Challenge the Dominant Mathematical Discourse by Privileging Multiple, Connected Representations

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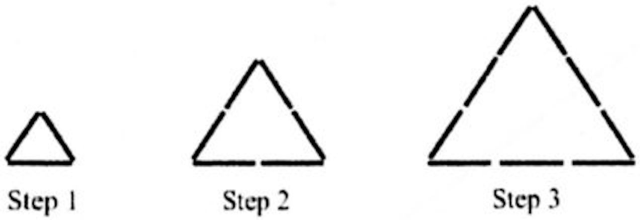
Two Tasks.

Express your answer in words, pictures, numbers, and any other representations you want to share. Add a picture of your work to the slides (see link below).



Use the pattern of toothpicks shown in the drawing above to figure out what the 10th drawing in the pattern looks like.

$2\frac{1}{4} - 1\frac{1}{2}$

Write a problem context that matches the expression above. Use other representations to illustrate your problem situation. Click on the images to access virtual manipulatives.

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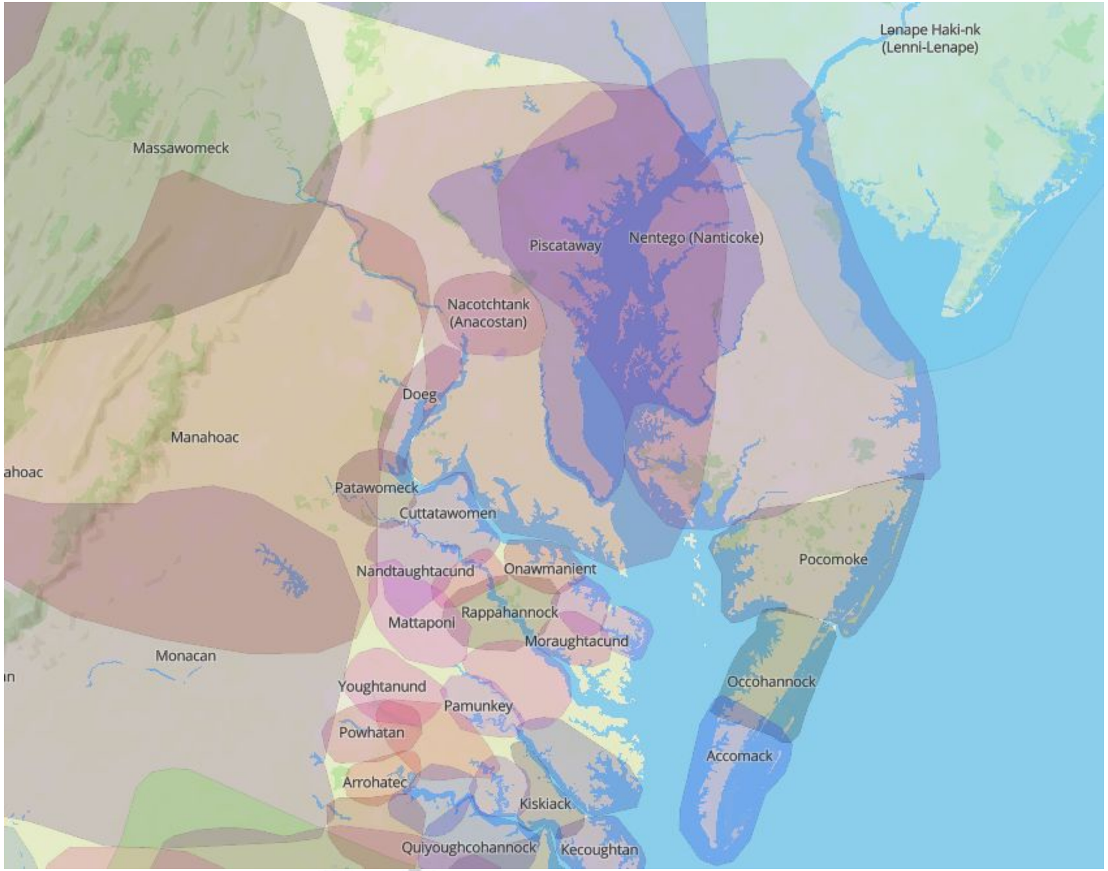
Communities often form around large rivers like the Potomac River. This was true before colonists crossed the ocean, as we see in this map.

While I currently "own" land where the Manahoac used to thrive, this region has also been claimed in the past by the Powhatan people, one of the eight Algonquian tribes that still have members thriving on tribal lands today.

