

# Fostering Empathy through Mathematical Modeling K – 12

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## What Is Mathematics For?

Without pressing “enter,” type your response into the chat. It’s okay to write either the first thing you think of, the thing you think is most important, or a few different responses.

What Is  
Mathematics  
For?

Now press enter.

## What Is Mathematics For?

Now press enter.

Read colleagues' responses.

- Identify one other response that aligns with your response.
- Identify one response that is distinct from yours.
- Do you notice any categories of responses?

## Transcendence and Wisdom

“

... three broad classes of reasons to study mathematics. Only the first and most basic class is **practical**. It pertains to job skills and the needs of science and technology. The second concerns the understandings that are essential to an **informed and effective citizenry**. The last class of reasons involves considerations of curiosity, beauty, playfulness, perhaps even **transcendence and wisdom**.

”

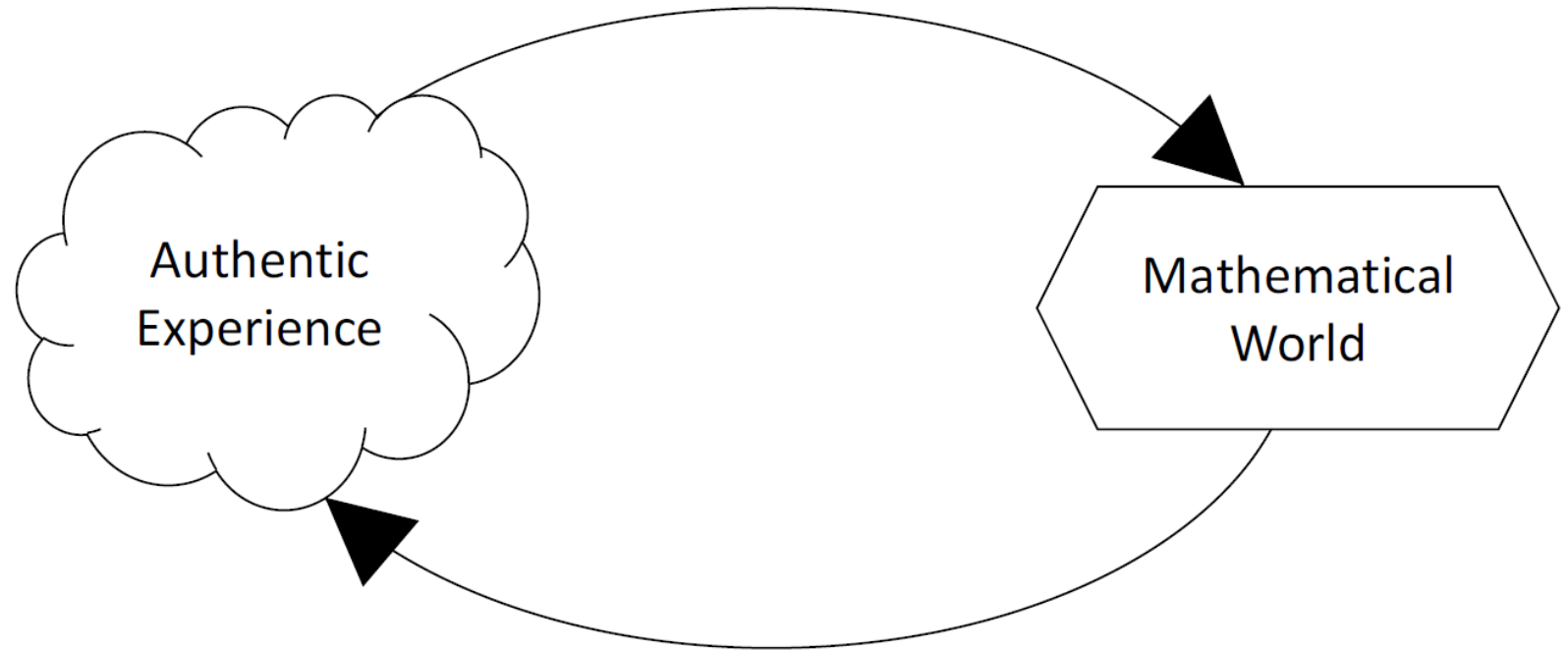
# What is Mathematical Modeling?

