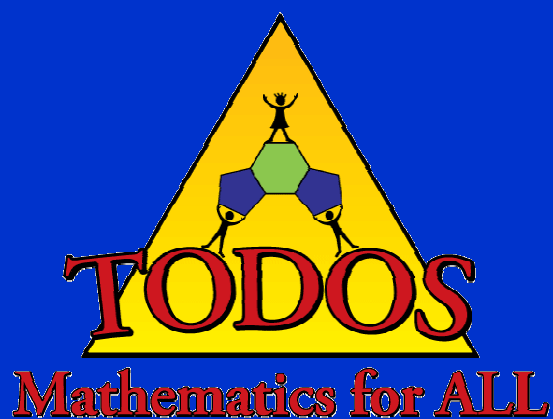


TEACHING FOR EXCELLENCE AND EQUITY IN MATHEMATICS

A PUBLICATION OF



an Affiliate Organization of the
National Council of Teachers of Mathematics

From the Editors

¡Bienvenidos! It is an exciting privilege to welcome everyone to the debut issue of this refereed TODOS journal: *Teaching for Excellence and Equity in Mathematics*.

Let us introduce our *TEEM* team of editors. **Cynthia Anhalt** has been an editor and contributor for *Noticias de TODOS* and is currently on the Instructional Faculty in Mathematics Education at The University of Arizona in Tucson, AZ. **Larry Lesser** has also been an editor and contributor for *Noticias de TODOS* and is an Associate Professor of Mathematics Education at The University of Texas at El Paso. **Miriam Leiva** is the founding president of TODOS, is Distinguished Professor of Mathematics Emerita at the University of North Carolina Charlotte, and is an author of Houghton Mifflin Harcourt Mathematics. Collectively, the editors have served on several national editorial or research boards, have published on equity/ELL issues in mathematics and statistics education, and have precollege teaching experience.

Since its launch in Spring 2005, the semiannual periodical *Noticias de TODOS* has served as much more than TODOS' newsletter by also including quality peer-reviewed and invited articles on pedagogical activities, curriculum, and issues on topics of interest. This past winter, the Board of TODOS voted to create *TEEM* as a separate (for now, annual) publication to focus on teacher-oriented articles to incorporate ideas for excellence and equity into teaching practices.

As stated in the flyer released at the 2009 NCTM annual meeting, *TEEM*'s intended audience includes math educators, practitioners, leaders, and administrators at all levels. The journal aims especially to engage mathematics education topics involving excellence and equity simultaneously (rather than either in isolation) in a way that connects research to classroom practice and can inform the practice of teachers or professional developers. One feature that helps ensure this accessibility and practicality is the "DARE" (Discussion And Reflection Enhancement) questions that bookend each paper. Information for prospective authors (and referees) may be found at <http://www.math.utep.edu/Faculty/lesser/TEEM.html> or by contacting us at teem@todos-math.org.

This inaugural issue is a bit unusual in that it consists of articles that were competitively selected by us (with input from key TODOS officers and former *Noticias* editors) as the most "*TEEM*-like" peer-reviewed papers from past issues of *Noticias de TODOS*, and then enhanced by the addition of a concise abstract, the thoughtful DARE questions, and some editorial processing. Future *TEEM* issues will generally consist of new papers submitted during the annual submission months (November and April) using guidelines at the website mentioned above.

We hope *TEEM* not only serves the current members of TODOS, but also brings in new members. We also hope *TEEM* serves as an inspiring pedagogical and scholarly resource for the broader mathematics education and education communities.

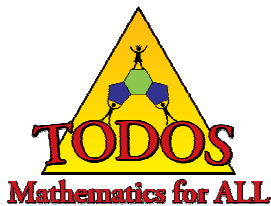
Enjoy the debut issue!

The Editors

The mission of TODOS: Mathematics for ALL is to advocate for an equitable and high quality mathematics education for all students— in particular, Hispanic/Latino students— by increasing the equity awareness of educators and their ability to foster students’ proficiency in rigorous and coherent mathematics.