I often think of these early residents as I travel the modern roads laid down on their ancient trails.



https://native-land.ca/

Attending an Online Session

- We will use another Slides document to engage with each other. Please share!
- Access Engagement slides and presentation slides at the TODOS conference webpage.
- When you see the Slides symbol there will be an opportunity to engage and share with other participants.
- Share on Twitter @kmorrowleong @TODOS2021

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Challenge the dominant mathematical discourse by privileging multiple, connected representations

Challenge the dominant discourse in mathematics by changing the perception of the privileged problem solution. Currently the symbolic representation is the end goal of much of mathematics instruction despite the fact that more information comes to us visually than ever before. The equitable approach is to privilege solutions that include a network of multiple, connected representations. Let's challenge ourselves to look differently at evidence of student thinking.

What is your role in mathematics education?

- School-based coach or teacher
- Central district leader
- Itinerant district leader
- Teacher educator
- Researcher
- Publishing

Use the **slides** to share.

Participate on Slides http://bit.ly/ConnectedRepresentationsTODOS2021

Session Goals

1. Describe what “representations” are.
2. Recognize a framework for connecting multiple representations.
3. Describe a translation and a transformation between representations.
4. Translate between representations
5. Examine student work samples for evidence of translation
6. Connect to practice.

Representations

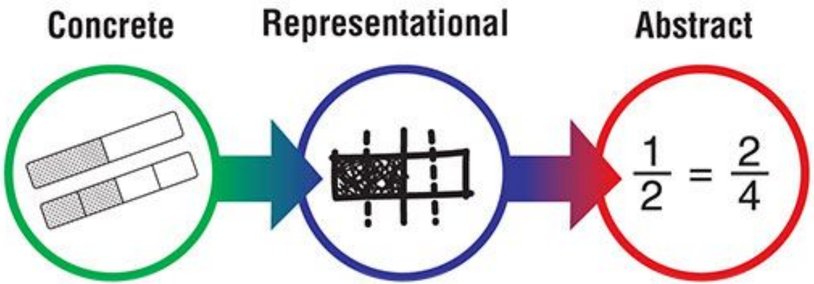
CRA (CPA): Concrete, Representational (Pictorial), Abstract

Bruner's original theory of learning included:

Enactive (Concrete)- referring to actions

Iconic (Pictorial)- referring to images that stand for something

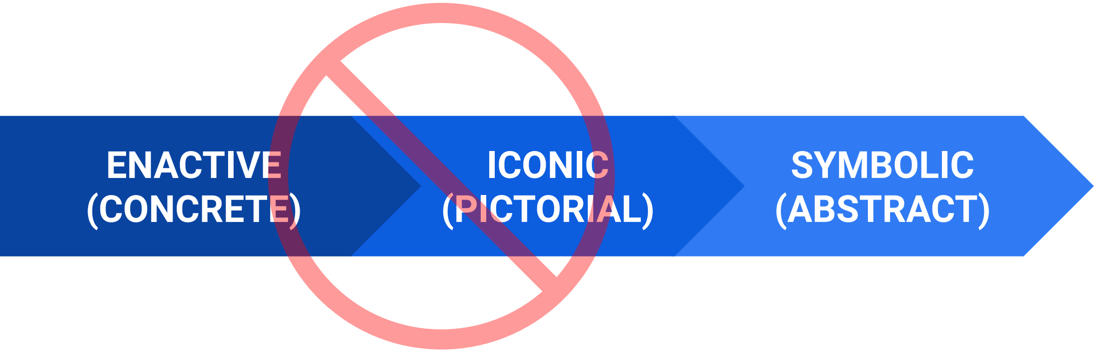
Symbolic (Abstract)- an orderly system of symbols

