to Questions 1-3. Next, they wrote responses to Questions 4 and 5.

1. What do you think this child will need during mathematics instruction?
2. How will you assess the child's needs?
3. How will you support this student during mathematics?
4. Do you anticipate that the needs would be different depending upon the continent of origin?
5. How comfortable would you feel about these two different situations?

To analyze the data, we coded the 51 PSTs' responses to Questions 1-5 to identify and examine (1) how PSTs would meet the needs of ELLs in general and (2) how they would meet the needs of ELLs from different countries of origin. The respondents were juniors enrolled in two sections of the only mathematics methods course required for their program and participating in field experiences in local schools prior to entering a year-long student teaching experience. The first author taught the only section of this course to 28 students and all of them participated in the study. Intrigued by the students' responses, she decided to replicate the study the following spring semester. That semester, there were three sections taught by adjunct faculty, two of which were taught by full-time elementary teachers who had never taught undergraduates. The first author believed that these inexperienced instructors would not be prepared sufficiently to handle the conversations that would follow and chose to have the prompts administered only in the section whose instructor had experience teaching elementary PSTs. All 23 students from this section participated. All 51 PSTs (48 women, 3 men) were Caucasian, native-English speakers who had no coursework or experience in teaching ELLs.

PST Responses to Meeting the Needs of ELLs

Based on the analysis across all 51 PSTs' responses, we found four major categories in relation to how PSTs would meet the needs of ELLs in general. We discuss each category in the following subsections.

Outsourcing to Meet ELL Needs

Although the PSTs were asked how *they* would support these children, 63% of the respondents did not focus on *their* own actions, but rather on the actions of others. Their responses revealed underlying assumptions that other people (e.g., ESL teachers, translators, tutors, other students, and parents) would be available to work with ELLs both inside and outside their classrooms. For example, they wrote:

- "A translator should be present in the classroom and perhaps a dictionary."
- "I think weekly conferences with the family would be beneficial."
- "Also any extra work could be sent home if needed for the parents and a tutor would be provided."

- "Bring in an ESL teacher; assess – taking pre test; have ESL teacher explain math problems."
- "Bring in an ESL teacher for assistance."
- "I would feel better with an interpreter because then I feel (or believe) that everyone would feel well represented and perhaps that their voice is heard."

They also indicated that they would pair these children with other children—children who were at a higher level mathematically, children who were from the same country of origin, or just "helpful" children.

Utilizing Resources to Support Students

Second, the PSTs suggested that they would utilize resources to support students. For example, they indicated that they would find games to help students learn English and provide manipulatives when appropriate. In addition, they explained that they would use an Internet translator or provide translated curriculum materials during mathematics instruction. The underlying assumption in some responses was that if a translator was not available, then they could at least provide translated materials to help ELLs learn mathematics and if translated materials were not available, then they would be able to use the Internet to create translated materials. However, the PSTs did not elaborate on how they would actually use the translated materials, how these materials would be incorporated into whole class discussion on mathematics concepts, or what role providing separate curriculum materials might play in engaging ELLs in math conversations with their peers.

The PSTs' responses revealed the belief that ELLs would be proficient in reading academic content in their native languages and that Internet translation engines would be able to provide grammatical and syntactically recognizable sentences for ELLs. Unfortunately, at this point, Internet translation search engines do not consistently provide translations that native speakers would use or recognize as typical everyday language (Rieha & Rieh, 2005; Savoy & Colamic, 2009). This has prompted Google to continue to refine this service.

Using Teaching Strategies

Third, some PSTs identified particular teaching strategies to support ELLs such as demonstrating and modeling, providing detailed explanations and clear instructions, using visual aids and concrete examples, and including examples from the child's culture. All of these strategies have the potential to be effective for ELLs. However, the PSTs considered using these strategies only while working with the child in isolation, rather than as excellent whole-class strategies that could be more efficient and effective at addressing the needs of ELLs without unnecessarily isolating them. In other words, PSTs did not consider using these strategies or other strategies as resources for engaging ELLs in a community of practice.

Providing Individualized Support

Fourth, the PSTs explained that they would make themselves available to provide individualized support to ELLs. For example, they stated that they would assess ELLs' English proficiency and provide English instruction (unrelated to math instruction). PSTs also stated that they would conference with the child one-on-one, visit the child's desk frequently during class, and spend time with the child before and after school and during recess. These future teachers explained that the ELLs would need more help than their native-speaking peers and this help would be provided by giving them extra time to complete class work, extra practice, and additional opportunities to demonstrate understanding. These PSTs also stressed that they would provide encouragement and patient guidance to help ELLs feel comfortable in school. Yet, they did not elaborate on the nature of this encouragement and support. Additionally, they seemed unaware of the difficulties associated with balancing this type of support with other students' needs.

It should be noted that several PSTs indicated that they did not know what to do for either case as the following quotes illustrate:

- "I honestly don't know what I'd do."
- "I think it would be hard. I would probably not know what to do and have to see what the students know to make decisions."
- "I would find this very challenging and would need outside resources for further support."

These honest statements capture the thoughts of a few PSTs who were willing to admit that they did not have sufficient knowledge or experience to address adequately the needs of ELLs in general. In the next section, we examine the PSTs' approaches to mathematics instruction for different ELLs based on country of origin.

PSTs' Approaches to Country of Origin

Our analysis of participant responses to prompts revealed that some PSTs would use different approaches for ELLs from different countries. Out of 51 respondents, seven indicated that ELLs from different countries would not have different instructional needs while 43 respondents indicated that these children would have different needs and these needs were directly related to their country of origin. (One PST did not indicate one way or the other.) Some PSTs elaborated and provided reasons for these differences:

1. Culture, and thus teaching and learning, varies between countries (16 PSTs)
2. The value of education and mathematics varies between countries (6 PSTs)
3. Each student has different background knowledge and needs (5 PSTs)

Of the 43 that indicated that these children would have different needs, eight indicated more specific differences related to country of origin as illustrated in Table 1.