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Framing Equity: Helping Students “Play the Game” and “Change the Game”

Rochelle Gutiérrez

Abstract

This article introduces a framework for equity that entails the dimensions of Access, Achievement, Identity, and Power. Beyond knowledge and skills, teachers need an “equity stance” that embraces and works to balance the tensions between these four dimensions.

Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. How do people in your working context define equity and what words do they use to discuss it?
2. How do *you* define equity and how do you know you are addressing it in your everyday practice?

”DARE” Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Spring 2008 *Noticias de TODOS*.

Rochelle Gutiérrez (rg1@illinois.edu) is an Associate Professor at the University of Illinois at Urbana-Champaign, and her research focuses on equity in mathematics education, race/class/language issues in teaching and learning, effective teacher communities, and social justice.

Framing Equity: Helping Students “Play the Game” and “Change the Game”

Rochelle Gutiérrez

“Equity” is a hot topic in mathematics education these days. However, for many people, addressing equity issues rarely moves beyond the goal of closing the achievement gap (Gutiérrez, 2008). For me, equity is ultimately about the distribution of power—power in the classroom, power in future schooling, power in one's everyday life, and power in a global society (Gutiérrez, 2002). I draw on the idea that equity must be framed with both dominant and critical definitions. In working with teachers, I have found it useful to explicate four key dimensions (Access, Achievement, Identity, and Power) and to highlight the relationships and tensions between them. Let me explain.

Access relates to the resources that students have available to them to participate in mathematics, including such things as: quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation, and infrastructure for learning outside of class hours. The *Access* dimension reflects the idea that students are affected by their “opportunity to learn.” However, a focus on access is a necessary but insufficient approach to equity, in part because it fails to redress past injustices. Besides giving students necessary resources, we also care about student outcomes, or what I categorize as *Achievement*. This dimension is measured by tangible results for students at all levels of mathematics, including such things as participation in a given class, course taking patterns, standardized test scores, and participation in the math pipeline (e.g., majoring in mathematics in college, having a math-based career). Moving from mere ac-

cess to achievement is important when considering that there are serious economic and social consequences for not having enough math credits to graduate from high school, not scoring high enough on a standardized achievement test to gain acceptance to college, or not being able to major in a math-based field that can confer a higher salary and prestige in society.

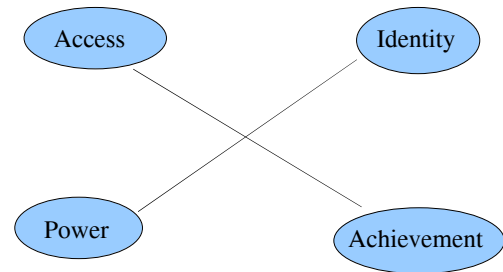
However, because many students find themselves down playing some of their personal, cultural, or linguistic capacities in order to participate in the classroom or the math pipeline and because some groups of students historically have experienced greater discrimination in schools, issues of *Identity* are also important to consider. For many mathematics educators, attending to students' identities means focusing on students' pasts (e.g., including the contributions of their ancestors). But, the identity dimension also concerns itself with a balance between self and others in a global society and acknowledges ways students are racialized (Martin, 2007), gendered and classed (Walkerline, 1988). It includes whether students have opportunities to draw upon their cultural and linguistic resources (e.g., other languages and dialects, algorithms from other countries, different frames of reference) when doing mathematics, paying attention to the contexts of schooling and to whose perspectives and practices are “socially valorized” (Abreu & Cline, 2007; Civil, 2006). The goal is not to replace traditional mathematics with a pre-defined “culturally relevant mathematics” in an essentialistic way, but rather to strike a balance between opportunities to reflect on oneself and others as part of the mathematics learning experience.

The Power dimension takes up issues of social transformation at many levels. This dimension could be measured in voice in the classroom (e.g., who gets to talk, who decides the curriculum) (Morales, 2007; Zevenbergen, 2000; Adler, 1998), opportunities for students to use math as an analytic tool to critique society (e.g., exploring “risk” in society) (Mukhophadyay & Greer, 2001; Skovsmose & Valero, 2001; Gutstein, 2006), alternative notions of knowledge (D'Ambrosio, 2006), and rethinking the field of mathematics as a more humanistic enterprise (Gutiérrez, 2002).