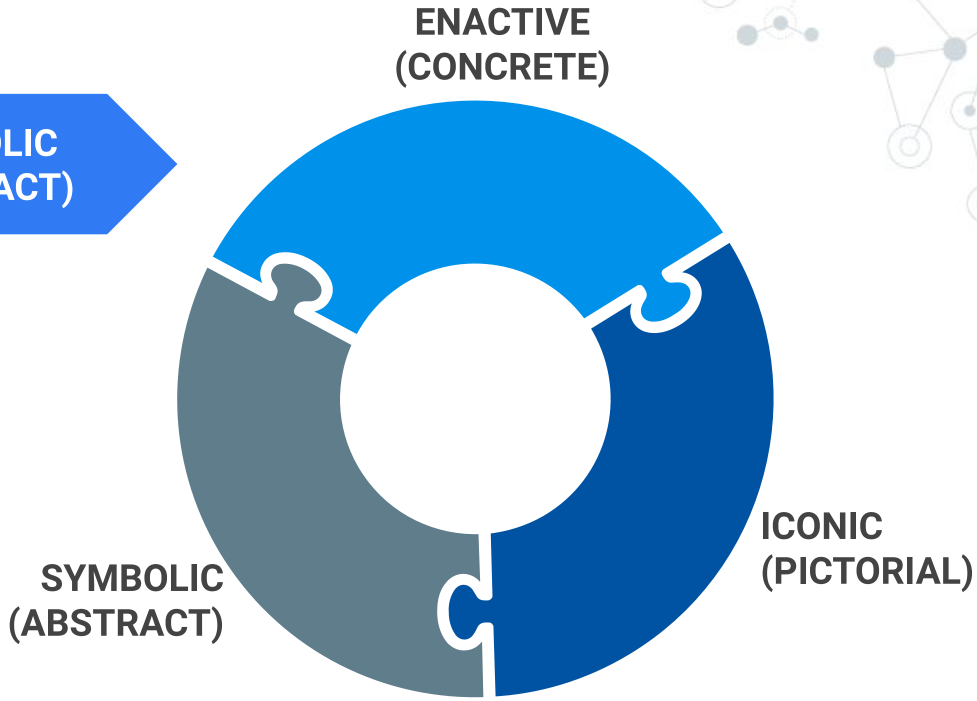


https://makingeducationfun.wordpress.com/2012/04/29/concrete-representational-abstract-cra/

For a good review of Bruner (1966), see (Leong, Ho, & Cheng, 2015).

Understanding Bruner's Conceptions of CRA

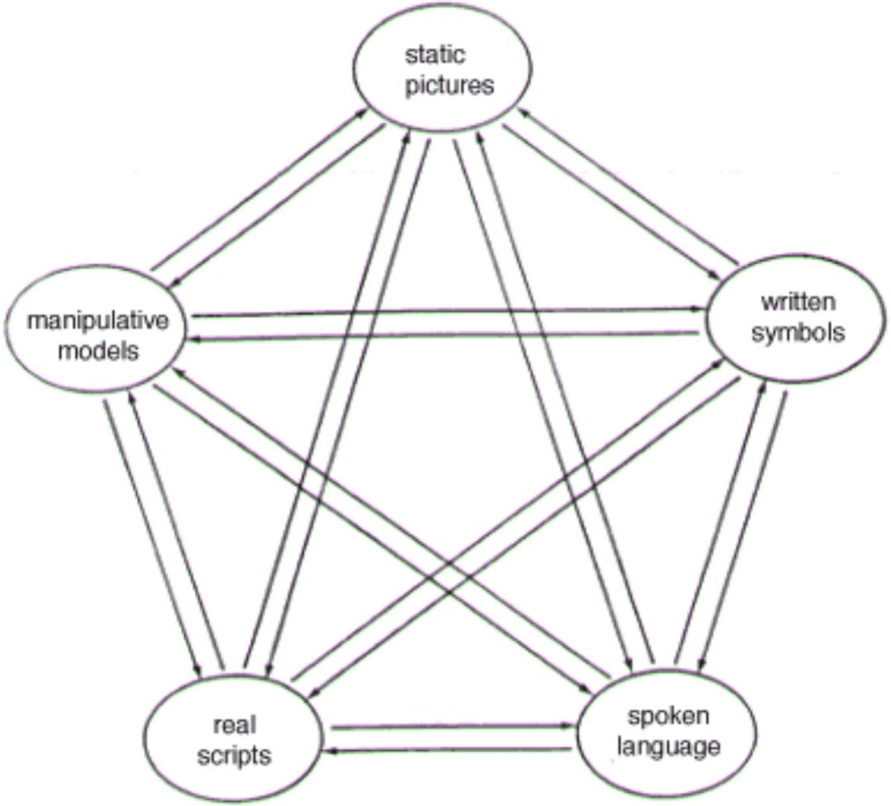




One misconception about the CRA interpretation of Bruner's theory is that it is a linear progression. It is NOT. The symbolic was meant to develop *alongside* the enactive and iconic representations.

Cyclical or *web-like* may better describe Bruner's model.

“Five Distinct Types of Representation Systems”



Real Scripts:

“...experience-based ‘scripts’- in which knowledge is organized around ‘real world’ events that serve as general contexts for interpreting and solving other kinds of problem situations (p. 33).

Written Symbols

“...like spoken languages, can involve specialized sentences and phrases

$x + 3 = 7$

A'UB' = (AnB)'

as well as normal English sentences and phrases.”

(Lesh, Post, & Behr, 1987)

Does the Use of Multiple Representations Empower Students to Learn?

- Meta-analysis of manipulative use: low to moderate effect size growth
- Concrete manipulative use in older elementary students (concrete operational, ages 7-11) show medium to large effect size
- Younger students showed ***less*** impact.
- "Scripted manipulations" of manipulatives are more likely to help students connect concrete with other representations.

(Carbonneau, et al., 2013)

Does the Use of Multiple Representations Empower Students to Learn?

- Multimodal environments helped students internalize new material
- Students taught using multiple modalities created mental models of fractions and used them to complete tasks.
- Students in multimodal environments demonstrated more pathways for finding solutions within a context.

(Chahine, 2013)

Is This True for ALL Students?