Discussion

The data collected from these pre-service elementary teachers suggests three major findings that need to be addressed in mathematics teacher education: 1) differential treatment

Table 1. Written responses from eight PSTs

PST	About Child from Central America	About Child from China
#1	Needs a lot of visual aids and concrete examples; check for understanding in personal interviews/conferences; provide manipulatives and detailed explanations.	Needs to be challenged; check for understanding in personal interview/conference; provide many opportunities to show understanding.
#2	The child will need extra help maybe from an ESL coach or volunteer.	The student will mostly likely excel at mathematics.
#3	The child will need extra help and support during mathematics instruction.	The child will need deeper instruction and manipulatives to work out problems.
#4	The child will need one-on-one help and some translation & explanation.	The child will need me to explain the directions, but based on previous experiences, my Asian students are excellent in math.
#5	I would guide the child through what we are learning and try to use lots of pictures. I would also allow him to have extra time.	I have a student like this in field. He is already far more advanced in science than the other students. During math time he leaves to go to ESL because he is already able to understand the concepts being covered. I would support him by giving him appropriate level work that will help him continue to grow.
#6	This child will need extra time to complete assignments and more practice. Support will come from me, as the teacher, the child's classmates, parents, and aide. Together we will help the child better learn and understand the English language and mathematics through encouragement, patience, drawings, and any kind of support necessary.	From my experience, Chinese students excel in mathematics, but learning the English language hinders their progress. I would assess them through conferences, assigned work, and other methods with support from an aide.
#7	Well the student is going to need some English instruction first. I would pair one of my higher-leveled students with that student to help the student during math.	Help child with the English, they more than likely have a good background in math, so math won't be a problem – but English will. Pair student with student from China.
#8	She would probably need to get used to our whole class discussions, so I would have the students guide the student and show her how it is done.	This child will probably need to be challenged more since China is ahead of the U.S. in math. So I will do my best to include more challenging material.

based on country of origin; 2) isolation rather than community; and 3) outsourcing to meet ELL needs.

Differential Treatment Based on Country of Origin

Some PSTs appear to make assumptions based on one isolated characteristic of a child (i.e., country of origin) rather than on any information about student knowledge, experiences, mathematical and language proficiencies, and performance on assessments. For instance, eight pre-service elementary teachers clearly expressed a discrepancy between how they would support children from Central America and China (i.e., students from China would need challenge while the students from Central America would need remediation). The data indicated that some PSTs had stereotypical ideas about students from both origins. Of course, it is problematic when teachers make assumptions of deficiency or proficiency without appropriate assessments. Mathematical knowledge, skill, or ability is not the province of a particular gender, race, or ethnicity.

When we designed the prompts, we selected Central America, rather than a specific country south of the United States such as México, Columbia, or Guatemala to avoid potential bias PSTs might associate with specific countries. For similar reasons, we used China rather than Asia or countries such as Japan or Singapore. Moreover, we avoided terms that would invoke SES stereotypes common to U.S. entertainment and media. As a result, we expected that some PSTs would realize that SES and educational backgrounds may influence ELLs' prior experiences and academic preparation. However, none of the 51 participants mentioned this notion.

Decisions teachers make about classroom practices can greatly facilitate student learning or serve as an obstacle (Wenglinsky, 2002). The assumption that children from different countries of origin should receive different mathematics instruction would lead to different teacher expectations, teacher-student interactions, and student participation in mathematics classrooms. Good (1987) documents how differential expectations as shown by these eight PSTs would lead to differential treatment in the classroom in terms of grouping practices, locus of responsibility for learning, feedback and evaluation practices, motivational strategies, public display of thinking, and cognitive demand of tasks posed to students. These assumptions reflect Ogbu and Simons' argument (1998) that, "the treatment of the minorities in the wider society is reflected in their treatment in education" (p. 161). The eight future teachers' statements are problematic and will perpetuate low expectations and ultimately, low achievement for Latino/a students.

Isolation Rather than Community

Overwhelmingly, PSTs expressed the notion that the best way to help ELLs learn is to isolate them in order to provide individual instruction. Not one PST indicated strategies for helping a child engage with peers to solve mathematics problems or to participate in classroom discussions related to mathematical ideas. This striking result is in sharp contrast to research that clearly indicates that teachers must (1) promote active ELL participation in mathematical discussions, and (2) recognize the resources that ELLs use to express mathematical ideas in order to facilitate participation and learning of ELLs, especially Latino/a ELLs in mathematics classrooms (Brenner, 1998; Brown et al., 1993; Khisty & Chval, 2002; Moschkovich, 2002). When instruction facilitates full engagement and participation in communities of practice (Lave &Wenger, 1991), Latino/a ELLs are successful (Chval, 2001). Thus, the PSTs' notions about isolated instruction contradict what we know about supporting the learning of Latino/a ELLs in mathematics classrooms. This finding suggests that PSTs need the knowledge and competencies to reframe instruction for Latino/a ELLs that values the construction of mathematical knowledge in social arenas, producing classroom norms that facilitate ELL participation in whole-class settings and during small-group peer interactions.

Outsourcing to Meet ELL Needs

The PSTs, with limited knowledge about how to meet the needs of ELLs, made assumptions that other educators with specialized knowledge and experience will be available to support ELLs in their future classrooms. The PSTs assumed they would not need to develop this specialized knowledge since they would have a host of assistants (i.e., translators, tutors, ESL specialists, parents, and peers) to help them meet the needs of their ELLs. They also assumed that translating curriculum materials to the child's first language would be helpful, not realizing that some ELLs may not be literate in the printed word in their native language, especially in the academic language of mathematics. The PSTs also assumed that peers would have the ability to interact with ELLs in academically productive ways; however, elementary students do not necessarily know how to engage ELLs so that they are equal partners who have a voice (see Chval et al., in preparation, and Van Lier, 2004, for other problematic issues with this assumption). None of the 51 PSTs indicated that they would need to develop knowledge or expertise to meet the needs of ELLs. This was not unusual given that at the time of the study the participants were enrolled in a certification program in a state that had not officially aligned ELL instruction with TESOL

(Teachers of English to Speakers of Other Languages) national standards. This finding is problematic and needs to be addressed in teacher preparation programs.

Implications for Classroom Teachers and Mathematics Teacher Educators

In most (if not all) U.S. teacher preparation programs, future teachers are not prepared to teach mathematics to Latino/a ELLs. The undergraduates in our study, less than a year away from entering the teaching profession, have little knowledge about the needs, resources, and supports required to effectively teach mathematics to Latino/a ELLs. Furthermore, U.S. teachers are not participating in professional development related to teaching ELLs (Wenglinsky, 2002). These problems suggest that as a field we need to address some important challenges:

- What are the critical elements that need to be added to teacher education programs so that teachers are better prepared and supported to make informed decisions regarding teaching mathematics to Latino/a ELLs?
- What are effective methods for identifying and challenging PSTs' assumptions and beliefs about teaching Latino/a ELLs?
- When we identify resources and supports for ELLs through research, how do we initiate implementation of their effective use in a larger number of classrooms with Latino/a ELLs?

First and foremost, we need to improve mathematics teacher education programs and professional development opportunities to address misconceptions related to teaching mathematics to Latino/a ELLs. Teacher preparation programs should include experiences specifically aimed at addressing the often implicit assumptions, beliefs, and expectations that teachers have in regard to students from linguistically and culturally diverse backgrounds. These experiences should not be isolated in one course, but rather should be purposefully integrated across the curriculum (i.e., these experiences should not be limited to one course such as "multicultural education" or "culture and society") and specifically addressed within the mathematics context. Moreover, these experiences need to include opportunities to expose assumptions PSTs might hold *about students* based upon their ethnicity, race, and educational background so that they can develop more critical and reflective pedagogy. Coursework that delves into multicultural curriculum is vital, but also must address the often "hidden beliefs" teachers hold about Latino student achievement and knowledge (Gutiérrez, 1999; Rolón-Dow, 2005).