# Mathematical Modeling

Modeling is the process of creating representations of real-world situations in search of better understanding or solutions to problems.



# Mathematical Modeling



Do we have  
enough pencils  
to last the  
school year?



Do we have  
enough pencils  
to last the  
school year?

Different from  
“How many are there?”,  
“Do we have enough?”  
is a question central to justice.

## Modelers Develop and Use Empathy

Translating problems that have consequences in the experiences of human beings to and from the mathematical world requires modelers to make decisions about what is important.

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Judgments about what is important are tied to what people value.

Understanding what people value involves seeing situations from others' perspectives.

# Empathy

- How can one know what another person is thinking and feeling?
- What leads one person to respond with sensitivity and care to others?

# Empathy

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- What leads one person to respond with sensitivity and care to others?

Type one idea into the chat. Don't press enter until prompted

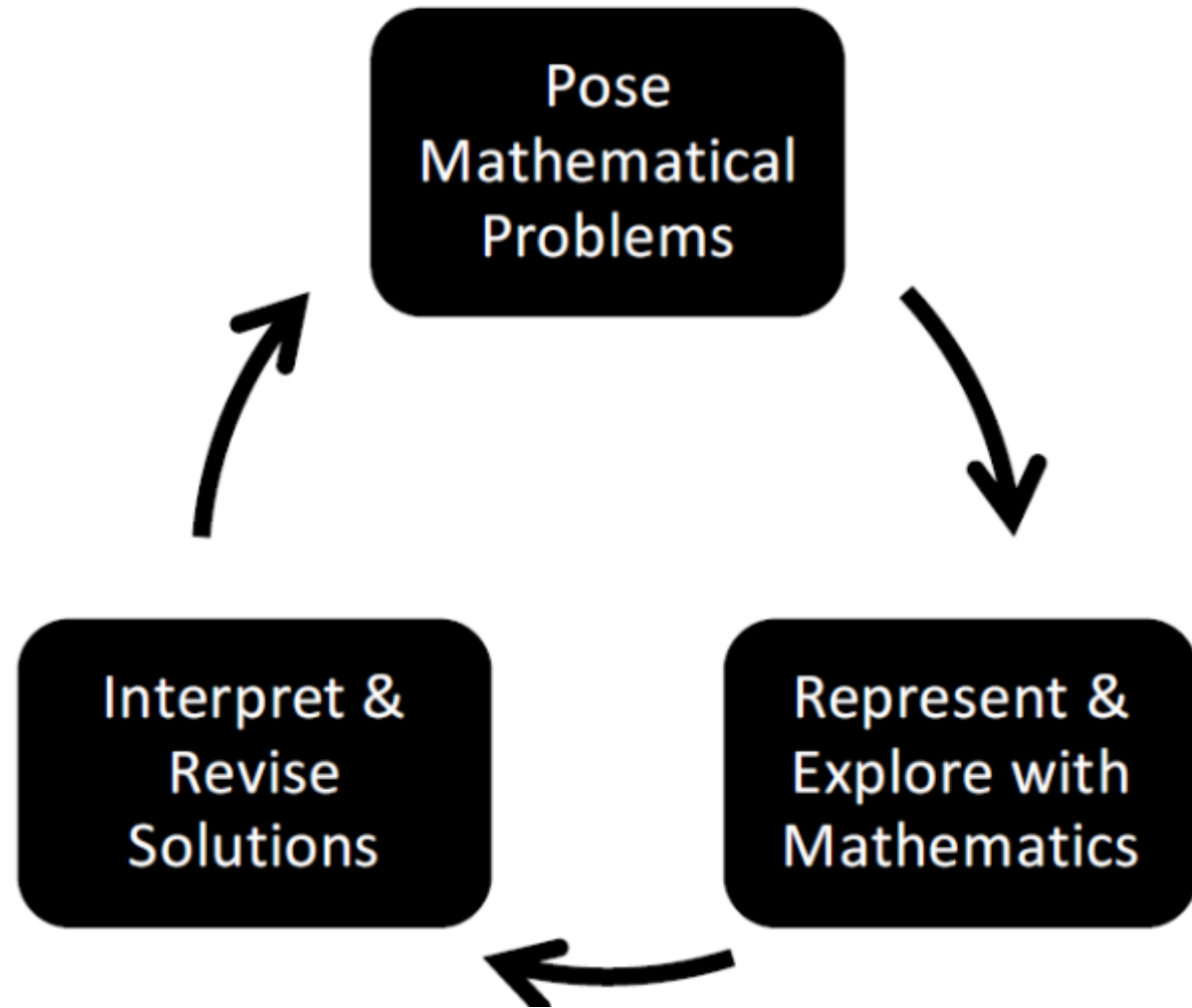


## The Process of Mathematical Modeling Includes

- Empathy in understanding situations
- Mathematics as a tool for reasoning and compassion

# Fostering Empathy through Modeling

Student  
modelers  
engage in  
three kinds of  
mathematical  
activity



# Wonder Care Ask Reflect

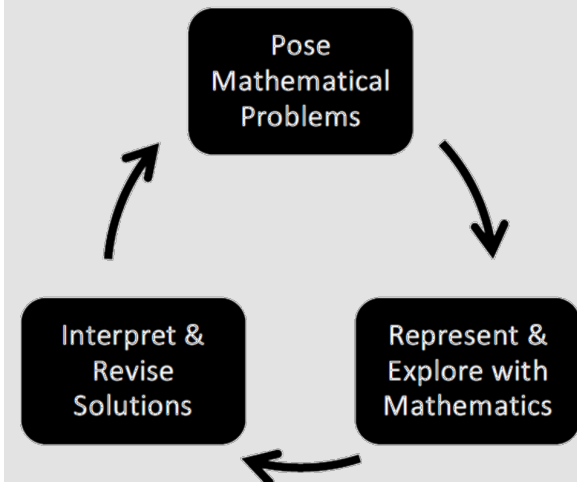
| Problem Posing   | Building Solutions  | Understanding Results  |
|--|---|--|