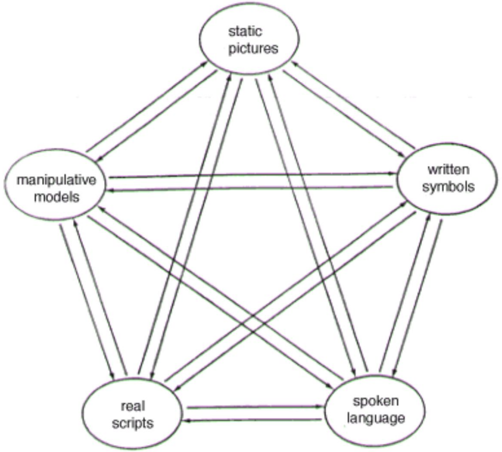
One study of mostly Latin@ students showed that the use of manipulatives made a significant difference.

- Two classrooms on the central coast of CA
- One classroom used manipulatives. The control class did not.
- Eureka curriculum, grade 4 (assessed using pre- and post-test)
- The study design has validity concerns related to consistent implementation, so it should be taken as one data point.

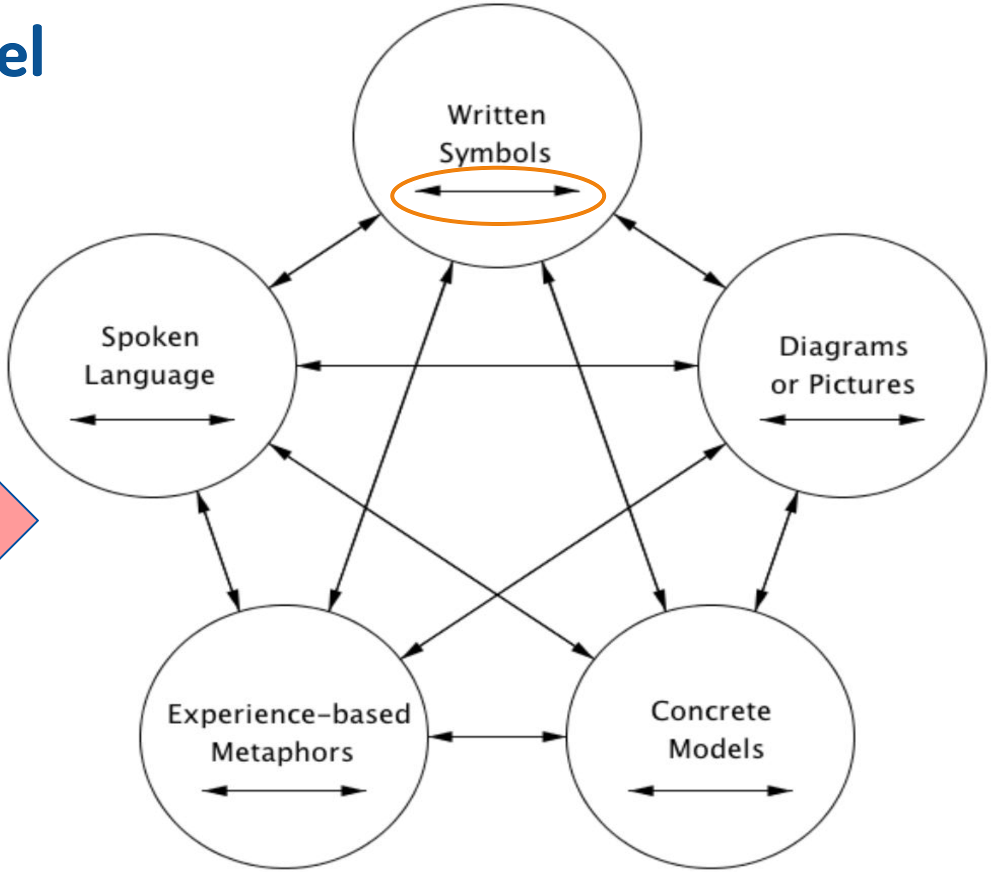
(Zandakis, 2019)