After administering the prompts to our PSTs, we compiled their responses and distributed a summary of comments back to the PSTs. We asked them to analyze and discuss what they noticed and what they still needed to learn about teaching mathematics to ELLs from different countries of origin. The instructor then fueled and facilitated what became a heated debate among students where assumptions and misconceptions were questioned and challenged. Next, the instructor provided a variety of experiences over the course of the semester that continued to address these assumptions and misconceptions. These experiences included the examination and discussion of (a) videos of successful Latino mathematics students who participated in a community of learners, (b) ELL student work, (c) research about ELLs, and (d) newspaper articles related to ideas such as ELL policies in U.S. schools and reasons for low mathematics achievement for Latinos. The instructor recognized that challenging PSTs' beliefs would take a significant investment of time and may not change during one 15-week course.

Second, teacher preparation programs must include opportunities for PSTs to learn how to establish and maintain productive learning environments for ELLs, and specifically Latino/a ELLs. PSTs need to view images, from video or field experiences, of effective mathematical learning environments. For example, they need to see:

- how Latino/a ELLs can participate in productive peer interactions as members of a community of learners;
- how teachers can support Latino/a ELLs without isolating them or outsourcing the work to others;
- how a mathematical task can be enhanced to grant access to all the students in the classroom, including the Latino/a ELLs; and
- how Latino/a ELLs can flourish when they are challenged mathematically.

Third, teacher preparation must include content that develops knowledge and competencies in teaching mathematics to Latino/a ELLs. For example, viewing or observing productive learning environments will not necessarily enable PSTs to facilitate these environments themselves. Shulman (1986) outlined a variety of knowledge bases that teachers need to develop including content knowledge, pedagogical knowledge, pedagogical content knowledge, and curricular knowledge. As a result, in recent years, there has been greater attention placed on developing pedagogical content knowledge, or content knowledge for teaching mathematics (Hill, Ball, & Schilling, 2008). Unfortunately, discussions in mathematics education have not given sufficient attention to developing teacher knowledge related specifically to teaching ELLs and Latino/a ELLs. The development of this knowledge should not be deferred to additional certification programs or professional development, but rather should be initiated early in the preparation process.

Finally, PSTs' responses indicated a strong preference for outsourcing ELLs to other personnel within the school (e.g., ESL teacher, translator, etc.), but no mention was made of the need for proactive collaboration with ESL teachers or bilingual colleagues. Given this, we strongly recommend that prospective and practicing teachers, along with their students, collaborate with ESL teachers and bilingual colleagues in order to draw upon their colleagues' expertise as well as build the capacity of the school to respond to the needs of ELLs.

The three critical misconceptions identified above (i.e., differential treatment of ELLs based on country of origin, isolation rather than community, and outsourcing to meet ELL needs) as well as the related implications for teacher preparation also need to be addressed with practicing teachers. The current teaching workforce in the U.S. has not had sufficient preparation for teaching mathematics to Latino/a ELLs. Achieving excellence and equity for Latino/a ELLs in current and future K-12 mathematics classrooms will require a significant investment in teacher preparation and professional development.

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Appendix: Writing Prompts Given to Pre-service Teachers

No names please. We just want your honest thoughts.

Imagine it is your first year of teaching. You have taken a position as a third grade teacher. One of your students has just moved to the United States from Central America. What do you think this child will need during mathematics instruction? How will you assess the child's needs? How will you support this student during mathematics?

Imagine it is your first year of teaching. You have taken a position as a third grade teacher. One of your students has just moved to the United States from China. What do you think this child will need during mathematics instruction? How will you assess the child's needs? How will you support this student during mathematics?

Do you anticipate the needs would be different depending upon the continent of origin?

How comfortable would you feel about these two different situations?

Thank you for sharing your thoughts.

Discussion And Reflection Enhancement (DARE)
Post-Reading Questions

1. What role might student country of origin play in your instructional strategies or expectations?
2. Are there norms in your classroom that facilitate isolation rather than community?
3. What role does outsourcing play in meeting the needs of Latino/a ELLs in your context?
4. How can we best prepare PSTs to teach mathematics to Latino/a ELLs?
5. How would prospective or practicing teachers you work with respond to the questions in the Appendix? Try it!

"DARE to Reach ALL Students!"

TODOS Mathematics for ALL

"[Equity is] being unable to predict students' mathematics achievement and participation based solely upon characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language."

-- Rochelle Gutiérrez (2007)

Investigating Mathematics Teaching Practices in Latin America: Reflections on Preparing U.S. Teachers of English Learners

M. Alejandra Sorto

Abstract

This article analyzes teaching practices in mathematics from two elementary classrooms in Costa Rica, a Spanish-speaking Latin American country. Insights and examples from the analysis may inform mathematics teacher educators and teachers of English learners in the U.S.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. Many Latino/a students come from and previously studied mathematics in Latin American countries. What do *you* think math lesson looks like in those countries?
2. How do you think teaching practices in Latin American countries are similar to or different from those in the U.S.?
3. In your everyday practice, have you had any students commenting about the way they learned mathematics in their native country? If so, what was your reaction?

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Investigating Mathematics Teaching Practices in Latin America: Reflections on Preparing U.S. Teachers of English Learners

M. Alejandra Sorto

International Comparisons of Mathematics Teaching

The inspiration for this paper comes from lessons learned observing the teaching of mathematics in other countries. By focusing on comparing and describing teaching practices and mathematical discourse, the work described in this article will expand our knowledge of countries about whose teaching practices we know very little of while addressing two of the most pressing challenges in mathematics education: educating *all* children and producing *qualified* bilingual teachers (NCTM, 2000; No Child Left Behind, 2002).

An extensive international comparison about the teaching of mathematics is the TIMSS 1999 Video Study (Hiebert et al., 2003), which built upon the TIMSS 1995 Video Study (Stigler & Hiebert, 2004). The 1995 Video Study examined teaching practices of Japan, Germany, and the United States in 8th-grade classrooms. One major finding was that Japanese teachers taught differently than those in the U.S. by having students engage in solving, presenting, and discussing problems. The 1999 Video Study was extended to high-achieving countries Australia, Hong Kong, Switzerland, the Netherlands, and the Czech Republic. This latter study found that countries using methods other than the Japanese methods could still produce high-achieving students in mathematics. One common finding in the higher scoring countries was the *effective* implementation of tasks or problems requiring higher-order thinking and reasoning.