| <p>(1) <b>We Wonder</b> – <i>What makes this situation important?</i></p> <p>(2) <b>We Care</b> – <i>Who else might care about this situation? What might make it important to them?</i></p> <p>(3) <b>We Ask</b> – <i>What questions do you have about this situation? What questions could be answered using mathematics or statistics?</i></p> <p>(4) <b>We Reflect</b> – <i>What is the right question, given what is important to us and to others? Is there more than one?</i></p> | <p>(1) <b>We Wonder</b> – <i>How are others in the classroom community thinking about this problem?</i></p> <p>(2) <b>We Care</b> – <i>How can I understand their reasoning?</i></p> <p>(3) <b>We Ask</b> – <i>What made them approach the problem that way?</i></p> <p>(4) <b>We Reflect</b> – <i>How might others' approaches influence how I am thinking about this situation?</i></p> | <p>(1) <b>We Wonder</b> – <i>Have we answered the question we set out to ask?</i></p> <p>(2) <b>We Care</b> – <i>What difference will our solution make to the community? What other communities might care about our solution?</i></p> <p>(3) <b>We Ask</b> – <i>Is our solution fair? Is it just?</i></p> <p>(4) <b>We Reflect</b> – <i>How could we modify our solution? What new questions do we have?</i></p> |

What  
mathematics  
or statistics  
might be  
involved in  
investigating  
this situation?



# Problem Posing

## Problem Posing



## Problem Posing

Suggest one or two mathematical or statistical questions that exist in the “mathematical world” that students might pose to address the problem from their daily life, “Do we have enough pencils?”