Connected Representations

Updated Translation Model

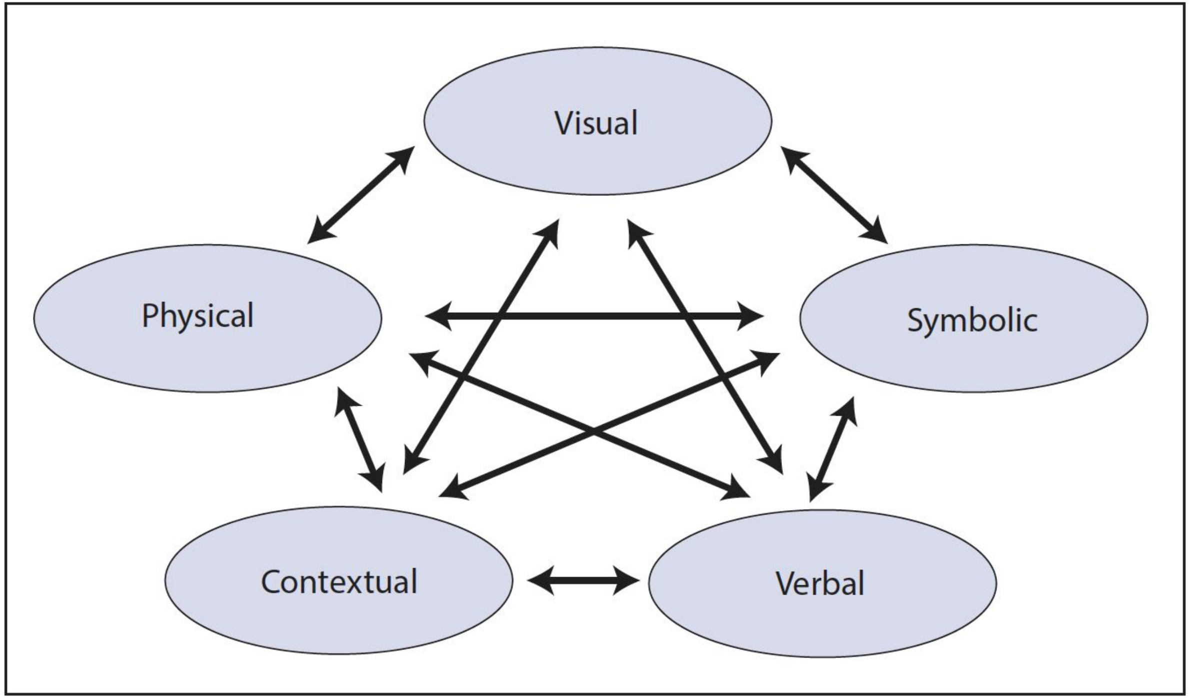


(Lesh, Post, & Behr, 1987)



(Lesh & Doerr, 2003)

Translation Model



(Principles to Actions, 2014)

Translations and Transformations

Translations

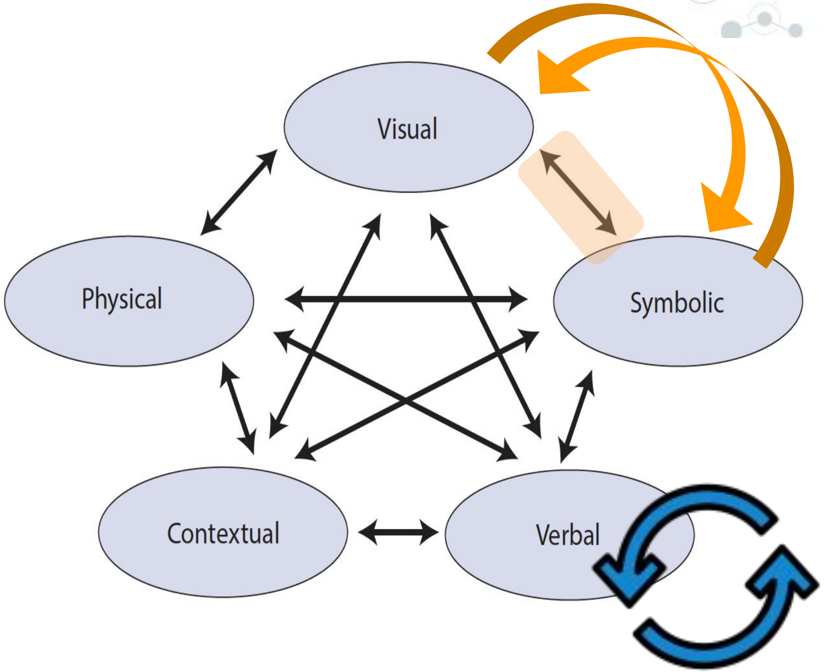
"between-system" mappings

Connecting one mode to another.