In a parallel 13-country study on third and fourth-grade student mathematics achievement in Latin America (UNESCO, 1998), the top-scoring country was Cuba, followed by Chile and México. The UNESCO also identified, based on a multivariate analysis of associated factors, seven countries (Argentina, Chile, Costa Rica, Cuba, Colombia, Bolivia, Venezuela) with 'outstanding schools' — "schools whose students demonstrated achievement in mathematics above that which would be expected, given the educational level of their parents" (UNESCO, 2002, p. 8). Carnoy, Gove, & Marshall (2007) followed up the achievement study with a comparison study that included a video examination of teaching practices in 10-12 third grade mathematics lessons each from Cuba, Chile, and Brazil. The comparison results are consistent with the TIMSS results. Cuban and, to a lesser extent, Chilean (private school) teachers engage students successfully in tasks that require students to reason and think, such as:

> If asked to indicate whether or not 430 is divisible by 10, Cuban students would be expected to explain that the zero in the units place is an indicator that 430 is a multiple of ten and is therefore divisible by 10 (Carnoy, Gove, & Marshall, 2007, p. 134).

The purpose of the TIMSS and Carnoy, Gove, & Marshall (2007) studies was to describe a pattern of teaching in high-achieving countries and, in the case of the latter study, to understand better the sources of between- and within-country variation that statistical models can detect but not illuminate (McEwan & Marshall, 2004). However, we can take further advantage of these two sets of studies and link them in natural ways to have a greater impact on the improvement of teaching in the U.S. classrooms. A natural connection is to look at what U.S. and Latin American classrooms have in common: children born in Latin America whose native language is Spanish. According to Jefferys (2007), México and Central American countries (after Cuba) are the leading regions of Spanish-speaking countries from where legal permanent residents come. School-age children of these populations also have the lowest mathematics achievement test scores (NCES, 2004). Even though México and some countries in Central America have (relatively) high or outstanding achievement, not much is written about what their mathematics lessons look like or how their practices compare to other countries.

The main goal of this paper is to illustrate the potential of the examination of teaching practices in mathematics classrooms of our Latino students' native countries by 1) *uncovering different ways children learn mathematical concepts* in their native countries; and 2) *providing authentic classroom situations* reflecting the quality of mathematics instruction for future use by prospective and practicing teachers as well as teacher educators.

Teaching Practices in Central American Countries

In 2006-2007, a large-scale study (involving 385 teachers at 97 randomly-selected schools) was conducted in two Central American countries: Costa Rica and Panama

(Carnoy, M., Gove, A., & Marshall, J. H., 2007). The purpose of this study was to document qualitative differences in both countries' educational systems in order to explain the large differences between countries in student achievement. Different measures were used for this purpose including teacher knowledge questionnaires and videotaped lessons. The results showed that Costa Rica's teachers performed better in the teacher knowledge measures. The lessons, especially at the third grade level, are characterized as being more conceptually focused, having higher levels of cognitive demand for the student tasks, and having longer lessons that allow for exploratory and discovery activities (Sorto, M. A., Marshall, J. H., Luschei, T. F., & Carnoy, M., 2009). An observation derived from video analysis of the lessons is that high quality teaching was achieved in rural classrooms even with very limited instructional resources. For example, in most rural schools in both countries, there was no evidence of any manufactured set of manipulatives, technological equipment, textbooks, prepared handouts, or educational posters. However, there were creative teachers that used what was available in the environment to teach effectively.

To illustrate further this observation, a more qualitative analysis of these practices is necessary. The deeper analysis is an attempt to extract more out of the rich source of data in terms of the mathematical discourse and the role of the teacher's pedagogical choices. Hence, two lessons from Costa Rica with the same instructional goal but different pedagogical approaches were selected, transcribed and translated to explore a mathematical discourse that relates to issues of language for Latino/a students when engaged in tasks with high-level cognitive demand.

Third Grade Lesson 1

This lesson corresponds to a rural public school and the number of students in this particular classroom was large (36 to 40), especially for the physical space. The teacher had the students sitting on benches arranged in continuous rows with enough space for her to circulate from the front row all the way to the back in a snake-like path. This arrangement also allowed students to interact with their partners at each side. The classroom had only one desk in front, a blackboard, and chalk. Teaching materials consisted of a large protractor and wooden sticks use to make popsicles and that are sold at local convenience stores. The main goal of this lesson was to identify angles, sides, and vertices in regular polygons. What follows are two episodes from the lesson (student names in this article are pseudonyms):

T1: Vamos a tomar tres paletitas y vamos a ver que figura nos sale. [*We are going to take three sticks and we are going to see what figure comes out].*

Students work individually on their desks with three sticks of the same length, they all form a triangle.

T1: ¿Que nombre le pondríamos a esta figura? [*What name would you give to that figure?*]

Sonia: Yo le puse triangulo [*I named it triangle*]

T1: Un tri-angulo. [*A tri-angle*] (The teacher emphasizes separation of letters.)

T1: ¿Que características tiene ese triangulo que me han formado ustedes allí? Obsérvenlo bien y luego me dicen porque se llama triangulo. ¿Que le ven ustedes a esa figurita que formaron para que se llame triangulo? [*What characteristics does that triangle that you have made have? Observe carefully and then tell me why it is called a triangle. What do you see in that figure that you have made for us to call it a triangle?*](Several students raise their hands saying "yo, yo" [*me, me*] and the teacher picks Karla in the front row.)

Karla: Tiene tres lados. [*It has three sides.*]

T1: Ahora con el dedito índice yo quiero que me señale esos tres lados. [*I want you to point out those three sides with your finger*]

T1: ¿Que otras características ven en ese triangulo? [*What other characteristic do you see on that triangle?*]

Mario: Tiene tres vertices. [*It has three vertices.*]

T1: Ha, tiene tres vértices, y ¿cuales son los vértices?, a ver señálemelos. ¿Y que son los vértices? Yo se que ya me lo dijeron, pero quiero que me lo recuerden. [*Ah, it has three vertices, and which ones are the vertices? Let's see - show them to me. What are the vertices? I know you already told me, but I want to be reminded.*]

Susana: Son las esquinitas. [*They are the little corners*]

T1: Las equinitas…si, pero anteriormente me lo dijeron en forma diferente. [*The little corners…yes, but you told me something different before.*]

Students had worked at the beginning of the class defining angle, sides, and vertex using only two sticks.

María: Es donde se unen los lados. [*It is where the two sides join.*]

T1: Ha, son las equinitas, o sea, donde se unen los lados. [*Ah, they are the little corners, that is, where the two sides join*]

T1: ¿Que mas ven? [*What else do you see?*]

Karla: Una línea horizontal y dos líneas inclinadas. [*One horizontal line and two slanted lines.*]

Julio: ¡Forma un ángulo! [*It makes an angle!*]

T1: ¿Un? [*One?*](Asking directly to Julio.)