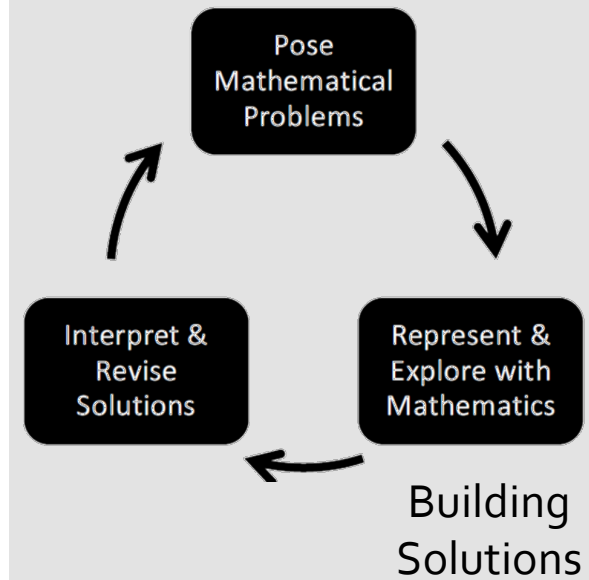
# Mathematics for Empathy: A Routine

## Problem Posing

1. We Wonder – *What makes this situation important?*
2. We Care – *Who else might care about this situation? What might make it important to them?*
3. We Ask – *What questions do you have about this situation? What questions could be answered using mathematics or statistics?*
4. We Reflect – *What is the right question, given what is important to us and to others? Is there more than one?*



# Building Solutions



## Building Solutions

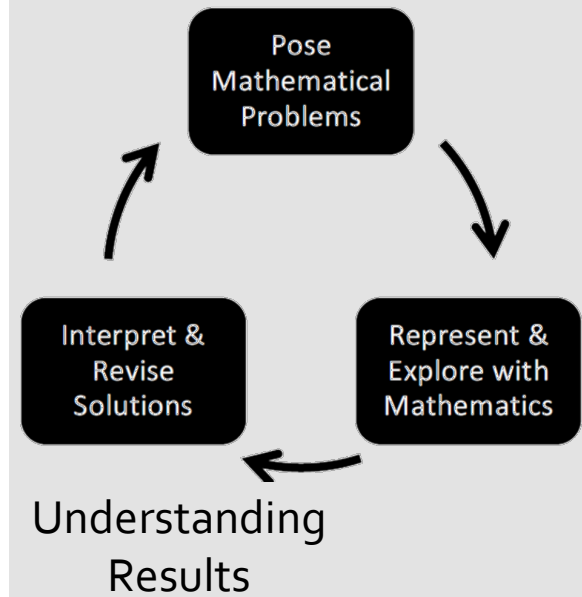
How might students go about answering your mathematical or statistical question? Try to come up with two or three different approaches.

# Mathematics for Empathy: A Routine

## Building Solutions

1. **We Wonder** – *How are others in the classroom community thinking about this problem?*
2. **We Care** – *How can I understand their reasoning?*
3. **We Ask** – *What made them approach the problem that way?*
4. **We Reflect** – *How might others' approaches influence how I am thinking about this situation?*

# Understanding Results



## Understanding Results

Imagine one or two of the solution strategies you identified was carried out.

# Mathematics for Empathy: A Routine

## Understanding Results

1. **We Wonder** – *Have we answered the question we set out to ask?*
2. **We Care** – *What difference will our solution make to the community? What other communities might care about our solution?*
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# Sample Student Solutions

Is 529 Pencils enough To Last Us The whole year?

By: Collic Brooke nit

4 weeks - 1 Month  
28 students  
529 pencils

week 1 week 2 week 3 week 4

1 Month

18 Pencils for each person

Each person has 10 Pencils

Each student has 8 pencils leftover

1 month 10 Month

The ☺ End.

Are gust is ☒ Yes we have enough

## What Makes a Situation a Potential Modeling Situation?

- Reflect a felt need or issue
- Open to multiple perspectives and approaches
- Authentic from the learner's perspective
- Make a difference to students and their community



# Teaching Practices that Foster Empathy

## Complex Questions Grow from Modeling

- What is fair?
- What is best?
- What might happen?