Transformations

"within-system" operations

Connecting representations within one mode

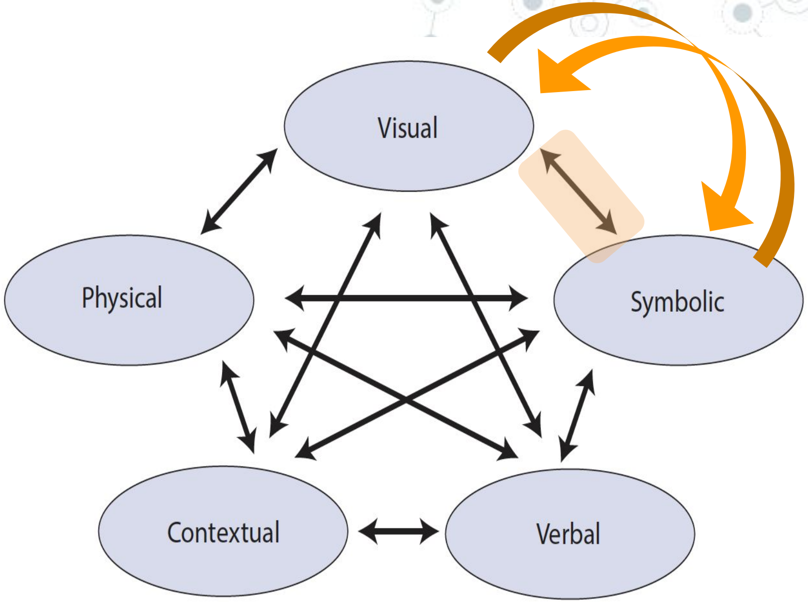


Translations

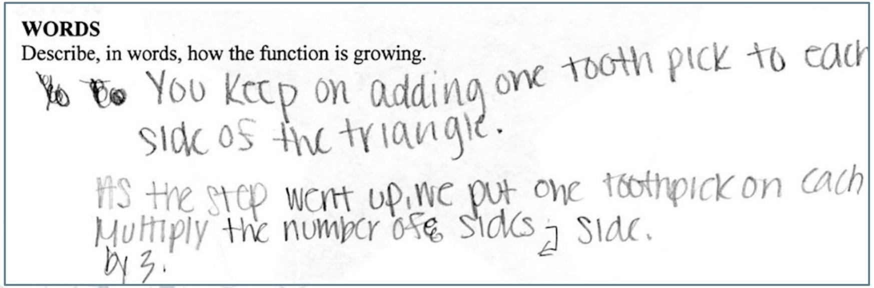
Translations

"between-system" mappings

Connecting one mode to another.



Symbolic


WORDS

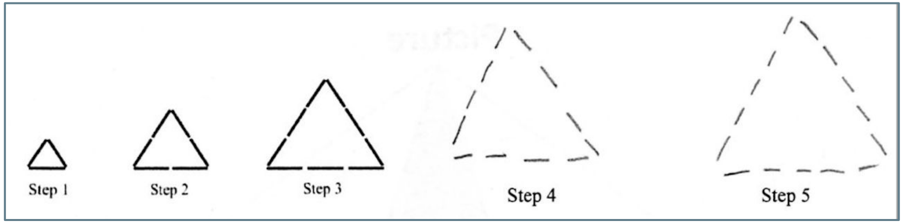
Describe, in words, how the function is growing.

You keep on adding one tooth pick to each side of the triangle.

As the step went up, we put one toothpick on each side.

Multiply the number of sides by 3.