Several: Dos [*Two*]

Julio: ¡Tres! [*Three*]

Students continue the lesson adding one more stick and making figures with four sticks. Here the majority of students formed squares but some formed non-square rhombi.

The class investigated the common characteristics of both figures and deduced the differences as well. One episode of this part of the lesson is worth mentioning because of the use of the expression "lados rectos" potentially to mean right angles and not necessarily straight sides (see further analysis of this episode in the next section):

T1: Vamos a ver si todos están de acuerdo con Karina. Ella dice que su figura tiene cuatro lados rectos. ¿El de ustedes también? [*Let's see if everyone agrees with Karina. She says that her figure has four straight (right) sides, do yours too?*]

Simon: No, el mío no porque yo forme un rombo. [*No, mine doesn't because I made a rhombus*.]

The last part of the lesson involved adding one more stick to form a pentagon and investigating its characteristics.

Third Grade Lesson 2

This lesson corresponds to an urban public school, the number of students in this classroom was also large (33 to 35) but with more physical space than the classroom in Lesson 1. The teacher had the students sitting in individual desks arranged in small groups. This classroom, unlike the rural classroom in Lesson 1, had a whiteboard, overhead projector, markers, and posters. Teaching materials consisted of large construction paper for each group, color markers, and rulers. As in Lesson 1, the main goal of this lesson was to identify angles, sides, and vertices in regular polygons. What follows is one episode of the lesson:

T2: ¿Quien recuerda que es un polígono? [*Who remembers what a polygon is?*]

Oralia: Una figura cerrada [*A closed figure*]

T2: Cuando estudiamos polígonos, dijimos que estaban compuestos por tres elementos, ¿Cuales son esos elementos? [*When we studied polygons we said they were composed of three elements. What one are those elements?*]

Alan: Líneas rectas [*Straight lines*]

Sam: Agudo [*Acute*]

T2: No, recuerden que los tres elementos son vértices, lados y ángulos. [*No, remember that the three elements are vertices, sides, and angles.*]

The teacher draws on the board a triangle, rectangle, pentagon, and hexagon, and then asks students to do the same on construction paper. While some are still drawing, the teacher gives the following color code: vértices - rojo, lados - azul, ángulos -verde. She then creates a triangle by making red dots at the vertices, coloring the sides in blue, and drawing green arcs for the angles. Then she sends one student at the time to the board to create the other polygons. Students cut and pasted the figures and were asked to leave space between each.

T2: Ahora quiero que me escriban abajo de cada figura lo siguiente: Numero de lados, numero de vértices, numero de ángulos. [*Now, I want you to write below each figure the following: number of sides, number of vertices, number of angles*]

T2: Yo les enseño como hacerlo para el triangulo. Numero de lados 3, numero de vértices 3, numero de ángulos 3. Ahora háganlo ustedes para el resto de los polígonos. [*I will show you how to do it with the triangle, number of sides 3, number of vertices 3, number of angles 3. Now you do it for the rest of the polygons.*]

Comparison of Teaching Practices in the Two Lessons

These particular examples show us why examining practices and mathematics discourse in Latino countries is worth investigating for three reasons. First, it *reveals* how children learn and how knowledge is acquired as a function of the instructional technique. In Lesson 1, children are given the opportunity to manipulate models of abstract objects, observe them, draw their own conclusions, and make generalizations. It further reveals, among other things, that the skill of recognizing angles in closed figures like a triangle is not trivial. First, even though the teacher emphasizes the word "tri-angle" from the beginning of the lesson, angles is the last thing the students "see" as a characteristic. One possible explanation for this phenomenon might be that the curriculum sequence places measuring angles after classification of figures by sides. Another reason could be the level of abstraction– it is harder to physically "see" an angle than see sides. Second, students first saw only one angle in a triangle, then two, and finally all three. This little episode shows us how the children construct their knowledge and make connections with previous knowledge, in this case, from knowing about angles involving two segments to angles formed by closed figures. By contrast, Lesson 2 gives the opportunity only to know the terminology, associate it with an object (a dot, a line segment, and an arc), and translate procedures.

Secondly, it *illustrates* the effectiveness of the teaching techniques presented in the two lessons. Lesson 1 uses sticks to model angles and polygons, and questioning techniques. Lesson 2 uses drawings, color codes, and student participation at the board. Neither of the teachers used printed materials, like worksheets or any type of technologies. In fact, from the video, it is not clear if these classrooms had any electricity at all. The pedagogical technique itself does not make one lesson better than the other. It suggests that it is a very special kind of teacher's knowledge that integrates the mathematics with the pedagogy in an effective way (Ball & Bass, 2000).

A third reason is that it *unveils* the role of language (in this case, Spanish) when teaching mathematics. In Lesson 1, children are looking for characteristics in squares and rhombi, one says "tiene cuatro lados rectos" [it has four right sides *or* it has four straight sides]. In Spanish the word "recto" has multiple meanings outside mathematics which I will not discuss here, but even within mathematics the word "recto(a)" is used to describe both "lineas rectas" or "segmentos rectos" [straight lines, straight segments] and "angulos rectos" [right angles]. It is not clear what the child means by "lados rectos" [straight or right sides]; it could be that she is now defining straight sides as those that form a right angle. We can conjecture only that the learners are using the word "right" in the sense of "up*right*" sides to mean sides that form 90-degree angles. (The term "right angle" comes from the Latin *angulus rectus*, where *rectus* means upright and relates to being perpendicular to a horizontal baseline, thus suggesting a person standing upright.) This would explain why the rhombus does not have "up-right" sides. If this is the case, the students do have the correct understanding but they are using a conventional word used for angles in an unconventional way. This less conventional way to talk about the characteristics of an angle in terms of the direction of its sides confirms the assertion that the concept of angle is a difficult concept to articulate for young children.

Reflections for English Learners Teacher Preparation

Future plans for this kind of analysis of teaching practices in Latin America are to help to the preparation of English learners in this country. One concern of using this approach to tackle mathematics education for English learners is that instructional practices occur in very different classroom, cultural and social environments. In particular, Latin American classrooms that are monolingual lack many of the complexities of U.S. mathematics classrooms with a large proportion of English learners. So, the question is, how can a study of monolingual classrooms from other countries help with the challenges teachers face in U.S. classrooms with a large proportion of English learners? The straightforward answer is grounded in research: students' previous knowledge of mathematics is fundamental to constructing new understanding, and the only way to know *what* and *how* students know mathematics before attending school in the U.S. is by observing and understanding the instruction in their native language (Moschkovich, 2007). Teachers can generally learn about their students' prior knowledge by experience, studying the curriculum at the lower levels, or attending school in their country of origin.