## Four Big Ideas in Modeling

1. Modeling begins and ends outside the mathematical world.
2. Modeling deals with situations that are open and complex.
3. Modelers exercise judgment when investigating problems. These judgments stem from a set of values.
4. Modelers decide when a solution is good enough.

## Access, Equity, and Empowerment

Modeling takes quantitative literacy a step further than developing critical consumers.

Students don't need to explicitly tackle a problem with a justice-oriented context to develop justice-oriented mathematical habits.

Through engagement in modeling, students become producers of mathematics, empowered to use their knowledge to enact change.

|                                   | Teachers are...  | Students are...  |
|-----------------------------------|--|--|
| <b>Pose Mathematical Problems</b> | <p>...sharing a problematic or interesting situation with students</p> <p>...eliciting students' ideas and questions about the situation</p> <p>...making public records of student questions</p> <p>...asking questions to support students in noticing the mathematics embedded in the authentic situation</p> | <p>...<b>brainstorming</b> and <b>sharing</b> ideas and questions about the context</p> <p>...<b>identifying</b> what questions can and cannot be answered using mathematics</p> <p>...<b>selecting</b> and refining mathematical questions</p> <p>...<b>posing</b> a mathematical problem that will guide their work on the modeling task</p> |

## **Represent and Explore Situations**

...anticipating what mathematical strategies students might use and what resources (e.g., calculators, graph paper, manipulatives) students might need to pursue those strategies

...asking students to explain and justify the representations and strategies they are using

...facilitating opportunities for students to share their developing solutions with the class

...providing scaffolding and support when students encounter new or challenging mathematics

...**deciding** what elements of the situation need to be represented in the mathematical problem and choosing how to represent those ideas

...**choosing** mathematical resources to support their work

...**documenting, explaining, and justifying** their mathematical work

...**listening** to their classmates' approaches

...**deciding** when and how to adjust or modify their mathematical strategies

## **Interpret and Revise Solutions**

...providing a structure or platform for students to share their solutions

...facilitating connections back to the authentic situation that motivated the modeling task

...asking questions about the scope of the solution (e.g., “Who else might our model help?” or “How could we change our solution to make it work for \_\_\_\_\_?”)

...**communicating** their solutions in a way that makes sense to themselves, to their classmates, and to other stakeholders

...**interpreting** their solution in light of the problem’s context

...**articulating** where their solution does and does not apply

...**deciding** whether or not the solution is “good enough” and revising it as needed