Visual

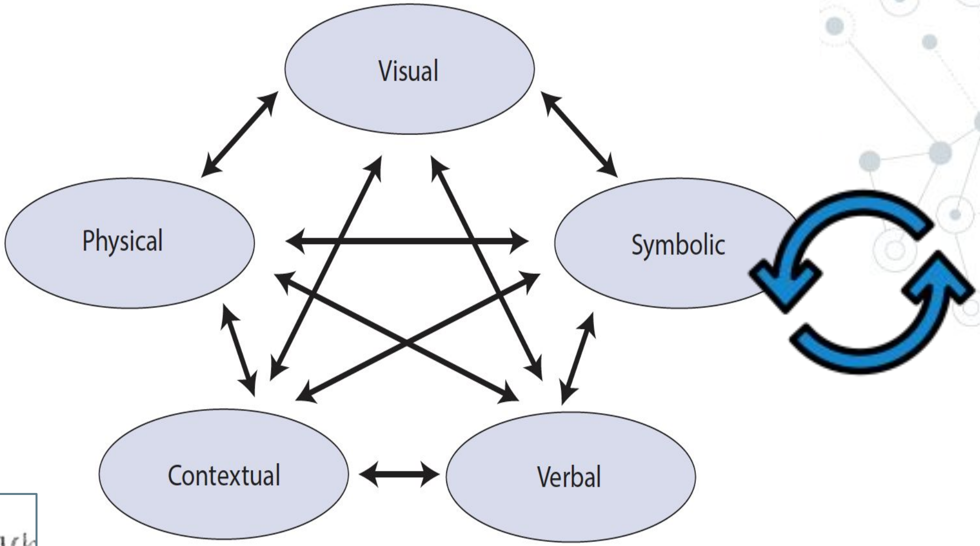


Transformations

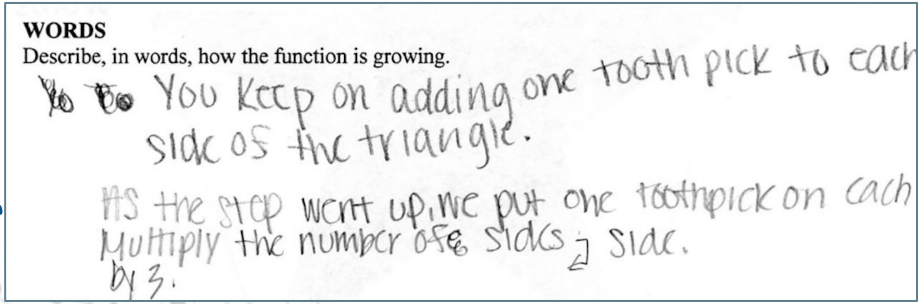
Transformations

"within-system" operations

Connecting representations within one mode



Symbolic



WORDS
Describe, in words, how the function is growing.

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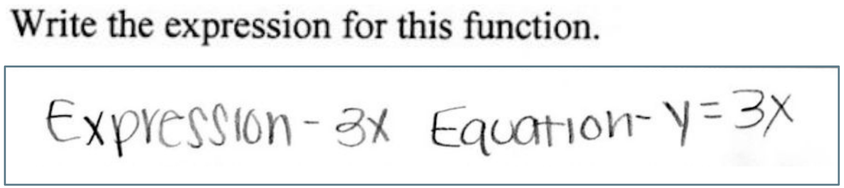
As the step went up, we put one toothpick on each side.
Multiply the number of sides by 3.

*Sometimes written language can be considered "verbal," but the fact that it is expressed in symbols (letters) also puts it in this category.

Symbolic



Write the expression for this function.

Expression - 3x Equation - y = 3x

What does it mean to "Understand"?

The student...

1. recognizes the idea embedded in a variety of qualitatively different representational systems,
2. accurately translates the idea ***between*** one system and another.
3. flexibly manipulates the idea ***within*** given representational systems

(Lesh, Post, & Behr, 1987)

Why is this important?

Let's Look at Some Student Work!

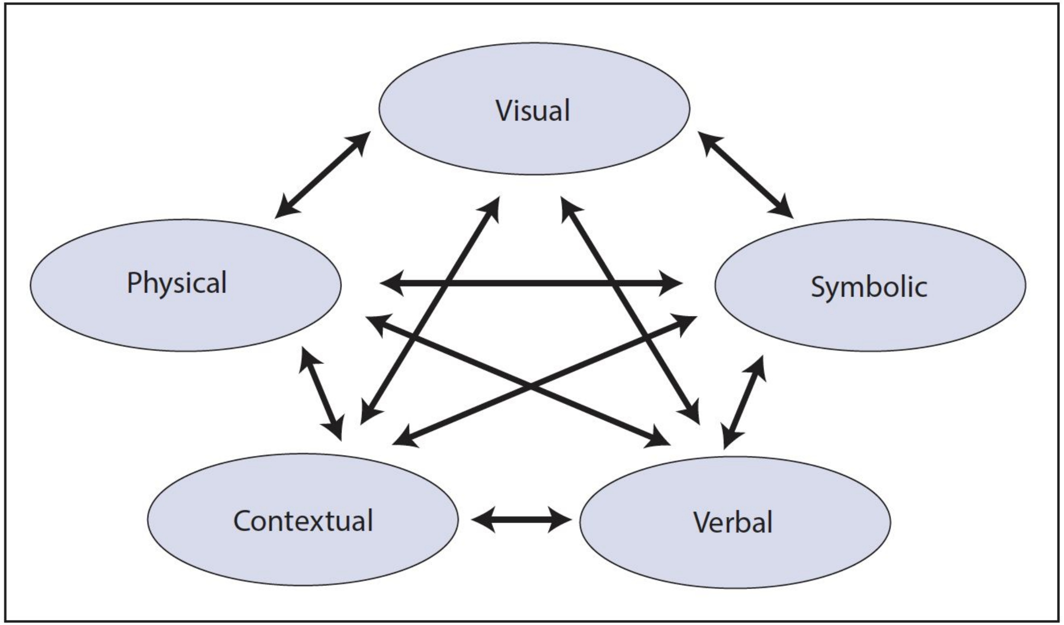
$2\frac{1}{4} - 1\frac{1}{2}$

"Show how you find the difference using words, pictures, and numbers."

1. Note what representation(s) each student used.
2. What mode of representation did they show?
3. What translations do you see between their representations?
4. What transformations do you see between their representations?
5. Other thoughts?