One way bilingual teachers acquire this knowledge is by informal talks with parents or with someone from the student's homeland. Even bilingual teachers that attended school in Latin America did not learn the mathematics with the purpose of teaching others - that is, they may know algorithms and general vocabulary, but they do not know necessarily how these early concepts were acquired and taught. Now, a second question arises: if knowing what and how children learned previously is important, why do we have to analyze entire mathematical lessons? In other words, what justifies the analysis of classroom practices? The answer is also grounded in research in that students' prior learning experiences are better understood when the learning is analyzed in the context of instructional dynamic (Ball & Forzani, 2007). Instructional dynamic takes into account the interactions among teachers, students, and content, in various environments. The common factor in the

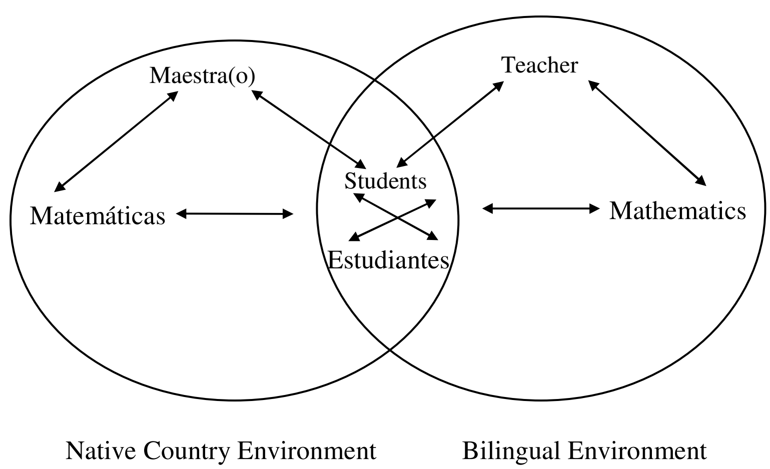

Figure 1. Instructional dynamic model (adapted from Ball & Forzani, 2007)

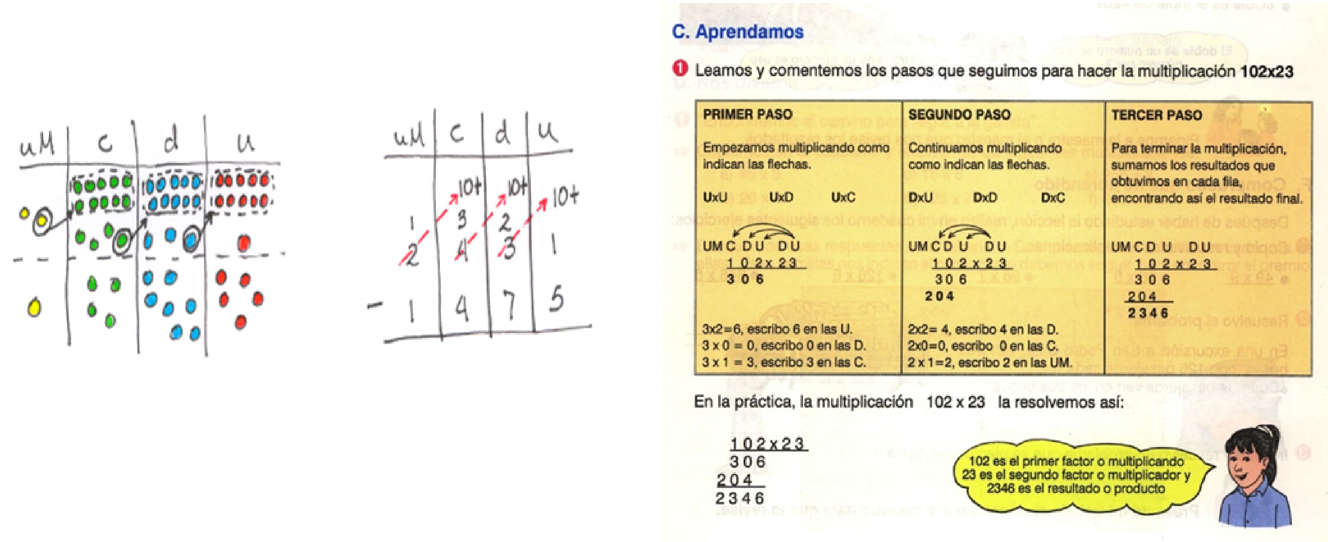

Figure 2. Subtraction algorithm work sample similar to *Matematica 3, Educacion Basica General*, Ministerio de Educacion (MEDUC), Panama, 2003. Multiplication algorithm from *Matematica 3, Escuela Morazanica*, Secretaria de Educacion, Honduras, 1997.

two instructional dynamics is the set of students (see Figure 1). The hypothesis for further research is that bilingual teachers' knowledge of students' instructional dynamic in their native country will help in the transition to the instructional dynamic in their current country of residence.

One more source of data that can be useful for understanding English learners' previous experiences in mathematics is the examination of textbooks. Figure 2 provides examples of the way third-grade Central American textbooks present the algorithms of subtraction and multiplication. Examining alternative algorithms can be a powerful tool to understand U.S. textbook procedures better. More instructional materials, lesson plans, and students' notebooks can be collected to enrich the other sources of data and to use them in the preparation of prospective and practicing mathematics teachers of English learners. This can be accomplished using the same type of activity from Sowder, Sowder, & Nickerson (2010) for preparing prospective teachers about the use of different algorithms by analyzing *nine* different ways children subtract 79 from 364, which includes the equal additions method used in many countries. The researchers claim, "when teachers realize that they must understand methods other than the ones they use, they are motivated to learn to attend to students' reasoning" (ibid, p. 54). The subtraction algorithm presented in Figure 2 can be added to the collection as yet another way to subtract using the abacus as a tool to demonstrate the regrouping process.

Closing Thoughts

Teaching quality in Latin American countries varies as in other parts of the world, and studying teaching practices suggests that variation is not necessarily associated with the students' social economic status or teachers' available instructional resources. This paper's lessons illustrate how teachers in rural areas lacking basic technology and manufactured teaching tools may use their knowledge of student concept development to support an environment for exploration in mathematics. More generally, analysis of teaching practices and materials can inform preparation of future teachers of English learners by increasing their awareness of the type of learning experiences some of their students may have had in their native countries.

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"Different solutions, interpretations, and approaches that are mathematically sound must be celebrated and integrated into class deliberations about problems. All members of the classroom group must accept the responsibility to engage with and support one another throughout the learning experience."

-- Position Statement on Equity in Mathematics Education (NCTM, 2008)

Discussion And Reflection Enhancement (DARE)
Post-Reading Questions

1. Describe the effect of the two teachers' pedagogical approaches on student learning. Which approach do you believe is more effective and why?