Access and Achievement can be thought of as comprising the dominant axis, preparing students to participate economically in society and privileging a status quo. The dominant axis, where access is a precursor to achievement, measures how well students can play the game called mathematics. Identity and Power make up the critical axis. The critical axis, where identity can be seen as a precursor to power, ensures that students' frames of reference and resources are acknowledged in ways that help build critical citizens so that they may change the game. All four dimensions are necessary if we are to have true equity. Learning dominant mathematics may be necessary for students to be able to critically analyze the world, while being able to critically analyze the world may provide entrance into dominant mathematics. It is not enough to learn how to play the game; students must also be able to change the game. As educators, we need to be clear on our stance—that we are advocates for our students to do both. Doing so requires situating ourselves in the tensions that exist in this work (Gutiérrez, 2009).

This equity diagram seeks not to simplify the complexity but rather to offer a useful “mapping space” for ideas when trying to reflect on one's practice. As a researcher, it is useful for me to see the kinds of ap-

Dimensions of Equity



proaches that teachers and families take to address equity. Take, for example, the issue of “power.” While teachers in interviews may say they “want to empower students,” they almost always mean it only as it relates to achievement, not with respect to helping students reach personal goals of excellence that may intersect with the doing of mathematics (e.g., helping their communities solve a local problem).

I am not implying that at the heart of all teachers' equity agendas is Access and/or Achievement while it is Identity and/or Power for most marginalized students and their families. Many educators already embrace the idea that students need to see themselves reflected in the curriculum and be offered opportunities to develop further agency in the world. My experience in working in urban communities is that some marginalized families do not want their students to develop “agency” in the ways that critical researchers seem to think is important, as they worry that it will take away from schools giving their students the tools to excel in school, or they feel they are already doing this “critical” work with their children at home.

As a researcher dedicated to equity, I attempt to situate myself in “Nepantla,” the crossroads of these tensions, to highlight the phenomena at hand. Being able to name the dimensions helps us move toward highlighting tensions between the dimensions so that we might be more reflective about how we can successfully balance attending to them all.

REFERENCES

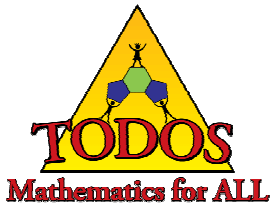
- Abreu, G. de & Cline, T. (2007). Social valorization of mathematical practices: The implications for learners in multicultural schools. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 118-131). New York: Teachers College Press.
- Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual mathematics classroom. *For the Learning of Mathematics*, 18(1), 24-33.
- Civil, M. (2006). Working towards equity in mathematics education: A focus on learners, teachers, and parents. In S. Alatorre, J. L. Cortina, M. Saiz, & A. Mendez (Eds.), *Proceedings of the 28th annual meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education*. Mérida, México. Universidad Pedagógica Nacional.
- D'Ambrosio, U. (2006). *Ethnomathematics: Link between traditions and modernity*. Rotterdam: Sense.
- Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance. *Democracy and Education*, 18(3), 9-16.
- Gutiérrez, R. (2008). A "gap gazing" fetish in mathematics education?: Problematizing research on the achievement gap. *Journal for Research in Mathematics Education*, 39(4), 357-364.
- Gutiérrez, R. (2007). Context matters: Equity, success, and the future of mathematics education. In T. Lamberg & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1-18). Stateline (Lake Tahoe), Nevada. University of Nevada, Reno.
- Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Towards a new equity research agenda. *Mathematical Thinking and Learning*, 4 (2&3), 145-187.
- Gutstein, E. (2006). *Reading and writing the world with mathematics*. New York: Teachers College Press.
- Martin, D. (2007). Mathematics learning and participation in the African American context: The co-construction of identity in two intersecting realms of experience. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 146-158). New York: Teachers College Press.
- Morales, H. (2007). The underlife of a high school mathematics classroom: Mathematical meaning-making among Latino/a students, paper presented at the annual meeting of American Educational Research Association. Chicago, IL.
- Mukhopadhyay, S., & Greer, B. (2001). Modeling with purpose: Mathematics as a critical tool. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 295-312). Mahwah, NJ: Lawrence Erlbaum.
- Skovsmose, O., & Valero, P. (2001). Breaking political neutrality: The critical engagement of mathematics education with democracy. In B. Atweh, H. Forgasz, & B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 37-55). Mahwah, NJ: Lawrence Erlbaum.
- Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. New York: Routledge.
- Zevenbergen, R. (2000). Cracking the code of mathematics classrooms: School success as a function of linguistic, social, and cultural background. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201-223). Westport, CT: Ablex Publishing.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What do you think it means to help students “play the game”? What do you think it means to help them “change the game”? In what way(s) do you believe you work to help students “change the game”? Why do we need to change the game? Is “the game” referring to mathematics or mathematics education or both?
2. Which of the four dimensions are addressed by an achievement gap focus? What is missing? Why is that important?
3. What are some examples of “past injustices” that an opportunity-to-learn view misses?
4. In what way(s) does Gutiérrez' concept of equity overlap with or depart from the way in which equity is articulated in the National Council of Teachers of Mathematics' position statement (accessed at <http://www.nctm.org/about/content.aspx?id=13490>)?
5. The four equity dimensions are written from the point of view of students and learning. Do these dimensions also apply to teachers and teaching? In other words, do teachers, administrators, and teacher educators need to be thinking about access, achievement, identity, and power with respect to teaching? If so, what might this involve?
6. Gutiérrez claims, “As educators, we need to be clear on our stance--that we are advocates for our students to do both. Doing so requires situating ourselves in the tensions that exist in this work.” What might it mean to situate oneself in the tensions that exist in this work?
7. Try this: The next time you teach, *make note of the ways in which you are attempting positively to address dimensions of identity and/or power in your classroom.* Would students agree with your list? What would it take to address more strategically these dimensions? Are there individuals in your working context that you feel are already doing this better with whom you can become an ally?
8. Read the Gutiérrez (2009) paper listed in the References. Come up with a tension you have experienced that is not highlighted in the article. Why is this tension important in an equity stance? In what ways do you embrace this tension and in what ways do you reject it?