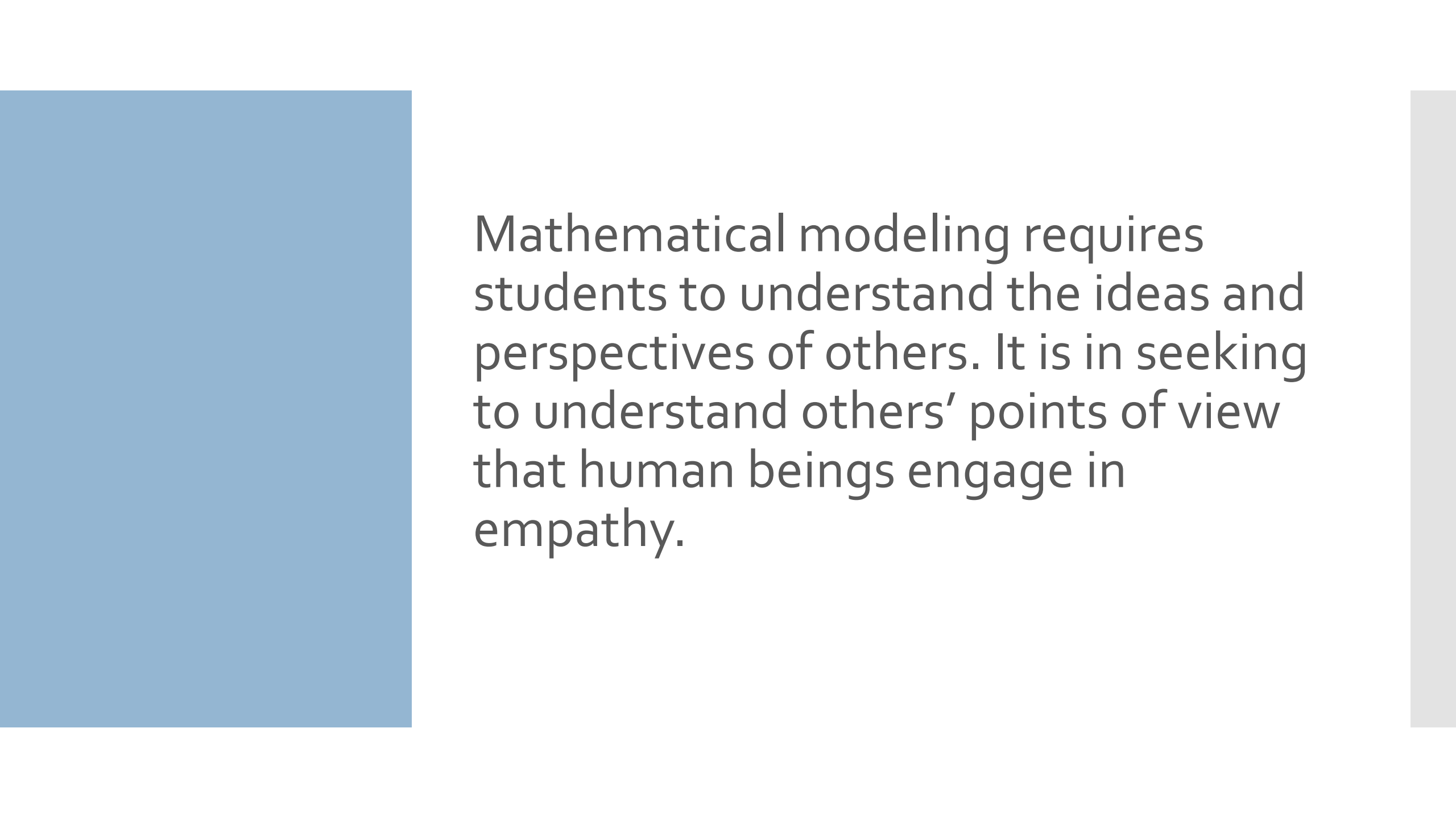
What's the role  
of teachers of  
mathematics?

The system of mathematics education and those of us in it bear a responsibility to ensure fair and equitable access to the knowledge, opportunity, and power that comes with the pursuit of mathematical ideas.




# Who has the authority to make mathematical decisions?


1. Who or what is the mathematical authority in your classroom?
2. How and where can you give students more authority to make mathematical decisions?
3. When might you use the “Wonder-Care-Ask-Reflect” routine in your classroom?



Mathematical modeling requires students to understand the ideas and perspectives of others. It is in seeking to understand others' points of view that human beings engage in empathy.

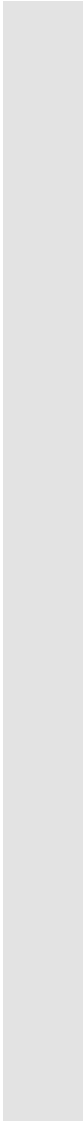


In the current era of political and social discord, many issues are framed as though they are dichotomous, with one right answer being fought over by two opposing sides.



Modeling provides a way for issues to be understood beyond right or wrong, and beyond even “agree to disagree.”

Modeling provides a way for students—human beings—to use their mathematical skills to examine different solutions to authentic problems based on different perspectives.



Modelers gain  
empathetic  
perspectives

A complex problem doesn't have a right or wrong answer; instead it has a problem statement and a proposed solution that capture something important about what another human being values.

# Individual Session Feedback

*We value your input. Please take a few minutes to provide TODOS with feedback on each of the sessions you attend. Answering will allow us to improve our conferences moving forward.*

*<https://bit.ly/3goyzgK>*