2. What are specific examples in the classroom dialogue excerpts of language issues that can inform the teaching of Latino/a students in the U.S.?

3. How can the use of teaching practices comparisons help teachers of Latino/a students?

4. What are some examples of alternative algorithms and how could they help Latino/a students learn the algorithms used in the U.S.?

5. Try this: Next time you teach geometry, give students two sticks and ask them to make a *right* angle. If any of your Latino students make a 180º angle, can you tell if they are interpreting the word "right" as in "straight" – since in Spanish (recto), the word "right" can mean straight? (Note: A straight angle in Spanish is called "ángulo llano o colineal.")

6. As a U.S. teacher, do you find it surprising and/or inspiring that Latin American classrooms with very limited resources have had high achievement? Why?

"DARE to Reach ALL Students!"

TODOS
Mathematics for ALL

"Students who are not native speakers of English, students with disabilities, females, and many nonwhite students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations."

– Principles and Standards for School Mathematics (NCTM, 2000)

Revisiting Board Games: A New Twist on a Familiar Activity

Margarita Jiménez-Silva, Conrado L. Gómez and Janel White-Taylor

Abstract

Through collaboration between university faculty, pre-service teachers, and local teachers, mathematical board games were developed to teach mathematical concepts to students, especially English Language Learners (ELLs). Guidelines, resources, and examples are provided.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. What barriers discourage ELLs in their mathematics learning?
2. What are mathematical board games and what are the potential benefits of playing them?

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Revisiting Board Games: A New Twist on a Familiar Activity

Margarita Jiménez-Silva, Conrado L. Gómez and Janel White-Taylor

In the last few years, 13-year-old Hispanic students have shown academic gains in mathematics achievement (National Center for Education Statistics, 2009). However, achievement differences still remain between Hispanic and White students (e.g., see National Center for Education Statistics, 2009; Arizona Department of Education, 2010). The purpose of this article is to revisit the familiar object of board games with a new twist: the use of board games created by pre-service teachers to motivate the learning of mathematics for Hispanic and ELL students in grades 4-8.

Humans categorically seek out reasonable challenges (Piaget, 1972). Once the challenge is identified, humans are motivated to behave in ways to surmount challenging activities (Brown, 2007). Challenges produce a state of mind that brings about focus and engagement, leading to improved task performance (Egbert, 2003). We applied these theories to the development of board games for students in grades 4-8 in hopes of having the games serve as a catalyst for increasing motivation in mathematics learning. This goal was a direct response from our conversations with teachers, who indicated the desire for strategies to motivate students who were bored with traditional drills and textbook approaches. Finding that prizes and even grades had limited power to improve students' motivation and attitude, teachers were seeking motivating mathematics activities "for which there is no apparent reward except the activity itself." (Deci, 1975, p. 23).

Intrinsic Motivation

In an age of accountability with standards, benchmarks, and standardized tests, teachers appear to have limited freedom and opportunities to develop intrinsic motivation in students. Yet, it is difficult to instruct students without addressing motivational variables (Brown, 2007). Brown recommends that if teachers are mandated to use unattractive textbooks, they should design interesting activities to engage students with the material. Brown (2007) makes the following suggestions to develop intrinsically motivating activities:

- Develop activities that appeal to the students.
- Present the activity in an enthusiastic manner.
- Make sure students are aware of the purpose of the activity.
- Provide choice of activities and how to fulfill the goals of the activities.
- Provide activities that encourage students to discover for themselves the principles and rules.
- Embed in the activity ways for students to develop and use effective strategies of learning and communicating.
- Design activities that develop students' independence and autonomy.
- Incorporate cooperative negotiation and interaction with other students.
- Make the activity challenging.
- Provide feedback on students' performance.

Most Americans fear mathematics, but enjoy mathematical puzzles (Burns, 1998) because games make mathematical thinking painless, interesting, and fun (Moscovich, 2001). Smith and Backman (1975) catalog a number of games and puzzles that have been used in teaching elementary and middle school mathematics. While there do not appear to be any research studies on middle school students playing mathematical games, the randomized experiment of Siegler and Romani (2008) demonstrated that playing a numerical game for as little as an hour can make a significant difference in mathematical understanding of low-income preschoolers. Teed (2010) used games in teaching entry level geoscience to encourage students to learn material outside of class while Kuiper (2010) used games to teach various statistical techniques.

Incorporating mathematics games from diverse cultures also can serve to motivate students. Bell and Cornelius (1988) and McCoy, Buckner, and Munley (2007) successfully used games from various cultures, including Hubbub (Native American), Mancala (African), Toma Todo (Mexican), Driedel (Jewish), Ashibi (Native American), and Lu-Lu (Hawaiian) to make connections across mathematics concepts. These games appealed to middle school students for several reasons. First, the games provided a rich and interesting context for applying probability concepts. Second, the games connected students to diverse cultures. Finally, the games were incorporated into pedagogically sound lessons. More recently, Lesser (2010) reported solid student engagement when he incorporated the Mexican game Toma Todo into a probability lesson he taught pre-service teachers.

Benefits to English Language Learners

Although board games can provide meaningful opportunities for engagement for all students, this game project can be particularly beneficial for ELLs and other students who need scaffolding to learn academic content in English. While acknowledging that not all Hispanic/Latino students are ELLs, many ELLs in the Phoenix metro area are from Hispanic/Latino backgrounds. It is a misconception that math is less difficult for ELLs because it is based on the language of number (Janzen, 2008). One of the barriers to learning in mathematics classroom for ELLs can be the issue of academic language (Echevarria, Vogt, & Short, 2010). Although mathematics vocabulary and everyday vocabulary can overlap, mathematics language is often used to express concepts not used conversationally (Ron, 1999). Faltis and Coulter (2008) refer to the academic language used in mathematics as a register that is acquired in school settings where specialized vocabulary and expressions are learned through apprenticeship. Students, especially ELLs, may need explicit scaffolding of the academic language required in mathematics. It helps to learn language that is embedded in the visual context provided by the equipment, other visual cues, and physical demonstrations (Gibbons, 2002).

Another challenge for ELLs, especially newcomers, is that the mathematical symbols and algorithms may vary from native countries to those taught in U.S. classrooms (Brown, 2005). For example, some Spanish-speaking countries use a period in place of a comma when writing numerals for multiples of a thousand (e.g., 4.232 versus 4,232) (Chamot & O'Malley, 1994). By engaging in games, ELLs can learn English as well as learn *in* English. Through games, they have opportunities to engage in mathematics activities that provide them practice and repetition in the language and symbols of mathematics. Although the venue of learning through games is not a new idea in education, the value added for students, especially ELLs, should not be underestimated. Learning has the potential to happen naturally as students play mathematics board games which reinforce and strengthen the academic language so critical to success in mathematics.