“DARE to Reach ALL Students!”





A Reflection on my Work with Latino Parents and Mathematics

Marta Civil

Abstract

This article describes research from different parental engagement projects in mathematics. Through Latino/a parents' voices, we learn about their beliefs and values about mathematics education and these findings can inform those who work with students or parents.

Discussion And Reflection Enhancement (DARE) Pre-Reading questions:

1. What experiences have you had or do you know of that involve families and mathematics teaching/learning?
2. Based on your prior experiences and/or readings, what are some resources (specific to mathematics learning) that Latino/a parents (and their children) bring to school?
3. What are some challenges that you think Latino/a parents (and their children) face in terms of the teaching and learning of school mathematics?
4. What is your definition of “parental involvement” for this context?

“DARE” Post-Reading questions appear at the end of the article. This article (without DARE questions) originally appeared in Spring 2007 *Noticias de TODOS*.

Marta Civil (civil@math.arizona.edu) is a professor in Mathematics at the University of Arizona. Her research in mathematics education spreads over two areas: teacher education and equity in mathematics education—in particular, on a socio-cultural approach to the mathematics education of ethnic and language minority school age students and adults.

A Reflection on my Work with Latino Parents and Mathematics

Marta Civil

I have been working on issues related to Latino parents' engagement in mathematics education for almost fifteen years. Research with parents is indeed one of the key areas of research in our NSF-funded center called CEMELA (<http://cemela.math.arizona.edu>; Center for the mathematics education of Latinos/as). Currently our main research study in this area is looking at Latino parents' perceptions of the teaching and learning of mathematics. In this reflection, however, I focus on some of the highlights of the work I did prior to CEMELA as a way to explain the background for our current work.