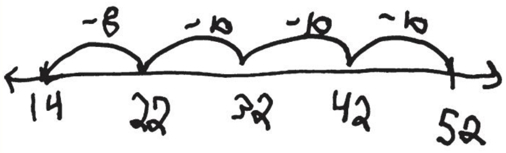
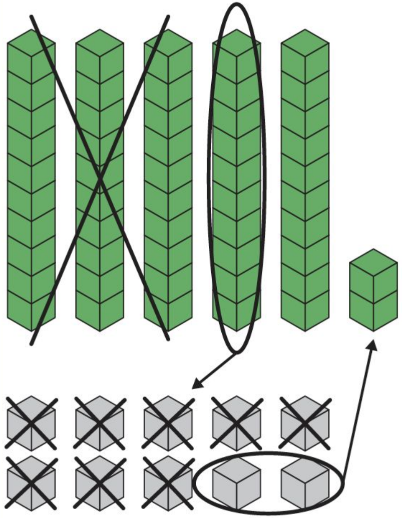
Active Translations and Transformations

- The arrows go in both directions.
- Each node is connected to every other node.
- Within each node are many different representations.



(Principles to Actions, 2014)

FIGURE 2.17 FIVE WAYS TO REPRESENT THE VALENTINES PROBLEM

REPRESENTATION	EXAMPLE
Contextual	Shruti bought 52 Valentines for her classmates. There are 38 kids in her class and she gave one to each. She wondered how many she would have left.
Pictorial	
Concrete	 **Video 2.3** Valentine's Cards With Base 10 Blocks resources.corwin.com/problemsolvingk-2

Look at the five different representations of the same problem situation.

Pick two and write a translation from one to the other, making connections between the representations.

(Morrow-Leong, Moore, Gojak, 2021)

FIGURE 2.17 FIVE WAYS TO REPRESENT THE VALENTINES PROBLEM

REPRESENTATION	EXAMPLE
Contextual	Shruti bought 52 Valentines for her classmates. There are 38 kids in her class and she gave one to each. She wondered how many she would have left.
Verbal	Shruti had 52 and she gave away 38. She gave away 10 at a time I think.
Symbolic (Equation)	52 −38 52 40 12 −30 −8 10 4 14

Look at the five different representations of the same problem situation.

Pick two and write a translation from one to the other, making connections between the representations.

(Morrow-Leong, Moore, Gojak, 2021)

Try it!

From:

To:

	Visual	Symbolic	Verbal	Contextual	Physical
Visual					
Symbolic	*SAMPLE* The drawing shows three toothpicks added for each stage. The student describes how the growth takes place.	*SAMPLE* The equation for adding one toothpick to each side is $y = 3x$. The 3 is how many toothpicks you add.			
Verbal					
Contextual					
Physical					

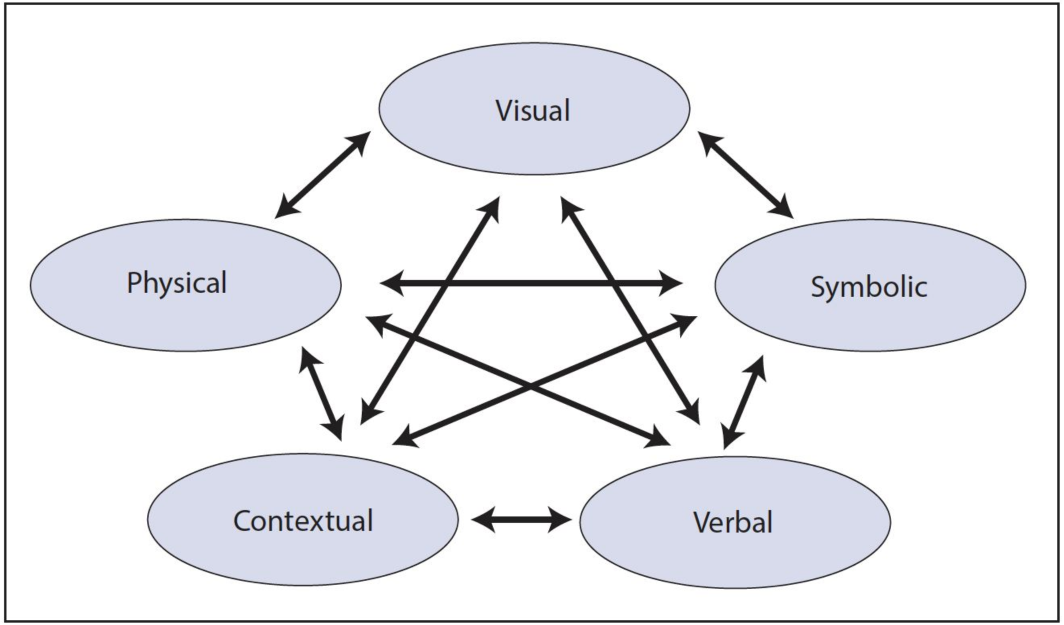
Active Translations and Transformations

Questions remain:

Which translations/transformations are more impactful?

Which translations/transformations are most challenging?

How robust does the "web" have to be?



(Principles to Actions, 2014)

How can use this information to challenge the dominant mathematical discourse ?

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