Designing and Utilizing the Mathematics Board Games

A partnership was established between a school district in the Greater Phoenix area and the Educational Technology Department at Arizona State University. The goal was to produce quality instructional materials to reinforce state mathematics standards as well as to increase academic achievement. To be fiscally sound, the plan called for using existing resources by training pre-service teachers how to produce mathematics materials using technology. In-service teachers were involved in evaluating the created materials using state academic content standards.

The task for developing games in mathematics using technology was appropriate for incorporating into an upper-division educational technology course for pre-service teachers because it used an authentic hands-on activity to help prepare them for the profession. In addition to the requirement to use technology to create the mathematics board games, the teachers in this educational technology class were also required to share their work with classmates by uploading the templates for their games to the project's Google Site.

Pre-service teachers read articles to learn about the reasons for using games (Kuiper, 2010) for instruction, and the steps in game design (Teed, 2010). They also read a collection of journal articles to learn about the effects of games in the classroom. Next, they reviewed the Edutopia (2010) video on commercially-produced games. Then, they reviewed games produced by others (Kuiper, 2010; Teed, 2010), and sample games designed by students from previous semesters. Research readings, sample games, etc., are at http://sites.google.com/site/mathgamesintheclassroom/.

For their design of the games, the pre-service teachers were given a rubric (available at the aforementioned website) that included categories such as creativity, use of graphics, appeal, and an evaluation of the rules, board and game cards. To help those who may want to create their own mathematics board games, we have included guidelines in the Appendix. (The set of guidelines differs from the rubric used with the pre-service teachers because the pre-service teachers were fulfilling a requirement of a specific course in designing games to meet specific learning objectives.) In-service teachers also should explore any commercially-manufactured (e.g., Quinn, Koca, & Weening, 1999) or technology-based games that address the identified learning objectives. If no suitable games are available, it may be best to design a game tailored to the learning objectives.

The pre-service teachers started designing their board games by selecting a grade-level strand and performance objectives. They then studied the directions from various commercially-produced and student-produced games. They knew beforehand that their games would stay at a school site. When the gameboards were finished, a "Game Day" was scheduled at several schools for pre-service teachers to set up their games in the school cafeterias to share with students and teachers. Game Day served to pilot the games with students to see if the students could readily learn the rules and remain engaged with mathematics content throughout the game.

Classes of K-8 students took turns playing the different games during the school day. Students often chose to spend their lunch time continuing to play the games. Game Day served as a valuable experience for the pre-service teachers because it provided an opportunity for them to observe how the students interact with the games. These observations provided an opportunity for them to see which games were popular with the students, and they often made notes of which games they wanted to obtain (from the Google Site) to use in their future classrooms. In addition, the teachers often provided helpful feedback regarding the games. This event also gave the teachers an opportunity to request which games they would like for their classrooms.

After Game Day, board games were graded by the instructor using the game rubric. Then, a team of master in-service teachers (selected by their principals for their commitment to innovative teaching) reviewed each game for educational soundness and readiness for use in a K-8 classroom, sometimes identifying what standards were the focus of the game. The in-service teachers appeared passionate about the games and spent significant time reflecting on the classroom usefulness of the games with many wanting to leave with the games that day, since the reviewing teachers were generally those who would be using the games in their classrooms. The board games that were rated as high quality were delivered to Title I schools, sometimes after modification by a team of interns. Games that were rated as not classroom-ready were returned to the student for a chance to modify and resubmit the game for consideration.

Example of a Created Mathematics Board Game: Extra Extra Exponents (E^3)

One example of a student-designed board game is Extra Extra Exponents (E^3). The game is designed to strengthen students' skills with exponents and learning conceptual patterns. The game exposes students to mathematical terms such as less than, greater than, coefficient, exponent position, and equations as well as gives students practice in writing expressions involving exponents. The game was intended for supplemental practice and for a creative way students could engage with exponents.

The E^3 gameboard is colorful and designed to be engaging to 6^{th}-grade students in order to increase their interest in using the mathematical concepts. The board is divided into seven vertical levels, each of which has its own set of question cards in one of three formats. In particular, some questions have a True/False format, while others involve rewriting an expression with an exponent to one without an exponent (or vice-versa). Students appeared to be engaged by needing to progress through the levels in order to try to win the game. E^3 provides playing instructions for students and also offers a "Helpful Hints Area" with implementation tips for teachers.

Conclusion

Incorporating mathematics board games into the classroom has the potential to benefit all students, especially ELLs. Board games can provide the motivation for students to grow in their understanding of mathematical concepts as well as reinforce skills previously learned. Additionally, through the use of board games, students are given the opportunity to interact and practice academic language that they would not typically use in peer interactions. Visit http://sites.google.com/site/mathgamesintheclassroom/ for updates such as a new math video game component, and may contact the third author to inquire about participating in the project. Additional context on this collaboration between a university and public school district appears in Jiménez-Silva, White-Taylor, and Gómez (2010).

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Appendix

Guidelines for Mathematics Board Games

When evaluating educational games:

- Identify the learning objective for the game
- Investigate if there is a commercially-produced board game or if a board game from another culture exists that meets the objective

If no game with your educational goals exists, use the following guidelines:

- Develop a concept for the game based on the learning objective
- Identify the goal for the game: How does the player win?
- Develop rules for the game
- Develop gameboard with appealing color and graphic design
- Develop or gather game pieces (i.e., tokens, game cards, dice)
- Teacher verifies game works and is free of spelling/ grammar errors
- Pilot the game with a small group of students and check that:
    - Ω students find game practical and easy to set up and implement;
    - Ω students find the game fun, engaging, and motivating;
    - Ω game is grade level / age-appropriate; and
    - Ω game is effective in helping students meet instructional objective.

Discussion And Reflection Enhancement (DARE)
Post-Reading Questions

1. What mathematics topics that you teach lend themselves most readily to a board game? Explain your reasoning.
2. From your experience in working with ELLs, what advantages do board games present in helping students develop their academic language in mathematics?
3. How might you combine assessment with the use of board games?
4. What ideas in this article apply to the website of mathematical games recently launched by NCTM at http://calculationnation.org?
5. What are the tradeoffs of using a commercially-produced game versus a game created by students or teachers and how would you decide which one to use for a particular topic?

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-- College Board president Gaston Caperton,
on Jaime Escalante (1930-2010), the HS teacher
who inspired the film "Stand and Deliver"

Born December 31, 1930, La Paz, Bolivia

Died March 30, 2010, Roseville, California, U.S.

http://en.wikipedia.org/wiki/Jaime_Escalante

http://commons.wikimedia.org/wiki/File:Jaime_Escalante.jpg

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