One of the original motivations for my current work was and continues to be the idea of mathematics teaching innovations that could help us bridge the gap between in-school and out-of-school learning. Our work takes place in Mexican / Mexican-American, working class communities in the Southwestern United States. Our efforts have been geared towards the development of learning environments that build on the students' and their families' knowledge and skills. But, how do we uncover that knowledge and those skills that all families have? Through the Funds of Knowledge for Teaching project and then later during the Bridge project, the teachers (sometimes accompanied by university researchers) visited the homes of some of their students. Using in-depth questionnaires (on family history, labor history, perspectives on education, uses of mathematics at home and at the work place), these teachers were able to uncover some of this knowledge and those skills/experiences that reside within the households. Our challenge

then was to use this knowledge in learning modules that would make mathematics more meaningful (both from a cognitive and from an affective point of view) for the school children. These household visits certainly had an impact on the teachers in that they saw the families and "the home as a real learning place, real learning environment, you know I didn't think it was so much a learning environment as it is" (teacher's interview).

But to me the most rewarding and eye-opening experience was another activity we developed towards this effort to bridge the gap between in-school and out-of-school, namely, our mathematics workshops for parents. Through these workshops, we do not only learn about the parents' ideas and perceptions about the uses of mathematics and about their children's mathematical education, but we engage in joint explorations that allow us to establish a two-way conversation grounded on the learning and teaching of mathematics. It is important to note that the success of this approach is in its continuity: these are not isolated workshops, but rather series of sessions with the same group of parents: sometimes we call them "math for parents courses", but our preferred term lately is "tertulias matemáticas" (mathematical circles). We (parents, university personnel, and sometimes teachers) come together to do and talk about mathematics. Our workshops are modeled after the work we have been doing for years in professional development for teachers. They are highly participatory, hands-on, and centered on what we view as meaningful school mathematics tasks.

What have we found out from these “tertulias matemáticas”?



Parents engaged in mathematics

Parents like being learners of mathematics

Although most of the parents originally come because they want to help their children, they soon become interested in the content as learners themselves. The parents in our projects have made it very clear that academic mathematics is important to them. They want to learn this type of mathematics to help their children, but also for themselves, as this excerpt from a mother’s reflection captures quite well:

I am so happy with all these mathematics workshops because I realize how to help my children understand mathematics in a different way, from a fun approach, all together as a family. ... And also for us, because one never knows when we may use it, and this way we move forward, and no one is going to mandate that it has to be the way they say, because we also think and solve problems.

Parents value teaching for understanding

Parents enjoy finding out the “why” behind the many things they had memorized as children in school.

[This project] has been very different from my previous experience (with math). I went through my whole life being told how things were not and not given any freedom to figure it out on my own. Being able to experiment with blocks or whatever is much more interesting.

I’m amazed because [I see] something that I didn’t see before, and it clicks in my mind and I understand why things are the way they are. I get excited because now I know, I’m not accepting it, now I know why that is the way it is.

We argue that if parents learn mathematics with an emphasis on understanding rather than rote memorization, they are more likely to become quite vocal about the importance of understanding for their children’s mathematics education. As one mother very eloquently said, “I don’t want them [teachers] to teach to the test. You have to be versatile in many things. If you don’t understand, what’s the point?”

Parents bring their own beliefs and values

Like everybody else, parents often have deep-rooted beliefs about the teaching and learning of mathematics. A clear example of this is with the algorithms for division. In all the workshops where we have parents who learned how to divide in México, this topic comes up. As they compare the methods traditionally taught in México and in the U.S., comments along the lines of “their” method being more efficient (because they write less as they do the subtraction in their heads) always come up. These differences in approaches are fantastic opportunities to engage in not only the mathematics behind the different methods but also in a conversation about issues related to the teaching and learning of mathematics. Another salient topic among immigrant parents is the

differences between the educational systems:

No, I'm not happy [with the system at her son's school]. I feel that there is repetition of a lot of things; I don't understand why the teaching is so slow, I don't like it, I don't like the system, I don't like it at all. When we go to México, my nieces and nephews or my husband's nieces and nephews, they are children that are more or less the same age as Jaime and I see that Jaime is behind. Here they tell me that Jaime is really excellent.

To me, this is not about discussing which system is “better.” But we have to be aware that parents are going to bring up these comparisons and that these perceived differences may lead to conflict between parents and teachers. We all bring our valorization of knowledge. But these differences may also lead to conflict between parents and children:

Last night my son said to me that school from México was not valued the same as school here, that is, it doesn't count. What I studied there doesn't count here. He knows that what is taught here is different from what is taught there and so he says, 'why would I ask my Mom for help if she's not going to know.' So, there is a barrier.

Parents have mentioned to us that when they try to help their children with the mathematics homework, they often run into two obstacles—the language (English / Spanish) and the mathematical approach. Several of these parents have mentioned how they know the content but they do not know it the way their children are learning it and then they (or sometimes their children) feel that they cannot help them:

He [her son] doesn't feel very sure that I am understanding him because the problem is

written in English. I don't know how to read it and he doesn't...know how to translate well for me because he speaks Spanish and reads Spanish, but we say different things for the same words and questions, I think he thinks I studied differently.

Parents value “confianza” (trust)

As I mentioned earlier, this kind of work requires time. It is not about isolated workshops but about establishing rapport and connections with the families. I want to end stressing the importance of the concept of “confianza” when working with parents.

When I joined this group, for me the most important foundation was the confianza that each one offered me.... I can say that all that I now know and have learned has been accomplished by means of the confianza (a mother reflecting on her experience with the math workshops).

At last, I also have someone that more than a teacher is a friend and most importantly inspires me: Confianza, the confianza that I in particular never had with any other teacher of mathematics. ...Thanks to the confianza that exists in the group we can work without problems and pose any sort of question without fear.

This concept of “confianza” is not only important for the parents in the workshops. The final excerpt below is from a fifteen-year old reflecting on the impact that these workshops had on his mother:

Now that she [his mother] is attending these workshops she is learning in a different way, understanding the why of the formulas and where they come from and how they can be applied in her life; she shares it with the entire family and we all get involved in a mathematical gathering that is

fun. We are all teachers and students at the same time, there is no difference and that there be much respect and confianza is most important.

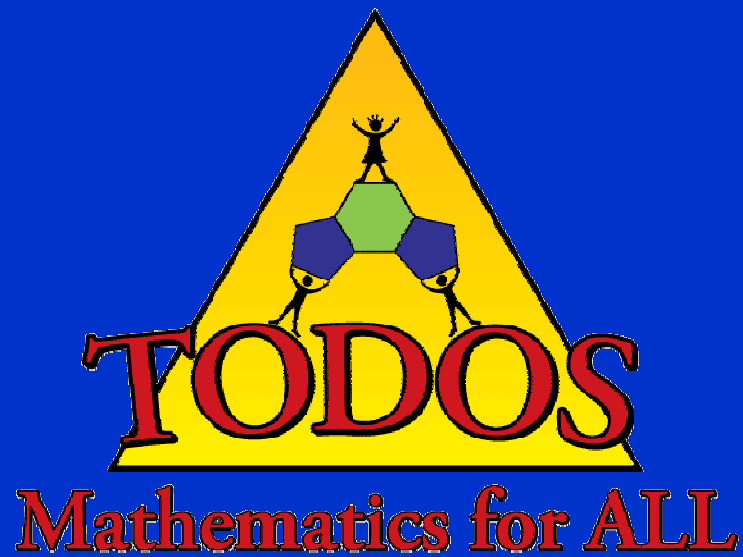
Acknowledgement: The work reported here was supported by the Educational Research and Development Centers Program, PR/Award Number R306A60001, as administered by the OERI (U.S. Department of Education) and by the National Science Foundation (NSF) under grants ESI-99-01275 and ESI-0424983. The views expressed here are those of the author and do not necessarily reflect the views of the funding agencies.

Discussion And Reflection Enhancement (DARE) Post-Reading Questions

1. What are some experiences you could develop to engage with parents in a two-way dialogue?
2. The article's author believes in encouraging the sharing of different approaches to doing mathematics such as multiple algorithms for the basic arithmetic operations. What experiences have you had (e.g., with your students) in sharing different approaches? What are some benefits and challenges involved with this?
3. What strategies do you (or could you) use to help overcome "the language barrier" that parents who do not feel comfortable with English may encounter when it comes to helping their children with homework or other school-related issues?
4. What are some implications of the concept of "valorization of knowledge" for the teaching and learning of mathematics?
5. What strategies could you use to learn more about the knowledge and experiences that your students and their families have? What implications would this additional information have for your teaching of mathematics?

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